論文 Modified Unified Concrete Plasticity Model and Its Variations

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ABSTRACT: The Characteristics of the plasticity model named the Unified Concrete Plasticity Model, proposed by Tanabe. et. al.[1] was modified by Gupta and Tanabe[2] because various problems were detected in the model. Though the original model was tested for various loading conditions, like Kupfer's biaxial experiments etc., the modified model[2] was not tested for various conditions. The modified model also has some interesting possible variations. In this paper, these possible variations are tested, analyzed and compared with Kupfer's experimental results.
KEYWORDS: concrete, plasticity, constitutive relations, Unified Concrete Plasticity Model

1. INTRODUCTION

For a general structural analysis of reinforced concrete structures, one needs a constitutive relation that is valid under all multi-axial situations. The Unified Concrete Plasticity Model was presented by Tanabe et al. [1] , aiming at tension stiffening effect and compressive strength reduction effects due to tensile strain to be incorporated into a single plasticity model. This model was numerically able to simulate various experimental results, under proportional loading by Kupfer[3], and uniaxial tension experiment by Petersson[4] within reasonable limits. However, when the model was applied in finite element analysis, problems were encountered. This lead Gupta and Tanabe[2] to further investigate the characteristics of this model and it was realized that the chosen constitutive model exhibits material instability, like snapback in stress-strain relation etc. because of inappropriate choice of set of material parameters. Defects of this model were also detected and major modifications were proposed by Gupta and Tanabe[2]. However, the proposed model was not tested for various multi-axial stress conditions.

In this paper, various possible variations of this model, based on the order of yield surface and definition of damage is described and examined by constitutive level calculations[2].

2. THE UNIFIED CONCRETE PLASTICITY MODEL

The Unified Concrete Plasticity Model[1] follows essentially the basic concept of classical theory of hardening plasticity. The subsequent failure surface is assumed to change its size continuously depending on the accumulated damage ω(Wp). Associated flow rule is assumed. The yield surface (fig. 1) is given by

$$f = f\left(\sigma_{ij}, \omega\left(W^{p}\right)\right) = J_2 - \left(k_f - \alpha_f I_1\right)^2 + \left(k_f - \alpha_f \eta\right)^2 = 0 \tag{1}$$

where

$$k_f = \frac{6c\cos\phi}{\sqrt{3}(3+y\sin\phi_1)}, \qquad \alpha_f = \frac{2\sin\phi}{\sqrt{3}(3+y\sin\phi_1)} \tag{2}$$

where $\phi_1 = 14°$ is a material constant. Cohesion c, friction angle ϕ and material parameter η depends on the damage of that point ω and are defined by material constants $\phi_0, \phi_f, c_0, \eta_0$ as

$$c = c_0 \exp\left[-(m\omega)^2\right], \quad \eta = \eta_0 \exp(-\omega / b), \quad \phi = \begin{cases} \phi_0 + \left(\phi_f - \phi_0\right)\sqrt{2\omega - \omega^2} & \omega \le 1 \\ \phi_f & \omega > 1 \end{cases} \tag{3}$$

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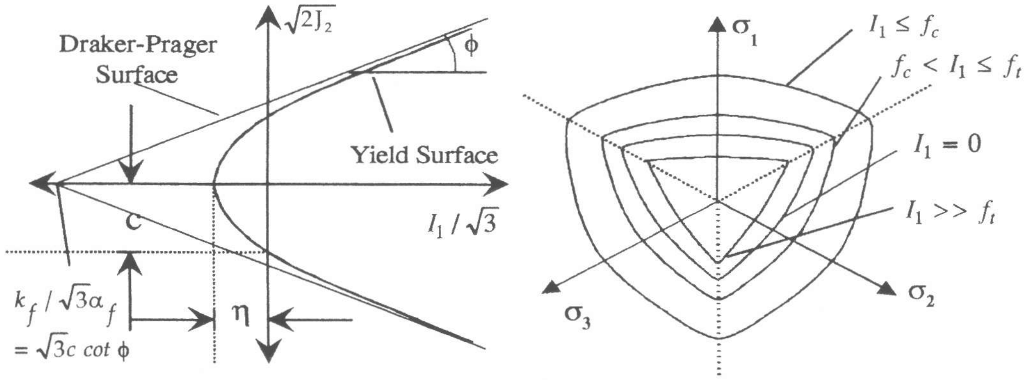

a) Meridian Plane b) Deviatoric Plane

Fig.1: The Unified Concrete Yield Surface

and y is a parameter that is used to match the Drucker-Prager based surface with Mohr-Coulumb core, both at the tensile and compressive meridian

$$y=\sqrt{a(\cos 3\theta+1.00)+0.01}-1.10,\quad a=0.5r^2+2.1r+2.2,\quad r=\begin{cases}3.14 & I\le f_c' \\ 2.93\cos(I_1\pi/f_c') & f_c'<I\le f_t \\ 9.0 & I>f_t\end{cases} \tag{4}$$

where $\cos 3\theta=\sqrt{2}J_3/J_2^{3/2}$ and I_1, J_2 and J_3 are the usual stress invariants

3. THE MODIFIED UNIFIED CONCRETE PLASTICITY MODEL

The state of stress-strain, or even the shape of the yield surface highly depends on the cohesion and friction angle. Assuming that the initial material parameters are constant, though no experimental verification exists, it is not difficult to understand that the rate of change of cohesion and friction angle should not be the same in the case of uniaxial tension and uniaxial compression. But Tanabe et al. [1], in an attempt to create a unified theory which can be applied to all cases, introduced a material parameter η, which is the distance between the tip of the yield surface in tension zone from the origin. By making the rate of change of η independent of the rate of change of c and ϕ, the tip of the yield surface, and hence the tension zone is reduced to zero at a faster rate, even though the value of c is not yet zero. Since the rate of change of η was independent of rate of change of c and ϕ, tension stiffening effect could be incorporated. Though, this gave excellent results in tension zone and compression zone. Gupta and Tanabe[2] found the assumption that the range of ϕ to be constant in all cases to be illogical because it presents illogical results for lateral behavior in case of uniaxial tension. Gupta and Tanabe[2] also found the fact that rate of change of η to be independent of rate of change of c and ϕ as not logical.

However ,the good results predicted by the original the Unified Concrete Plasticity Model could not be neglected. The tension behavior is largely controlled by the rate of change of the distance between the tip and the origin(value of η). So we have to maintain similar $\eta-\omega$ relation. We know that the tip of the Draker-Prager surface model is given by $\sqrt{3}c\cot\phi = AA^*k/(\sqrt{3}\alpha)$ (fig. 2a). Gupta and Tanabe[2] assumed the same η_o as before[1], and hence η is assumed to be

$$\eta = AA^*\sqrt{3}c\cot\phi\ ,\quad AA^*=\sqrt{3}c_0\cot\phi_0/\eta_0 \tag{5}$$

The modified yield function looks like

$$f=J_2-(k-\alpha_f I_1)^2+(k_f-\alpha_f\eta)^2$$
$$=J_2-(k-\alpha_f I_1)^2+(k_f-AA^*k_f/\eta_o)^2 \tag{6}$$
$$=J_2-(k-\alpha_f I_1)^2+k_f(1-AA^*/\eta_o)^2=0$$

Though not shown here, it was found that good

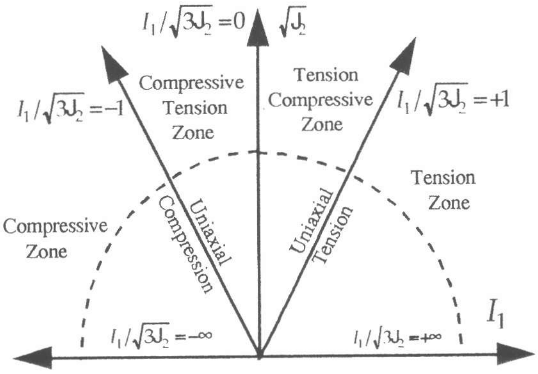

Fig.2: Different Zones in $(I_1,\sqrt{J_2})$ space

results can be obtained in case of uniaxial tension and compression if the following assumptions are made

a) the initial material parameters are constant,
b) ϕ to be constant for uniaxial tension,
c) rate of change of c in tension zone similar to the rate of change η as proposed by Tanabe et. al.[1].
d) ϕ to change from ϕ_o to ϕ_f in case of pure compression,
e) rate of change of c in compression zone similar the proposal of Tanabe et. al.[1].

These assumptions basically keeps the shape of the *initial yield surface* to be similar to the model proposed by Tanabe et. al.[1]. With eq. 7a for the uniaxial tension and eq 7b for uniaxial compression, the biggest question for Gupta and Tanabe[2] was how to define the material parameter for all conditions

$$\phi = \phi_o \qquad c = c_o \exp(-\omega / b) \qquad (7a),$$

$$c = c_0 \exp\left[-(m\omega)^2\right], \quad \phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{2\omega - \omega^2} & \omega \le 1 \\ \phi_f & \omega > 1 \end{cases} \qquad (7b)$$

Gupta and Tanabe[2] proposed a gradual variation between the tension to compression zone based on the parameter $X = I_1 / \sqrt{3J_2}$ whose variation is shown in fig. 2. This is based on a well established way[5] of defining various zones in stress space in the plane of $(I_1, \sqrt{J_2})$. Hence the material parameters c and ϕ was defined as

$$c = c_0 \exp\left[(-m_1\omega)p_1(X) + (-m_2^2\omega^2)p_2(X)\right],$$
$$\phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{(\omega + k)(2\omega_f - \omega - k)}\,p_2(X)/\omega_f & \omega \le \omega_f - k \\ \phi_0 + (\phi_f - \phi_0)p_2(X) & \omega > \omega_f - k \end{cases} \qquad (8)$$

where k=10^{-3} (small value) is introduced to get rid of the singularity caused by at ω=0 when $\frac{\partial\phi}{\partial\omega} = \infty$.

Though this can be a topic of further research, Gupta and Tanabe[2] proposed the following relation

$$\{p_1(X) \quad p_2(X)\} = \begin{cases} 1 & 0 & X \le -1 \\ \frac{1}{2}\sin\left(\frac{X\pi}{2}\right) + \frac{1}{2} & \frac{-1}{2}\sin\left(\frac{X\pi}{2}\right) + \frac{1}{2} & -1 < X \le 1 \\ 0 & 1 & X > 1 \end{cases} \qquad (9)$$

Moreover, r in eq. 4 is modified to make it continuous in all ranges and is redefined as

$$r = \begin{cases} 3.14 & I \le f_c' \\ 2.93\cos\left(\frac{f_t - I_1}{f_t - f_c'}\pi\right) & f_c' < I \le f_t \\ 9.0 & I > f_t \end{cases} \qquad (10)$$

4. VARIATIONS OF THE MODIFIED UNIFIED CONCRETE PLASTICITY MODEL

In this paper, variations based on the order of yield surface and definition of damage are examined.

4.1 IN CONSTITUTIVE EQUATION

The Unified Concrete Plasticity Model is a *second order yield function* $f(\sigma,\omega)$ (eq. 6 or the one proposed by Tanabe et. al.[1]). However the same model can also be rewritten in the first order variation. In the original paper[1], the reason for selecting the second order variation was not specified. Hence in this present research, behavior of both the variations are analyzed and compared. The yield function as given in eq. 6 can be rewritten as

$$J_2 - (k - \alpha_f I_1)^2 + k_f(1 - AA^*/\eta)^2 = 0 \quad \Rightarrow J_2 + k_f(1 - AA^*/\eta)^2 = (k - \alpha_f I_1)^2$$
$$\Rightarrow \sqrt{J_2 + k_f(1 - AA^*/\eta)^2} - (k - \alpha_f I_1) = 0 \qquad (11)$$

So we can define $g(\sigma,\omega)$,the *first order yield function* as

$$g(\sigma,\omega) = \sqrt{J_2 + k_f(1 - AA^*/\eta)^2} - (k - \alpha_f I_1) = 0 \qquad (12)$$

where α_f and k_f as defined in eq. 2. Here *order of yield function* implies the order the yield function with respect to stress terms.

4.2 IN THE DEFINITION OF DAMAGE

In the original paper[1], the classical plasticity approach was adopted. The damage parameter $\omega(Wp)$ was defined as

$$d\omega = \frac{\beta}{\sigma_e \varepsilon_0} dW^p = \frac{\beta^{'}}{\sigma_e} dW^p \qquad (13)$$

where $dW^p = \sigma_{ij} d\varepsilon_{ij}^p = \sigma_e d\varepsilon^p$ and $d\varepsilon_{ij}^p$ is the plastic strain, dW^p is the plastic work, σ_e and $d\varepsilon^p$ is effective plastic stress and strain, β and ε_o $(\beta^{'} = \beta / \varepsilon_0)$ are material constants. Though in the original paper[1], the basic method used is a little different, basically the definition of effective plastic strain $d\varepsilon^p$ is similar to

$$d\varepsilon^p = \sqrt{d\varepsilon_{ij}^p . d\varepsilon_{ij}^p} = d\lambda \sqrt{\frac{df}{d\sigma_{ij}} . \frac{df}{d\sigma_{ij}}} \qquad (14)$$

This means the definition of damage given in eq. 13 can be rewritten as

$$d\omega = \beta^{'} d\varepsilon^p = d\lambda \beta^{'} \sqrt{\frac{df}{d\sigma_{ij}} . \frac{df}{d\sigma_{ij}}} \qquad (15)$$

This is the typical definition of *strain hardening* damage. We can also calculate the damage in a simpler fashion as below. Hereafter, eq. 15 is referred as *strain hardening* (SH) *damage* and eq. 16 as *simple damage* model.

$$d\omega = \beta^{'} d\lambda \qquad (16)$$

5. ANALYSIS AND DISCUSSION

As we can see above , we can get 4 (eq. 6/12 x eq. 15/16) type of variations. The author tried to fit these variations to the Kupfer's experimental results. The different cases that are studied are presented in tabular form, and a reference name is attached to them. It was found that it was only enough to change β^1 the parameter to get similar results in all the cases. The parameters that are constant are: E = 32500 MPa, μ = 0.22, $f_t' = 3.25$ MPa, f_c=-32.5 MPa, c=28, $\phi_1 = 14°$, $\phi_o = 5°$, $\phi_f = 32.7$, m_1=10.0, m_2=1.0, η_o =5.52 MPa, k = 1.0 x 10^{-3}, ω_f=1.0. Table 1 shows the material parameters β^1.

5.1 COMPARISON WITH KUPFER'S EXPERIMENTAL RESULTS

Figure 3 shows the comparison of four cases with that of Kupfer's experimental results for three different biaxial stress conditions. It might look strange, but all these results were obtained by changing only the value of β^1, which controls the rate of development of damage. Since postpeak behavior depends of the rate of formation of the damage, each of the four cases act differently. So the author find it logical that all four cases have independent post-peak softening behavior. Cases 1A, 1B and 2B shows reasonable softening diagram, where as case 2A shows very sharp decline, which look not reasonable to be used for concrete and hence not considered

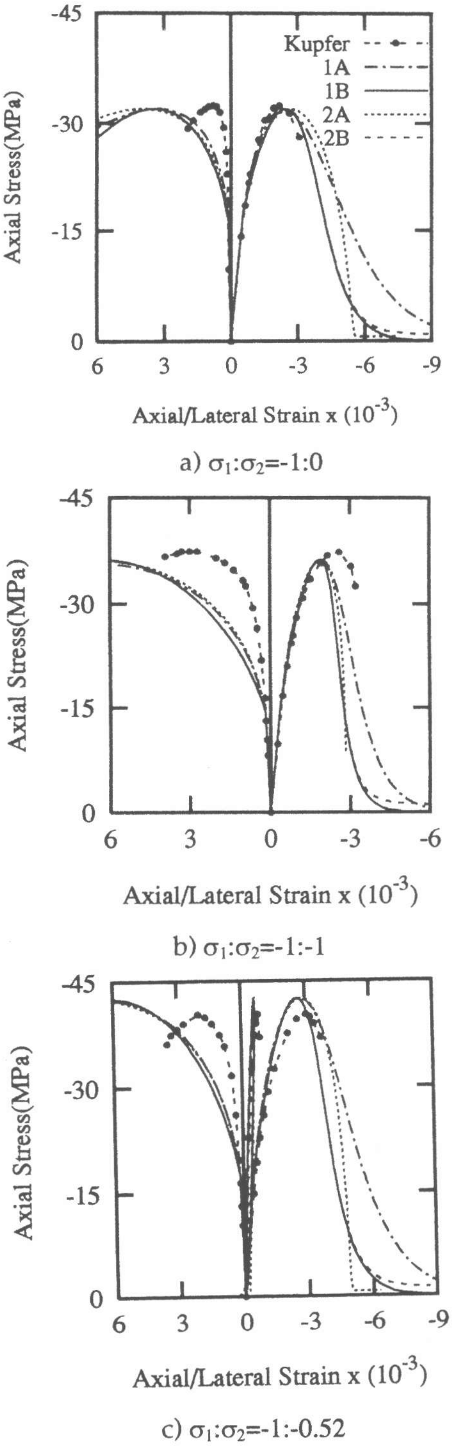

Fig. 3: Comparison with Kupfer's Experiment

henceforth. Figure 4 shows the comparison between the peak stresses of the analytical results with that of Kupfer's experimental results[3] for case 1A. The results of the other three cases were not presented because results of the four case were almost exactly similar with a maximum difference of about 5%. Therefore, if they are drawn, it would be indistinguishable and are not drawn. Figure 4 also shows the stresses at initially yields point.

5.2 RELATIVE DEVIATION FROM THE YIELD SURFACE

The constitutive level calculation shown in figures 3-5 are conducted with very small steps so that the error is minimized. Only results of uniaxial compression are used for simplicity to check the relative tendency of deviation from the yield surface. Figure 5a shows the axial stress-strain, which is same as that of fig. 3a. In this calculation, rate used is 14705 steps for 0.0152 axial strain increment. In this case (fig. 5a) we can note that all stresses comes down to almost zero value. Figure 6a used bigger increment of 149 steps for 0.0152 strain increment so that the deviation becomes prominent. However in this case

Table 1: Different types of variations

Name	order of equation	definition of damage	β^1	Equation numbers
1A	First	simple	30	12,16
1B	First	SH	66	12,15
2A	Second	simple	1700	10,16
2B	Second	SH	66	10,15

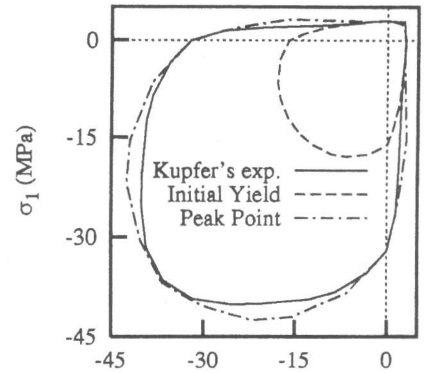

Fig. 4: Peak and Yield Stress: Experimental and Numerical Analysis

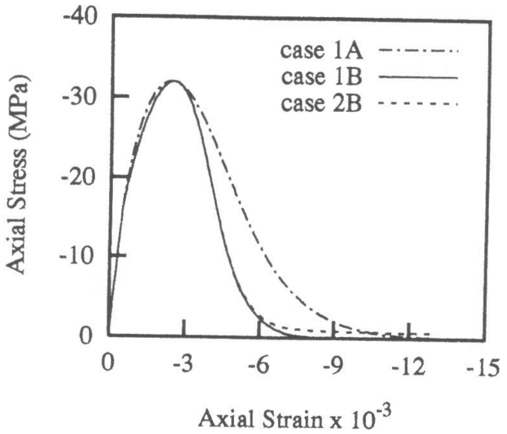

Fig. 5a: Uniaxial Stress-Strain: fine step

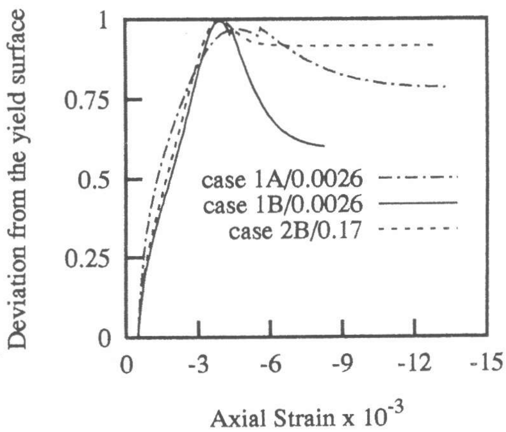

Fig. 5b: Deviation from the yield surface

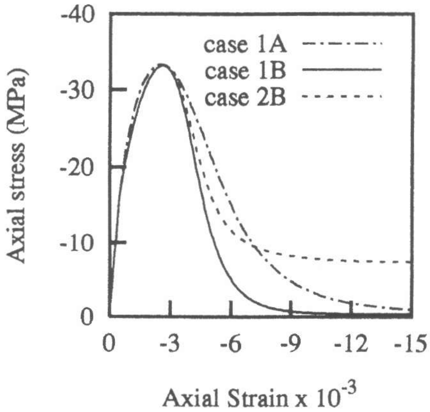

Fig. 6a: Uniaxial Stress-Strain: big step

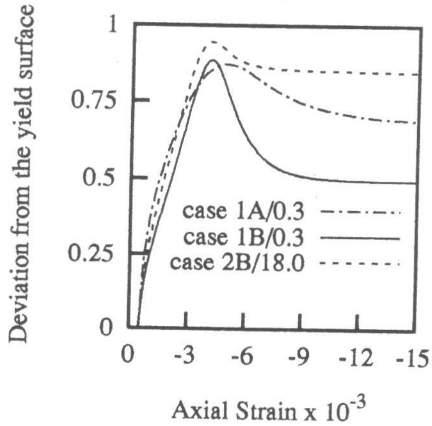

Fig. 6b: Deviation from the yield surface

(fig. 6a), the stress of case 2B remains significantly high where as the stress in cases 1A and 1B tended to zero. This type of result in case 2B is probably because of the deviation from the yield surface.

Figure 5b and 6b, shows the normalized error(deviation from the yield surface) with increase in axial strain. Error in all the 3 cases increased (0.0026 → 0.3 for case 1A and 1B, and 0.17 → 18.0 for case 2B) with increase in step size. Case 1A and 1B showed similar error behavior. Direct comparison between case 1A and 1B with that of case 2B is not possible because there is a difference in order of the yield surface. However, since the stress for case 2B could not come to zero, we can conclude that the first order yield function is better, in the sense that it produces lesser deviation from the yield surface.

5.3 SOFTENING BEHAVIOR OF CONCRETE IN TENSION

In case of uniaxial tensile experiment of reinforced concrete member, the crack spacing, hence the slope of the stress-strain curve in the smeared crack model should change with reinforcement ratio[6]. This effect is often referred to as tension stiffening effect. Hence it is important to be able to get different softening slopes for tension without effecting the compression behavior so that the tension stiffening effect can be simulated in the actual implementation in finite element analysis in a rational way. This implementation is a topic that will be dealt in future research.

This can be very easily obtained by changing the value of m_l of eq. 8. This is possible because of the introduction of independent variation of c and ϕ based on parameter $X = I_1 / \sqrt{3J_2}$ in tension and compression zone. The results can be seen in fig. 7. The peak of the uniaxial tension can be directly controlled by changing η_o .

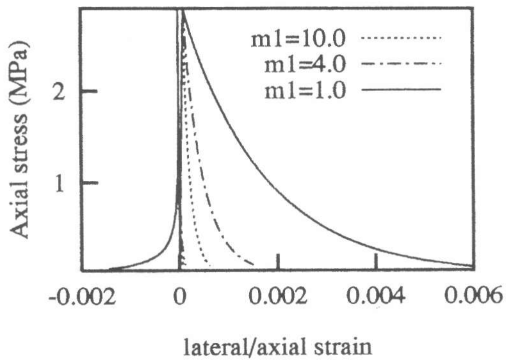

Fig. 7: Effect of m_1 on uniaxial tension of case 1

6. CONCLUSION

The various variations of the unified concrete model is shown. The original model utilized the second order variation with the strain hardening hypothesis (eq. 11) for calculation of damage. In this paper various variations of this proposed model are analyzed. We can conclude that:

1. First order yield surface with simple damage rule can also simulate the concrete behavior equally well as that of second order yield surface with strain hardening damage model.
2. First order yield surface provides less deviation from the yield surface in comparison to second order yield surface.
3. Since the comparison of Kupfer's experimental results were proved successful in intermediate range, hence it was proved that the variation of c and ϕ based on $X = I_1 / \sqrt{3J_2}$ to be logical.

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