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論文 Micromechanical Constitutive Relationship of Fiber Reinforced Concrete

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ABSTRACT: A micromechanical constitutive relationship that can describe the mechanical behavior of fiber reinforced concrete is presented. The constitutive properties are characterized separately on a microplane. The composite mechanical behavior which is affected by fiber and concrete matrix properties of each microplane is modeled. The material parameters are determined from test data and formulated as functions of the volume fraction, as well as the aspect ratio of fibers in concrete. By comparison with numerous experimental results from the literature, a good agreement is confirmed.
KEYWORDS: fiber, matrix, composite material. micromechanical constitutive relationship, steel fiber reinforced concrete

1. INTRODUCTION

Fiber reinforced concrete (FRC) has improved mechanical properties such as increased ductility and strength compared to plain concrete. The improvement is achieved by the bridging effect of fibers across microcracks of the matrix. Therefore, to obtain high performance for the mechanical behavior of concrete, fiber reinforcement is considered to be effective.

Although many studies have been conducted to understand the mechanical properties of fiber reinforced cement-based composites, a constitutive model of steel fiber reinforced concrete considering the microstructural behavior has not been accomplished. Therefore, in this study, a micromechanical constitutive model of steel fiber reinforced concrete (SFRC) subjected to multiaxial loading is investigated and its appropriateness is examined. The constitutive properties are characterized separately on small planes of various orientations within the material, referred to as microplanes. The state of each microplane is characterized by normal deviatoric and volumetric strains and shear strain. The behavior of fiber and concrete on each microplane is modeled and the material parameters are determined from experimental data.

2. MICROMECHANICAL CONSTITUTIVE MODEL

2.1 MODELING OF FIBER AND CONCRETE

The following assumptions are made to develop a micromechanical constitutive model for SFRC by incorporating the microplane model of concrete [1].

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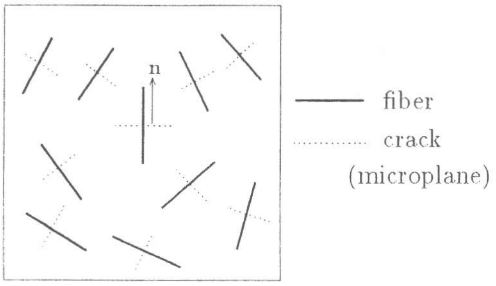
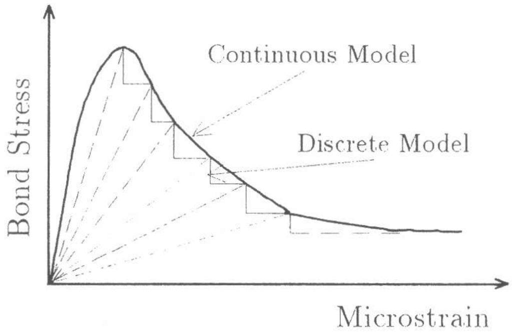


(a)Fibers and Microplanes



(b)Bond Stress–Strain Relationship

Fig. 1 Modeling of Fibers in Concrete

1. Fibers are uniformly distributed in concrete (see Fig.1(a)).
2. Fibers are bonded perfectly in concrete at the initial state.
3. Fibers are normal to microplanes.
4. Fibers are effective in tension and in the normal direction to a microplane.

The relationship between the microstrain or the strain at a microplane and the combined bond stress of several fibers at the microplane is modeled as shown in Fig.1(b) [2]. Then, by ignoring shear resistance, the microscopic stress-strain relationships for fiber are modeled as follows:

$$\sigma_V^f = C_V^f(\epsilon_V)\epsilon_V \quad ; \quad \sigma_D^f = C_D^f(\epsilon_D)\epsilon_D \tag{1}$$

$$C_V^f = \overline{C}_{V0}^{\,f}\ exp\left[-\frac{\epsilon_V}{a_1^f}\right]^{p_1^f} \quad ; \quad C_D^f = \overline{C}_{D0}^{\,f}\ exp\left[-\frac{\epsilon_D}{a_1^f}\right]^{p_1^f} \tag{2}$$

The microscopic stress-strain relationships for concrete matrix are expressed as follows:

$$\sigma_V^m = C_V^m(\epsilon_V)\epsilon_V \ ; \ \sigma_D^m = C_D^m(\epsilon_D)\epsilon_D \quad ; \quad \sigma_T = C_T(\epsilon_T)\epsilon_T \tag{3}$$

$$C_V^m = \overline{C}_{V0}^{\,m}\ exp\left[-\frac{\epsilon_V}{a_1^m}\right]^{p_1^m} \ ; \ C_D^m = \overline{C}_{D0}^{\,m}\ exp\left[-\frac{\epsilon_D}{a_1^m}\right]^{p_1^m} \quad ; \quad C_T = C_{T0}\ exp\left[-\frac{\epsilon_T}{a_3}\right]^{p_3} \tag{4}$$

where $\overline{C}_{V0}^{\,f}$,$\overline{C}_{D0}^{\,f}$ and C_V^f,C_D^f are the initial elastic and secant moduli for fiber, $\overline{C}_{V0}^{\,m}$,$\overline{C}_{D0}^{\,m}$ and C_V^m,C_D^m,C_T are the initial elastic and secant moduli for concrete matrix. a_1^m,a_1^f,p_1^m, p_1^f,a_3 and p_3 are material parameters. Subscripts V, D and T express the volumetric, deviatoric and shear components and superscripts m and f stand for matrix and fiber, respectively. For SFRC which is considered as a composite material[3], the secant moduli become as follows:

$$C_V = C_V^m(1 - V_f) + C_V^f V_f \quad ; \quad C_D = C_D^m(1 - V_f) + C_D^f V_f \tag{5}$$

where V_f is the volume fraction of fiber. The unloading and reloading moduli for fiber are defined in the same way as those for concrete matrix [1].

2.2 ELASTIC MODULI OF FIBER AND CONCRETE

For a straight fiber of a length L embedded in a solid matrix, the elastic modulus is reduced by the ratio λ given by the following equation [4].

$$\lambda = 1 - \frac{\tanh(\frac{\beta L}{2})}{(\frac{\beta L}{2})} \tag{6}$$

The term $\frac{\beta L}{2}$ can be related to the aspect ratio $\frac{L}{d}$ by the following equation [4]

$$\frac{\beta L}{2} = \frac{L}{d}\sqrt{\frac{4G_m}{(E_f - E_m)\ln(\frac{1}{V_f})}} \tag{7}$$

where G_m is the elastic shear modulus of concrete matrix, E_f, E_m are the elastic moduli of fiber and concrete matrix, respectively, and d is the diameter of fiber.

Then, the initial elastic moduli of fiber and concrete can be expressed as follows by composite material consideration.

$$\overline{C}_{V0}^{f} = (C_{V0}^{f} - C_{V0}^{m})\lambda + C_{V0}^{m} \quad ; \quad \overline{C}_{D0}^{f} = (C_{D0}^{f} - C_{D0}^{m})\lambda + C_{D0}^{m} \tag{8}$$

$$\overline{C}_{V0}^{m} = (C_{V0}^{f} - C_{V0}^{m})V_f\lambda + C_{V0}^{m} \quad ; \quad \overline{C}_{D0}^{m} = (C_{D0}^{f} - C_{D0}^{m})V_f\lambda + C_{D0}^{m} \tag{9}$$

$$C_{V0}^{f} = \frac{E_f}{E_m}C_{V0}^{m} \quad ; \quad C_{D0}^{f} = \frac{E_f}{E_m}C_{D0}^{m} \tag{10}$$

$$C_{V0}^{m} = \frac{E_m}{1 - 2\nu_m} \quad ; \quad C_{D0}^{m} = \eta_0 C_{V0}^{m} \tag{11}$$

where ν_m is the Poisson's ratio of the concrete and η_0 is a material parameter.

2.3 MACROSCOPIC STRESS-STRAIN RELATIONSHIP

Using the principle of virtual work to approximately enforce the equivalence of forces on the microscale and macroscale, the following macroscopic stress-strain relationship can be obtained [1].

$$d\sigma_{ij} = C_{ijkm}d\epsilon_{km} - d\sigma_{ij}^{''} \tag{12}$$

where C_{ijkm} denotes the incremental stiffness tensor (elastic modulus tensor)

$$\begin{aligned} C_{ijkm} = & \frac{3}{2\pi}\int_S [(C_D^t - C_T^t)n_i n_j n_k n_m + \frac{1}{3}(C_V^t - C_D^t)n_i n_j \delta_{km} \\ & + \frac{1}{4}C_T^t(n_i n_k \delta_{jm} + n_i n_m \delta_{jk} + n_j n_k \delta_{im} + n_j n_m \delta_{ik})]\ f(n)\ dS \end{aligned} \tag{13}$$

and $d\sigma_{ij}^{''}$ denotes the inelastic stress increments

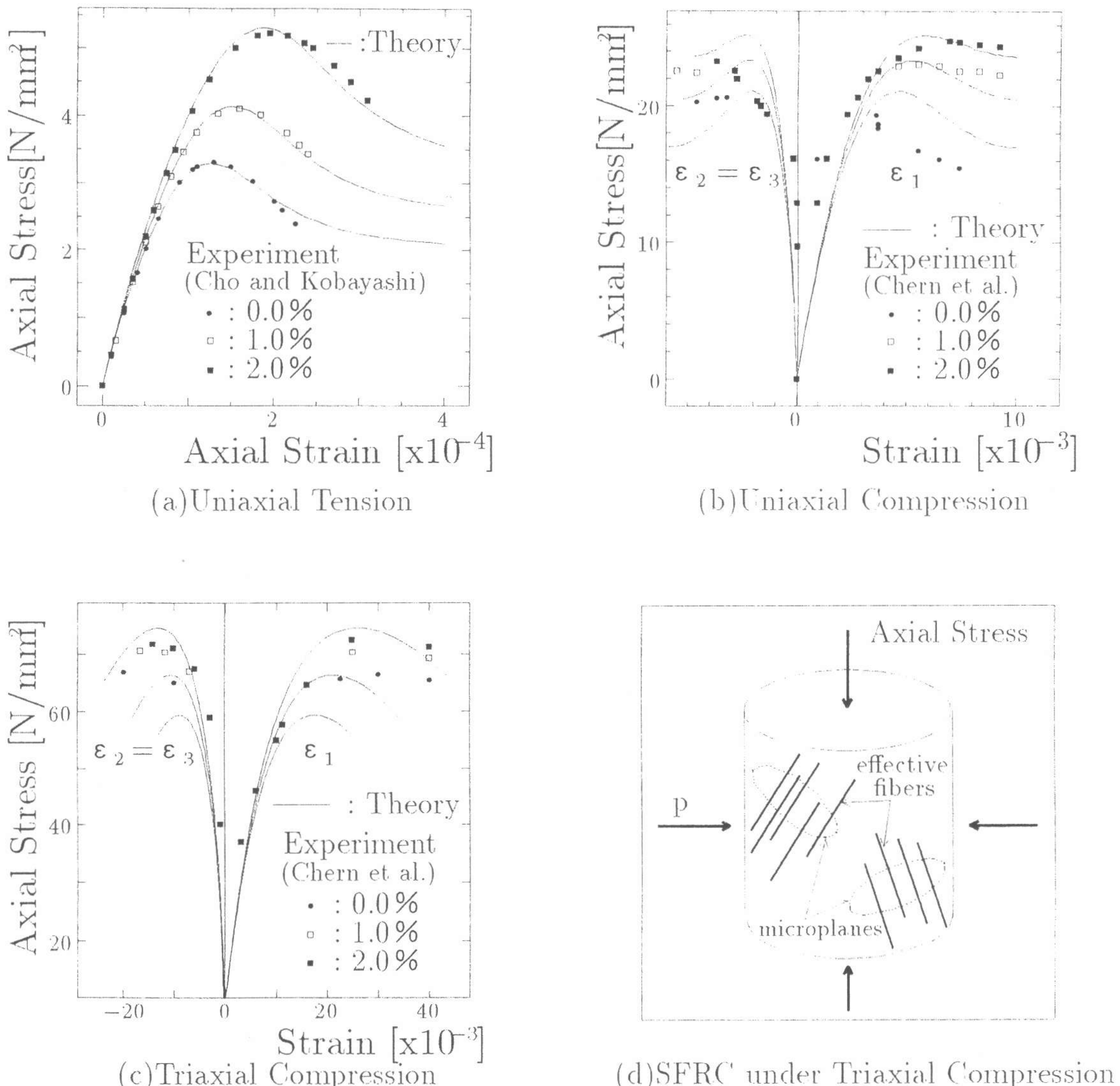
$$d\sigma_{ij}^{''} = \frac{3}{2\pi}\int_S \left[n_i n_j d\sigma_N^{''} + \frac{1}{2}(n_i\delta_{rj} + n_j\delta_{ri} - 2n_i n_j n_r)d\sigma_{T_r}^{''}\right] f(n)\ dS \tag{14}$$

where C_V^t, C_D^t and C_T^t are the incremental elastic moduli for the current loading step for a microplane and n_i is the microplane direction cosine, $d\sigma_N^{''}$ and $d\sigma_{T_r}^{''}$ are the inelastic normal and shear stress increments, S is the surface of a unit hemisphere and $f(n)$ is the weighting function of the normal direction n. The unloading and reloading criteria are assumed to follow those of the microplane model [1].

3. VERIFICATION OF MICROMECHANICAL CONSTITUTIVE MODEL

3.1 EFFECT OF VOLUME FRACTION OF FIBER

For uniaxial tension, the test data by Cho and Kobayashi[5] are used. Specimens are made with shearing steel fibers. The aspect ratio of fiber is 54 and the volume fractions are 0, 1 and 2 %.



(a)Uniaxial Tension

(b)Uniaxial Compression

(c)Triaxial Compression (Confining Pressure p=10N/mm ^{2})

(d)SFRC under Triaxial Compression

Fig. 2 Stress-Strain Curves for Uniaxial and Triaxial Loadings

For uniaxial and triaxial compression, the test data by Chern et al.[6] are used. In this test, straight carbon-steel fibers with an aspect ratio of 44 are used. Specimens are made

Table 1 Optimum Values of Material Parameters in Terms of Volume Fraction

Test Data	V_f	$a_1^m = a_1^f$	$p_1^m = p_1^f$	a_3	p_3
Uniaxial Tension	0%	0.3x10^{-4}	0.8	0.5	1.5
	1%	0.33x10^{-4}			
	2%	0.378x10^{-4}			
	V_f	(2.97+39V_f)10^{-5}			
Uniaxial Compression	0%	4.0x10^{-4}	0.5	1.8	
	1%	40x10^{-4}	0.4		
	2%	400x10^{-4}	0.3		
	V_f	4x$10^{100 V_f}$	$0.5 - 10 V_f$		
Triaxial Compression	V_f	4.0x10^{-4}	0.5		

Table 2 Optimum Values of Material Parameters in Terms of Aspect Ratio

Aspect Ratio	$a_1^m = a_1^f$	$p_1^m = p_1^f$	E
11	2.0x10^{-5}	0.600	4.9x10^5
23	2.5x10^{-5}	0.550	5.0x10^5
52	3.0x10^{-5}	0.475	5.1x10^5
$\frac{L}{d}$	(0.72+ln($\frac{L}{d}$))6.43x10^{-6}	(627-0.296($\frac{L}{d}$))10^{-3}	(35.58+ln($\frac{L}{d}$))1.29x10^4

with fiber volume fraction of 0, 1 and 2 %. Using the spherical integration formula with 21 microplanes, the optimization by which the fits have been achieved was carried out simply by a trial-and-error approach. The optimum values of the material parameters corresponding to these test data are listed in Table 1 and the stress-strain curves are shown in Fig. 2.

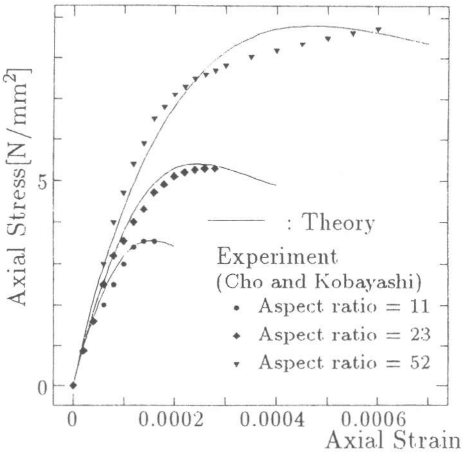
3.2 EFFECT OF ASPECT RATIO OF FIBER

The effect of the aspect ratio of fiber is examined by using the test data for various aspect ratios by Cho and Kobayashi[7]. In this test, cut-wire fibers with aspect ratios of 11, 23 and 52 are used and the specimens are made with 3.5 % of fiber volume fraction. The optimum values of the material parameters corresponding to each test data are listed in Table 2 and the stress-strain curves are shown in Fig.3(a).

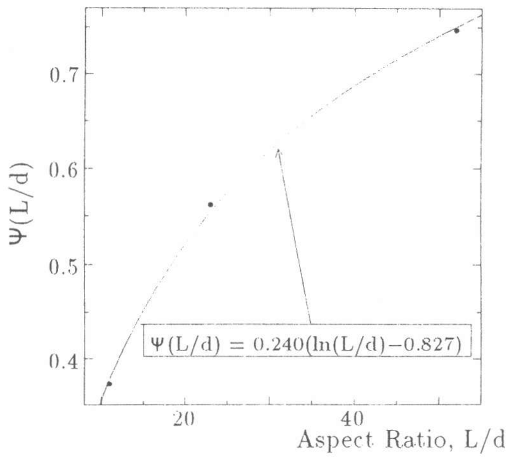
Considering Eq.(7) and by integration over all spatial directions of the microstructures, the micro-constitutive properties of Eq.(9) can be expressed in terms of a macroscopic composite elastic modulus E as follows:

$$E = (E_f - E_m)V_f \; \Psi(\frac{L}{d}) + E_m \tag{15}$$

where Ψ is the elastic modulus reduction factor of fiber and depends on the value of aspect ratio. By fitting the experimental data by Cho and Kobayashi[7], the following



(a)Stress–Strain Curves for Uniaxial Tension



(b) Elastic Modulus Reduction Factor

Fig. 3 Effect of Aspect Ratio of Fiber

relationship is obtained.

$$\Psi(\frac{L}{d}) = 0.240\ (\ln(\frac{L}{d}) - 0.827) \tag{16}$$

4. CONCLUSION

A micromechanical constitutive model based on the microplane model is proposed for SFRC. It is confirmed that the present model is suitable to express the mechanical behavior of SFRC in tension and compression under multiaxial loading by the comparison with experimental data. Several empirical material parameters are summarized as functions of fiber volume fraction and aspect ratio. The present model can be used in a finite element analysis of SFRC.

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