

論文 Comparison on Two Optimal Shape Design Methods-Sensitivity Analysis and Simulated Biological Growth Method

Kai-Lin HSU*¹ and Taketo UOMOTO*²

ABSTRACT : In the field of shape optimization, one of the interesting fields is focused on how to achieve the uniform stress distribution on the surface of design structures. The structures satisfying this design criterion can be observed in many biological tissues, such as trees or bones. In this paper, the authors attempt to conduct a comparison between a new optimal shape design method called "simulated biological growth method" and a structural optimization approach using sensitivity analysis also developed by the authors. Based on the same design criterion, the efficiency and difference of the two methods are illustrated by couples of examples.

KEYWORDS : optimal shape design, simulated biological growth method, stress concentration, optimality criterion method.

1. INTRODUCTION

Generally speaking, the approaches for structural optimization can be categorized into mathematical programming and optimality criterion method; the former one is related to mathematical techniques without considering the physical essence of the design problems and the latter one is intuitively devised by designers based on their observation on the physical characteristic of problems. As it is known, the methods by mathematical programming are developed on rigorous theoretical background but possibly inefficient for finding out optimal solution while the methods by optimality criterion method commonly lack the sound theoretical support with the potential advantage of higher efficiency on search for optimal solution. One well-known method of the optimality criterion methods is full-stressed design method. In addition, sensitivity analysis is essential for structural optimization for evaluating and updating the design variables by gradients of structural response with respect to design variables. However, the time-consuming process for obtaining the sensitivity coefficients seriously deteriorates the efficiency of the optimization process. In order to avoid the problem, gradientless optimization is considered to be one of the solutions. Based on the results of those researches on gradientless optimization, it indeed offered the designers some effective and convenient alternatives for structural optimization.

Among the proposed methods for gradientless optimization, one new proposal was highly noticed. According to the observation of Mattheck and Burkhardt [1] on natural biological tissues, e.g. trees or bones, one optimal shape design method called "simulated biological growth method (or adaptive growth method)" was presented by swelling the overloaded parts or shrinking the underloaded parts of the design structure. And similar concept of this method was also suggested in the work of several other research groups, such as Hsu and Uomoto [2] and Suh, Anderson and McDonald [3]. In essence, the idea of simulated biological growth method is also derived from the concept of optimality criterion method for considering the reduction of stress concentration. As mentioned in [1], the stresses along the design profile, e.g. notches or sharp edges, can be effectively reduced by computer simulation of tree growth. Meanwhile, for very similar optimization problems, some other researchers could also find out the optimal solutions by their proposed methods. By observing the results acquired by these different methods, it seems the results solved by the new proposed method (simulated biological growth method) show better efficiency but no direct comparison verified this observation. However, as it is recognized, there exists in this field no universally accepted solution method. As a result, in this paper, the authors would investigate the efficiency of this simulated biological growth method by comparing with a conventional optimization method also developed by the authors, both of which are based on the same optimality criterion for reducing the stress concentration.

*¹ Graduate Student, University of Tokyo, Member of JCI

*² Professor, Institute of Industrial Science, University of Tokyo, Member of JCI

In the following context, section 2 is given for defining the formulation of the objective function. And the algorithm of simulated biological growth method is briefly explained in section 3. For the interested readers, please refer to [1,2] for detailed description of this method. As for the sensitivity-analysis optimization method developed by the authors, the features of sensitivity analysis and the optimization procedure are respectively given in section 4 and 5. Finally, for the purpose of comparison, one analytical example for the case of a square plate under biaxial isotension load with central diamond-shape notch is illustrated in section 6. Through the comparison, the efficiency of simulated biological growth method can be confirmed due to its excellent performance on saving computational cost.

2. FORMULATION OF THE PROBLEM

As indicated in [1], the biological structures self-optimize their shapes by growth with respect to the natural load applied. In this case, the word "optimum" means that within all the design structures, a state of constant stress at the surface of the biological components should be observed for all the natural loading case applied. By considering this effect into the effort of optimization process, the optimization problem in this research can be defined as the problem to find out the state of constant stress distribution along the design profiles; that is, the optimality criterion considered here is to change the shape of the design profiles for reducing the stress concentration. This objective can be alternatively achieved by minimizing ratio of maximum stress to the object stress along the design profile, which is expressed in eq.1, where Γ_K : the design profile, s : the design point along the design profile, $\sigma_{eq}(s)$: equivalent stress of each design point and σ_{obj} : object stress, that is, average equivalent stress at each iterative step. By the same definition of objective function for different optimal methods, the effect of optimization for each method can be clearly clarified.

$$\text{Min } \Omega = \left\{ I \mid I = \frac{\text{Max}(\sigma_{eq}(s))}{\sigma_{obj}}, s \in \Gamma_K \right\} \quad (1)$$

3. SIMULATED BIOLOGICAL GROWTH METHOD (GRADIENTLESS METHOD)

The design idea of the simulated biological growth method was originated from the observation on biological structures which can adapt themselves to external loads for reducing stress peak with growth or atrophy. The basic procedure for an simulated biological growth approach is composed of two stages: the first one is FEM static analysis for obtaining the stress distribution over the design domain, which can be one part or the whole domain of the design structure according to the need of the designer. Then, in order to execute the second stage, the growth law using fictitious strain, which was proposed quite divergently by different research groups (referred to [1, 2]), is introduced to produce the fictitious equivalent nodal loads. Then, an incremental growth analysis based on this growth law can be carried out to generate the incremental displacements for updating the design structure. After the design structure is updated, the above-mentioned process is repeated again until the convergence of eq.1 can be recognized. The growth law, the governing equation of the incremental growth analysis and the generated fictitious equivalent nodal loads are given in eqs.2, 3 and 4 respectively. Here, the definition for $\langle \Delta g \rangle$ follows the one given in [2]. However, as indicated in [1], it also can be defined as thermal loading for executing the second stage as thermal analysis.

$$\Delta \epsilon_{kl}^B = \left\{ \frac{\sigma_{obj} - \sigma_{bas}}{\sigma_{bas}} \right\} \delta_{kl} \quad (2)$$

where $\Delta \epsilon_{kl}^B$: fictitious strain, σ_{bas} : reference equivalent stress and δ_{kl} : Kronecker delta

$$[K] \langle \Delta u \rangle = \langle \Delta g \rangle \quad (3)$$

where K : stiffness matrix, Δu : nodal transformational vector

$$\langle \Delta g \rangle = \int_{\Omega} [B(x)]^T [D] \langle \Delta \epsilon_{kl}^B \rangle d\Omega \quad (4)$$

where $[B(x)]^T$: transpose of strain-displacement matrix, $[D]$: elastic modulus matrix

4. SENSITIVITY ANALYSIS

The simulated biological growth method is derived in order to avoid the process of sensitivity analysis. However, for most of the existing optimal shape design method, the acquirement of sensitivity coefficients is inevitable. For the evaluation on the efficiency of the simulated biological growth method, one optimal shape design method using sensitivity analysis was also developed by the authors, the details of which is explained in the next section. As mentioned earlier, the objective of the optimization problem is to reduce the stress concentration. As a result, the sensitivity coefficients of stresses with respect to the design variable is essential. The sensitivity analysis in this paper adopted the semi-analytical method by a finite difference method [4]. The algorithm of the sensitivity analysis is briefly described here. By the modeling of FEM analysis, the governing equation in terms of displacement, can be expressed in eq.5, where $[K]$ is the stiffness matrix, $\{u\}$ is the nodal displacement and $\{R\}$ is the nodal force vector.

$$[K]\{u\} = \{R\} \quad (5)$$

In general, both $[K]$ and $\{R\}$ are dependent on the design variables. Hence, the derivatives of eq.5 with respect to any design variable a_m will be

$$[K]\left\{\frac{\partial u}{\partial a_m}\right\} + \left[\frac{\partial K}{\partial a_m}\right]\{u\} = \left\{\frac{\partial R}{\partial a_m}\right\} \quad (6)$$

by solving eq.6, the derivatives of $\{u\}$ with respect to design variables a_m can be obtained. Then, the derivatives of stresses with respect to design variables a_m can be procured as follows:

$$\left[\frac{\partial \sigma}{\partial a_m}\right]_e = [D] \left([B]|J| \left\{\frac{\partial u}{\partial a_m}\right\}_e + \frac{\partial [B]}{\partial a_m} |J| \{u\}_e \right) \quad (7)$$

where $[D]$: elastic modulus matrix, $|J|$: determinant of Jacobian matrix $[J]$,
and $[B]$: strain-displacement matrix.

5. OPTIMALITY CRITERION APPROACH (GRADIENT METHOD)

As stated in section 3, in order to evaluate the gradientless optimal shape design method derived from the concept of biological growth, one optimal shape design method using sensitivity analysis was also proposed by the authors. For brevity of the context, only the features of this method are explained in the following. This method also belongs to the optimality criterion approach. In order to reduce the stress concentration of the design structure, the nodes on the design profile are selected as the evaluation points used for the calculation on the objective function as shown in eq.1. The stresses for each evaluation point can be represented by the equivalent stress (here, the von Mises stress is used); that is, the equivalent stress is the functional of the stress tensor and the stress tensor is the function of nodal coordinates in the case of static analysis. These relationships can be expressed in eq.8 in the case of two-dimensional elasticity. By averaging the object stress on each evaluation point, average stress σ_{avg} can be obtained; that is, the average stress is the functional of equivalent stress on each evaluation point.

$$\sigma_{eq} = \frac{\sqrt{2}}{2} \left(\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\sigma_{xy}^2 \right)^{1/2} = f(\sigma_x, \sigma_y, \sigma_{xy}) = f(\sigma_x(x,y), \sigma_y(x,y), \sigma_{xy}(x,y)) \quad (8)$$

By Taylor's first-order expansion, the average stress can be approximated as follows:

$$\sigma^* = \sigma(\sigma_x, \sigma_y, \sigma_{xy}) + \frac{\partial f}{\partial \sigma_x} \delta \sigma_x + \frac{\partial f}{\partial \sigma_y} \delta \sigma_y + \frac{\partial f}{\partial \sigma_{xy}} \delta \sigma_{xy} + \Xi(\epsilon) \quad (9)$$

and we differentiate the components of stress tensor with respect to the design variables (i.e., X and Y), as follows:

$$\delta\sigma_x = \frac{\partial\sigma_x}{\partial x}\delta x + \frac{\partial\sigma_x}{\partial y}\delta y \quad (10)$$

$$\delta\sigma_y = \frac{\partial\sigma_y}{\partial x}\delta x + \frac{\partial\sigma_y}{\partial y}\delta y \quad (11)$$

$$\delta\sigma_{xy} = \frac{\partial\sigma_{xy}}{\partial x}\delta x + \frac{\partial\sigma_{xy}}{\partial y}\delta y \quad (12)$$

where $X = x_1, x_2, \dots, x_N$; $Y = y_1, y_2, \dots, y_N$ and N is the number of design points, By substituting eqs.10 - 12 into eq.9, eq.9 can be rearranged as

$$\begin{aligned} \sigma^* &= \sigma(\sigma_x, \sigma_y, \sigma_{xy}) \\ &+ \underbrace{\left(\frac{\partial f}{\partial \sigma_x} \frac{\partial \sigma_x}{\partial x} + \frac{\partial f}{\partial \sigma_y} \frac{\partial \sigma_y}{\partial x} + \frac{\partial f}{\partial \sigma_{xy}} \frac{\partial \sigma_{xy}}{\partial x} \right)}_{\alpha_{ij}} \delta x + \underbrace{\left(\frac{\partial f}{\partial \sigma_x} \frac{\partial \sigma_x}{\partial y} + \frac{\partial f}{\partial \sigma_y} \frac{\partial \sigma_y}{\partial y} + \frac{\partial f}{\partial \sigma_{xy}} \frac{\partial \sigma_{xy}}{\partial y} \right)}_{\beta_{ij}} \delta y + \Xi(\varepsilon) \end{aligned} \quad (13)$$

where $\Xi(\varepsilon)$ is the error functional and i, j is the node number of design point, $i, j = 1..N$. It can be expressed in the form of matrix in eq.14 by rearranging eq.13. As you can observe in eq.14, the right side of equation can be approximated by multiplying one constant (ϕ) for diminishing the effect of error functional.

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} & \beta_{11} & \dots & \beta_{1n} \\ \alpha_{21} & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \alpha_{n1} & \dots & \dots & \alpha_{nn} & \dots & \dots & \beta_{nn} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \dots \\ \delta x_n \\ \delta y_1 \\ \delta y_2 \\ \dots \\ \delta y_n \end{bmatrix} = \sigma^* - \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \dots \\ \sigma_n \end{bmatrix} - \Xi(\varepsilon) = \phi \left[\sigma^* - \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \dots \\ \sigma_n \end{bmatrix} \right] \quad (14)$$

As shown in eq.14, the order of coefficient matrix $[\alpha_{ij}, \beta_{ij}]$ is $N * 2N$. As a result, the solution of (dX, dY) cannot be solved by only N equations. Hence, some other information related to (dX, dY) needs to be introduced into eq.14. For solving that, the updating vector for each design point is assumed to be in the direction of the bisector of the angle defined by the design point with its neighboring points, as presented in Fig.1 (Here, the angle is defined by $j - i - i^*$ and the direction of updating vector is along the direction of $i - i'$). In order to shift the node i to i' , one ghost point i^* is introduced for keeping the same Euclidean distance for $i - j$ and $i - i^*$. The location of i^* can be expressed by eqs.15 - 17.

$$x_{i^*} = x_i + \gamma(x_{i-1} - x_i) \quad (15)$$

$$y_{i^*} = y_i + \gamma(y_{i-1} - y_i) \quad (16)$$

with

$$\gamma = \sqrt{\frac{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}{(x_{i-1} - x_i)^2 + (y_{i-1} - y_i)^2}} \quad (17)$$

Therefore, by these predicated geometrical conditions on updating vectors, the relationship for (dX, dY) can be established as

$$\delta Y / \delta X = \frac{x_{i+1} - x_{i^*}}{y_{i^*} - y_{i+1}} \quad (18)$$

by substituting eq.18 into eq.14 and rearranging eq.14, the order of coefficient matrix $[\alpha_{ij}, \beta_{ij}]$ becomes $N * N$; that is, after obtaining the derivatives of $\{u\}$ with respect to design variables a_m

by solving eq.14 and using the relationship given in eq.18, the solution for the updating vector can be obtained. With these updating vectors, the shape of the design structure will be changed by adding these updating vectors to the coordinates of design points. This process will be repeated until the convergence of eq.1 can be verified. However, due to the possible existence of unsmoothed design profile after update, some spline function will be used to smooth the design profile at each iterative step.

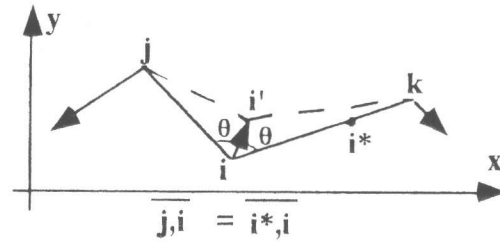


Fig.1 Local Shifting of Node i to i'

6. ILLUSTRATION OF THE COMPARISON ON THE TWO APPROACHES

As the purpose of research mentioned previously, one example for comparison is illustrated here. The example is a square plate under biaxial equi-tension with central diamond-shape notch, the FEM model of which is simplified in Fig.2 due to its symmetry. The points on the notch (i.e., design profile) were chosen as design points. Fig.3 indicates the optimized shapes obtained from the two methods with few difference. The convergence of objective function is shown in Fig.4 with good acceptance for both methods. Due to the excessive distortion of elements during the optimization process by sensitivity analysis method (SA), the smooth process by use of spline function led to the abrupt change of the continuity, represented by ellipsics in Fig.4. Fig.5 gives the variation of structure area, in which the structure optimized by SA has similar area to the one optimized by simulated biological growth method (SBG) for the deviation of area is only 0.288%.

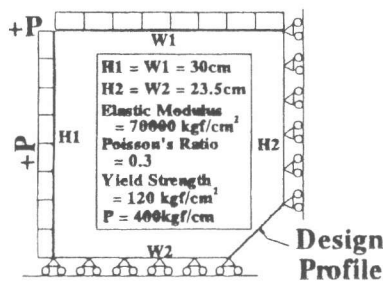


Fig.2 FEM Model of Design Structure

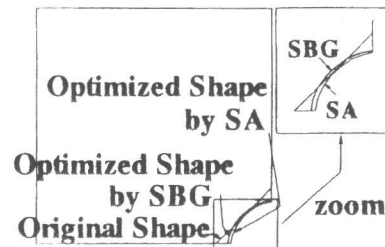


Fig.3 Comparison on Shape of Design Structure

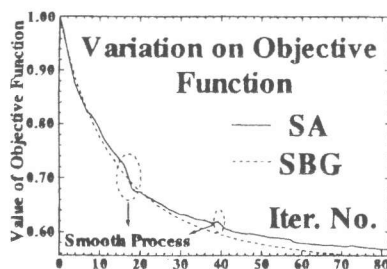


Fig.4 Comparison on Convergence of Objective Function

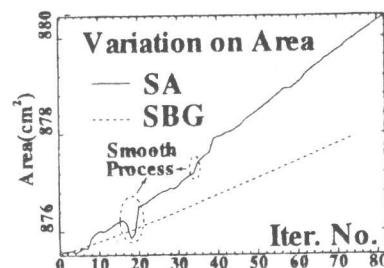


Fig.5 Comparison on Area of Design Structure

The strain energy within the design structure can be effectively reduced as shown in Fig.6, which indicates the optimized structure can undergo higher external loading. As emphasized in section 1, the effect of reducing stress concentration by the two methods can be clearly observed in Fig.7 (Here EQS is equivalent stress in the form of Von Mises stress). The stresses along the design profile after optimization can effectively tend to be uniform, which meets the requirement of the design problem. The reduction of stress concentration performed by the two method in the process of optimization are given in Figs.8 and 9

respectively. By observing Figs.7 - 9, there really exists some variation between the two methods but the deviation can be regarded acceptable.

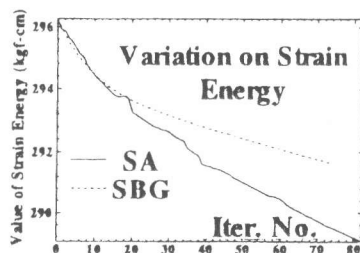


Fig.6 Comparison on Strain Energy of Design Structure

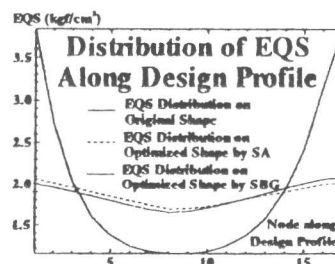


Fig.7 Comparison on Reduction of Stress Concentration

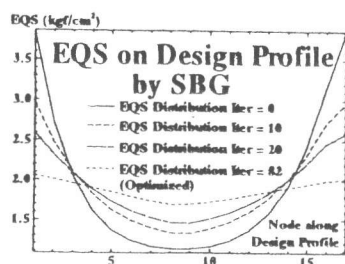


Fig.8 Performance on Reducing Stress Concentration by SBG Method

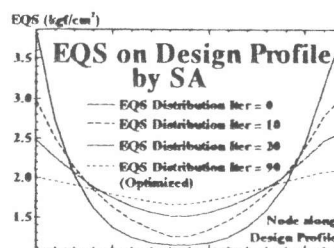


Fig.9 Performance on Reducing Stress Concentration by SA Method

Indeed, through the above figures, the agreement of the optimized results by the two methods can be clearly observed. However, on the other hand, the computational cost spent by the two methods was quite different. For the FEM model of this analytical case, in which there are 153 nodes and 128 elements, the programs were executed on SUN Sparc station 10. For SBG, one iterative step only took 1.2 sec while about 198 sec spent by SA; that is, only 98.4 sec was needed for SBG to find out the optimum (82 iterative loops) while about 17820 sec (about 4.95 hr) needed by SA. As a result, though similar optimal solutions could be achieved by the two methods, the high efficiency of the simulated biological growth method for saving a lot of computational cost was obviously confirmed.

7. CONCLUSIONS

By the observation on the two methods discussed in this research, the issues clarified in this study can be concluded as follows : (1) the high efficiency of simulated biological growth method could be confirmed with the comparison on another optimality criterion method for its excellent saving on computational cost; (2) the availability of the method using sensitivity analysis suggested in the research was also verified for its optimal solution highly similar to the result by the simulated biological growth method. However, the efficiency of this method was seriously influenced by sensitivity analysis. And such disadvantage makes this method quite unattractive when compared with the simulated biological growth method.

REFERENCES

1. Mattheck, C. and Burkhardt, S., " A New Method of Structural Shape Optimization Based on Biological Growth, " *Int. J. Fatigue*, Vol.12, No.3, May 1990, pp.185 - 190.
2. Hsu, K.L. and Uomoto, T., " A New Method for Optimal Shape Design of Structures, " *Proceedings of 49th Annual Conference of JSCE*, Sec. 5, 1994, pp.832 - 833.
3. Suh, M. W., Anderson, W. J. and McDonald, J. P., " Application of Geometric Strain Method to Shape Optimization of Spring Retainer Redesign, " *Computer Aided Optimum Design of Structures: Recent Advances*, edited by Brebbia, C. A. & Hernandez, S., 1989, pp.189 - 198.
4. Kibsgaard, S., " Sensitivity Analysis - The Basis for Optimization, " *Int. J. Num. Methd. Engng.*, Vol. 34, 1992, pp.901 - 932.