論文 Size Effect Analysis for Shear Stregth of Reinforced Concrete Beams

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ABSTRACT: This paper deals with simulating the shear strength size effect in large scale reinforced concrete beams by a nonlinear FEM program **WCOMR**. In order to predict the shear behavior of RC beam with small reinforcement ratio, a cracked concrete model for plain concrete which is in the RC structure but away from the reinforcement effecting volume was proposed. The experimental results on shear strength size effect for reinforced concrete beams (depth varied from 10cm to 300cm) were used for verification. The comparison of analytical and experimental results has shown that the analysis using the proposed models can predict the shear behavior of RC beams of different size.

KEYWORDS: Size effect, Shear Failure, RC beam

1. INTRODUCTION

In recent years, the scale of concrete structures is becoming larger and larger owing to the advances made in materials, and improvement in design and construction techniques. One of the problems of increases in size is the evaluation of nominal shear strength. The nominal shear strength of a reinforcement concrete beam has been found in experiments to be gradually reduced as the beam depth increases, this is generally regarded as the size effect in shear. In order to estimate the accurate shear strength of large reinforced concrete structures, the experiment for large reinforced concrete beams without shear reinforcement was conducted[1] where the effective depth ranges from 10cm to 300cm. On the other hand, the progress of numerical procedures based on the finite element method for reinforced concrete structures in the past twenty years is remarkable[3] and shear behaviors of reinforced concrete structures has been studied. Because concrete not only exhibits nonlinear deformational behavior but also fracture is a major feature of interest to engineers, the cracked concrete model and associated size effect on shear will be a major topic addressed in this study.

2. FEM PROGRAM WCOMR

In this paper, FEM program **WCOMR**[3] is used for analyzing reinforced concrete structures, to predict the experimental shear strength for RC beams of large scale. In this program, the smeared crack model is employed by combining the constitutive laws of concrete and reinforcing bars. The concrete model is composed of tension stiffening model, compression model and shear transfer model. These models are given as the relationship between average stress and average strain in reinforced concrete control volume. The crack spacing, or density, and diameter of reinforcing bars have negligible effect on spatially average stress-strain relation defined on RC control volume[2]. Therefore, the continuum damage model of concrete encompasses the reduction of compressive capacity of cracked concrete in relation to the mean strain normal to the crack.

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In some cases, such as large reinforced concrete beams without shear reinforcement which will be hereafter analyzed, as the reinforcement ratio is very small and steel bars are located at the bottom of the beam, some volume of concrete is outside the RC control volume in which tensile stress is not transferred through the bond mechanism with reinforcement. The spatially averaged behavior of concrete far away from reinforcing bars is supposed the same as plain concrete, showing sharp strain-softening character, as the tensile stress is transferred only through the bridge action at crack surface. To predict shear behavior of this kind of beams, it is needed to propose a modified cracked concrete model for plain concrete(PL) zone which is the component of reinforced concrete but outside the RC control volume(Fig. 1).

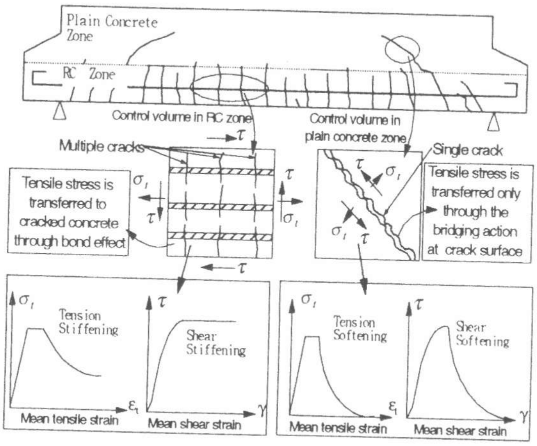


Fig. 1 Spatially averaged response defined on the control volume(element) for RC zone and plain concrete zone

3. CRACKED CONCRETE MODEL FOR PLAIN CONCRETE ZONE IN RC STRUCTURE

3.1. MODEL OF CRACKED CONCRETE UNDER TENSILE STRESS

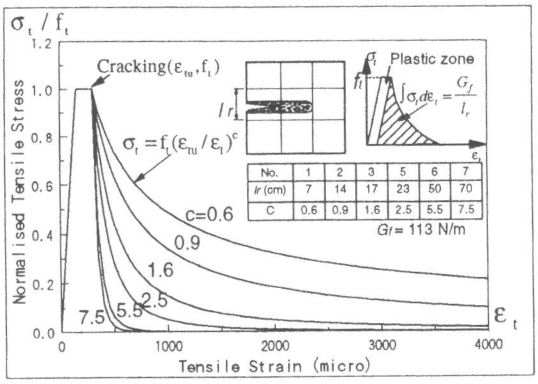
Because of the bond mechanism between concrete and reinforcing bars, the concrete continues to support a part of the tensile force even after cracking. This constitutive law for reinforced concrete derived for a control volume of 30-50 cm sized concrete element with distributed cracks are independent of crack density and crack numbers owing to the trade-off mechanisms among these effects[8]. It is proved that in the case of normal concrete and two-way reinforcement with the ratio over 0.1% to 2 %, this two dimensional constitutive law is independent of the size of the control volume. In this model, a simple relation for the tension stiffness in concrete is used[2]:

$$\sigma_t = f_t \left(\frac{\varepsilon_{tu}}{\varepsilon_t} \right)^c \tag{1}$$

where, σ_t is tensile stress of cracked concrete normal to cracks; f_t is tensile strength of concrete; ε_t is tensile strain normal to cracks; ε_{tu} : cracking mean strain of concrete; c is a constant, set equal to 0.4 for deformed bars and 0.2 for welded mesh[2].

In the case of plain concrete without the bond action between concrete and reinforcing bars, the concrete will not continue to support much tensile force after cracking. As mentioned above, the cracked plain concrete shows strain-softening characteristic in tension and shear comparing with the concrete confined by reinforcing bars. The stress-strain curve is decided by the fracture energy G_f and the crack band width[4]. In the finite element method, the crack band is simulated by the discrete finite elements. The crack band width in computation becomes the reference length related to the element size. The crack band energy is treated as a material property and needed to be kept constant regardless of the element size. Based on the fracture energy, the stress-strain curve defined in element needs to be modified according to the reference length l_r (l_r is the square root of the element area).

In the proposed model, the same mathematical form for reinforced concrete is used for plain concrete but the mean stress-strain curve is much sharper after the cracks occur. In this case, the constant c will change with the element size by getting the constant fracture energy. Fig.2 gives a series of tensile stress-strain curves for the computation of beams with depth varying from 10cm to 300cm which were used in size effect test[1].



No.	1	2	3	5	6	7
lr (cm)	7	14	17	23	50	70
C	0.6	0.9	1.6	2.5	5.5	7.5

Fig. 2 Tension model for cracked concrete in plain concrete controlled volume

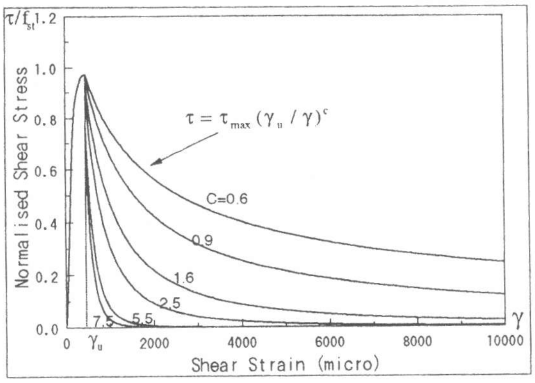


Fig. 3 Shear model for cracked concrete in plain concrete controlled volume

(tensile strain=0.00005, ultimate shear strain γ_u=0.0004)

3.2. MODEL OF CRACKED PLAIN CONCRETE UNDER SHEAR STRESS

The shear model of concrete in RC shows the shear stiffening after cracking[6]. On the other hand, for plain concrete, there is no effect of embedded reinforcing bars, so it is needed to express the shear softening of concrete in the shear model. This means when shear stress would reach the peak, the shear stress will decrease sharply as the shear strain increases. The shear stress and strain relationship will follow Eq.2 after shear strain reaches the ultimate value(Fig.3). Actually the shear softening curve should also decided with the element size based on the shear fracture energy. As the study of shear fracture energy is still no enough, the same coefficient *c* with tension curve is used here for different element size.

$$\tau = \tau_{\max} \left(\frac{\gamma_u}{\gamma} \right)^c \qquad \text{when } \gamma > \gamma_u \qquad (2)$$

where, γ is shear strain; γ_u is ultimate shear strain; τ_{max} is shear stress at peak point when $\gamma = \gamma_u$.

3.3. THE COMBINATION OF RC MODEL AND PLAIN CONCRETE MODEL IN BEAM

There is a further point which needs to be considered when a structure consists of several parts which are of a different nature and refer respectively to the reinforced concrete model or the plain concrete model discussed above. It is needed to combine the two models in the computation of beam, by dividing the beam into RC control zone and plain concrete zone. In order to decide the size of RC zone, the following method was conducted.

The size of RC zone is related to the bond effect of reinforcing bar and should be decided by the bond characteristics. Inside the bond effect zone, the steel bar is capable of providing crack control in this limited concrete volume. When the area of the concrete surrounding a certain steel bar is becoming larger, the steel bar will yield faster after one crack is formed, as much stress carried by concrete will be transferred to the steel bar(Fig.4a). In one critical case, if a reinforced concrete member that is subjected to tension contains only a very small amount of reinforcement, the reinforcement crossing the crack will yield just after cracking and most deformation will be rooted in this single crack. This is a critical point whether adequate crack control capability can be obtained or not. Hence, for one certain steel bar, the maximum size of crack control area in concrete, A_{cmax}, which represents the bond effect zone, can be calculated as,

$$A_{c\max} = \frac{A_s \bullet f_y}{f_t} \qquad (3)$$

where, A_s is the area of steel bar, f_y is yielding strength of steel bar.

In two dimensional computation, it is more convenient to use the relation of bond effect zone height (h, $A_{cmax}=h^2$) and steel bar's diameter,

$$h = K \bullet d_b \bullet \sqrt{\frac{f_y}{f_t}} \tag{4}$$

where: d_b is the diameter of steel bar; K equals to $\sqrt{\pi}/2$ in the case of deformed bar with enough covering concrete. The total height of RC zone for one beam can be calculated by the arrangement of steel bar, the thickness of covering concrete and the height of bond effect zone of each steel bar(Fig.4b).

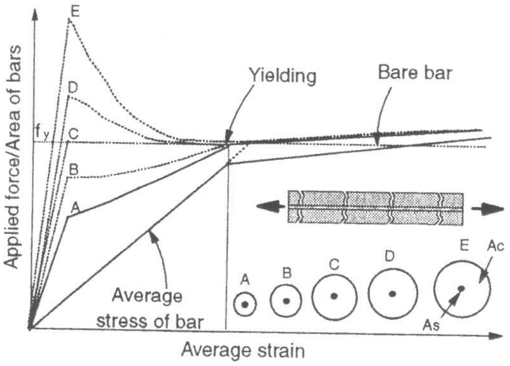


Fig.4a Crack control capacity and concrete area

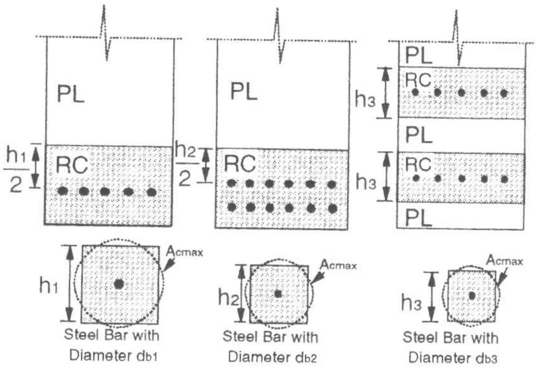


Fig.4b RC zone definition in computation

4. SIZE EFFECT SIMULATION ON SHEAR STRENGTH OF LARGE RC BEAMS

4.1. EXPERIMENTAL OUTLINE

The size effect experiment conducted by Iguro and Shioya[1] consists of RC beams without shear reinforcement of different depths "*d*"; 10cm, 20cm, 60cm, 100cm, 200cm, 300cm as shown in Fig.5. The ratio of loading span "*l*" to "*d*" is to be l/d=12 in order to give a value close to the lower limit of strength. The main reinforcement ratio in the vicinity of supporting point where shear failure would occur is defined as 0.4%. The beams were loaded by uniformly distributed hydraulic pressure until failure. The observed failure modes were flexural failure for beam No.1 and No.2 and the shear failure for the other beams left.

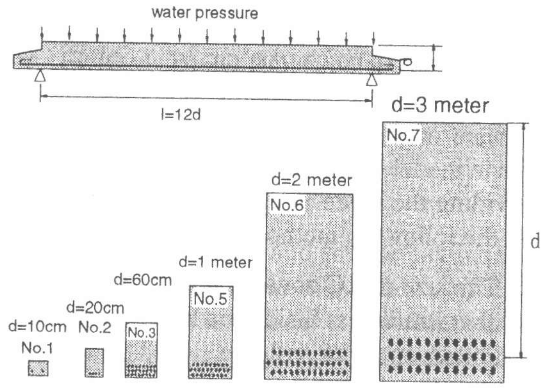


Fig.5 Details of cross sections and arrangement of reinforcing bars

4.2. ANALYTICAL RESULTS

The reinforced concrete beams without shear reinforcement was analyzed using the program **WCOMR**, in which the discussed models are adopted.

(1) Load deflection relation

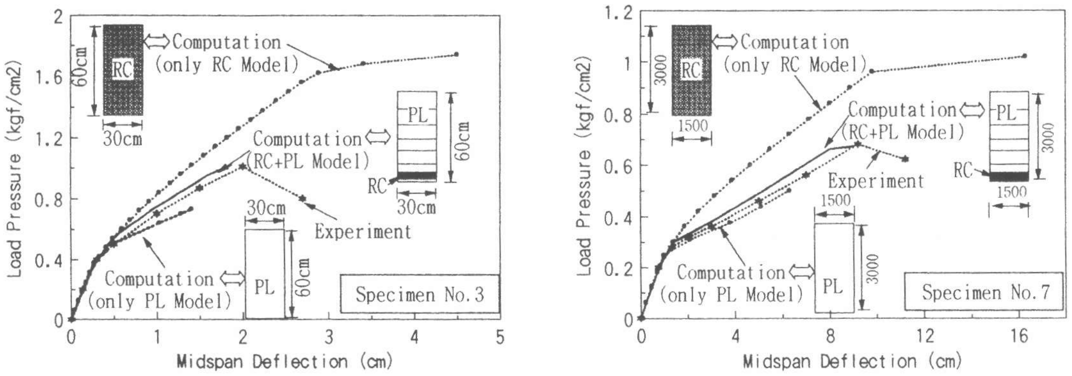


Fig.6 Load-deflection relation for different specimen at midspan

It can be seen in Fig.6 that, when the proposed plain concrete(PL) model is combined with the RC model in the computation, the analytical results are very close to the experiments. If only the RC model is used, the analytical results are much higher than the experiment data, and if only the plain concrete model is used, the beam will fail with much lower load comparing to the real results. These analysis can also show that the stiffening part of the cracked concrete model can affect the shear capacity, obviously.

(2) Size effect on shear strength

Fig.7 shows the shear strength results of the comparison between analytical results and experimentals. The lines in this figure show the calculated shear strengths which are obtained by applying standard codes of ACI and Okamura-Higai equation[7]. According to this figure, the analytical results which have good agreement with the experiment results, show the tendency of shear strength to decrease with an increase in beam size. This agreement ascertained the validity of proposed cracked concrete model and capability of the computational scheme to predict the size effect on shear strength.

The specimen No.1, and No.2 failed due to flexural mode, and the experimental results are higher than the analytical results. In fact, the tested bending capacity for these two small beams is much higher than the bending capacity which is computed by RC beam theory of cross section. It is guessed that the loading membrane made of rubber would act as fiber resistant component and affected the experimental results. Actually, the program can simulate the shear capacity of small beams very well according to many computation examples[5].

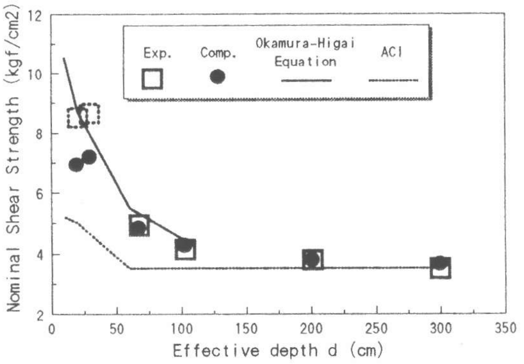
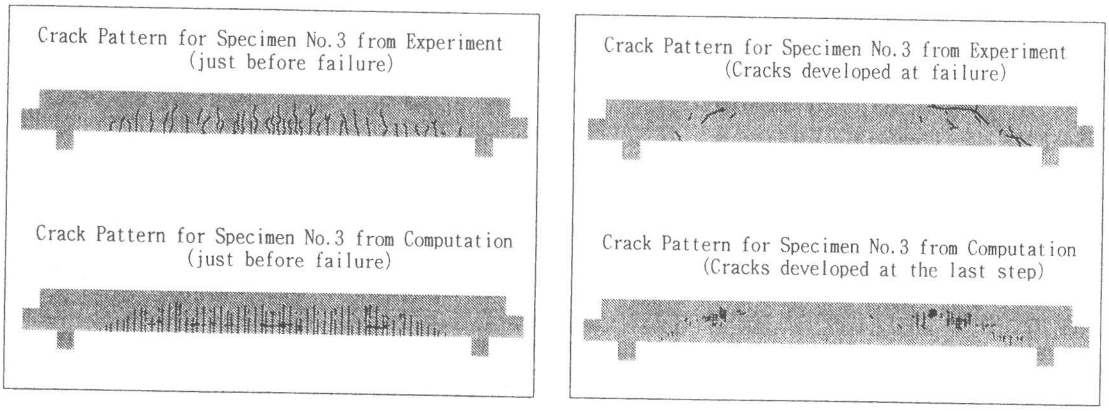


Fig.7 Shear strength and effective depth

(3) Crack pattern

Fig.8 shows the crack pattern of specimen No.3. The short lines in computed crack pattern picture should not be associated with individual discrete cracks. They represent the smeared cracks, whereas the thickness indicates the magnitude of opening of the cracks and from the direction of the short lines, the shear cracks and tension cracks can be distinguished. In order to confirm the failure mode and location, only the cracks occurred in the failure step are drawn in the right side of the figure. It can be seen that the computation can get the shear cracks almost at the same place with the experiments and the crack opening is bigger near the right side support. Also the cracks which developed at failure in the experiment are similar to the cracks which occurred in the last computation step. It shows that the program can simulate the sudden shear failure successfully, in the unstable propagation mode of diagonal cracks.



Crack Pattern for Specimen No.3 from Experiment (just before failure)

Crack Pattern for Specimen No.3 from Computation (just before failure)

Crack Pattern for Specimen No.3 from Experiment (Cracks developed at failure)

Crack Pattern for Specimen No.3 from Computation (Cracks developed at the last step)

Fig.8 Crack patterns for specimen No.3

5. CONCLUSIONS

The shear behavior of reinforced concrete beams without shear reinforcement was investigated using the program WCOMR. From this study, it was concluded that even for the very large beam, the shear behavior of reinforced concrete beam without shear reinforcement can be predicted well by the proposed concrete models applied to both RC and plain concrete domains.

It can be noted that the size effect in shear of reinforced concrete beams can be analytically shown by using the finite element method. The shear capacity which is predicated by the finite element method shows good agreement with the result of experiment. Both of the analysis and the experiment exhibited that the shear strength behavior of reinforced concrete beam is generally affected by the beam size. The shear strength is gradually decreased as the depth of beam increases.

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