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論文 Influences of the Position of the Inflexure Points on the Ultimate Shear Capacity of Indeterminate Reinforced Concrete Beams

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ABSTRACT: This paper summarizes the test results of reinforced concrete indeterminate beams without shear reinforcements with the different values of shear-span-to-depth ratios. It was found that, due to the combined effects of the inflexure points and the indeterminacy, there were the increases of the ultimate shear capacity of the indeterminate beams compared with that of the determinate beams. Finally, the comparison between the test results and the estimated values of the ultimate shear capacity by the different practical design formula was discussed.

KEYWORDS: shear, inflexure point, reinforced concrete beam, estimation of ultimate shear capacity, indeterminacy.

1. INTRODUCTION

In the reinforced concrete beams of indeterminate structures, there exist the inflexure points (the points of zero moment) within the span of the beams which have a major effect on the shear capacity of the beams [1]. In case of the indeterminate beams, the distribution of the internal actions, i.e. shear forces and bending moments are quite different from the determinate beams. In this case, a shear capacity is affected by the inflexure points as well as by the indeterminacy of the beams. However, most of the design codes adopt the design formula which was obtained by the tests on simply-supported beams subjected to two-point concentrated loading [2]-[4]. Hence, the codes seemed to neglect the effects of inflexure points and indeterminacy on the shear capacity. In the past, very few studies on the effects of inflexure points on shear strengths of the determinate beams were performed [1], [6], [7], and almost none on the indeterminate beams.

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In this study, the experimental investigation has been conducted on the reinforced concrete two-span continuous beams without shear reinforcement. A practical design method proposed by Aoyagi et. al. was derived by modifying the JSCE code [1]. The shear capacities of the tested beams obtained by the formula proposed by Aoyagi et. al. were compared with the experimental results in order to verify the applicability of the proposed formula in comparison with other current practical design formula such as JSCE, ACI codes and Australian Standard (AS) [2]-[4].

2. OUTLINE OF THE EXPERIMENT

A total of ten specimens were tested in the present study. Eight indeterminate continuous beams were subjected to two concentrated loads within each span (Fig. 1) with the different positions of the inflexure points, and two basic beams of a determinate type with no inflexure points in the shear span were subjected to a concentrated load at the mid-span [5]. The dimensions of the beams are 17 cm × 29 cm section and 180 cm span length.

The ratios of the maximum equivalent shear span (a') to depth (d) of the beam were within the range of 1.5 ~ 3.5, where the maximum equivalent shear span is defined as a maximum value between the distance from the nearest loading point to the inflexure point and that from the nearest support point to the inflexure point (see fig. in Table 1). The concrete compressive strengths were within the range of 270 ~300 kgf/cm^2. The main reinforcement ratio was 2.5%, and the steel yield strength was 4000 kgf/cm^2. This reinforcement was designed so that the beams would not fail by a flexural mode. In order to localize the shear failure occurring only in the portion between the left loading point and the mid-support, the beams were reinforced with the stirrups in other portions except this left portion of the mid-support (Fig. 2).

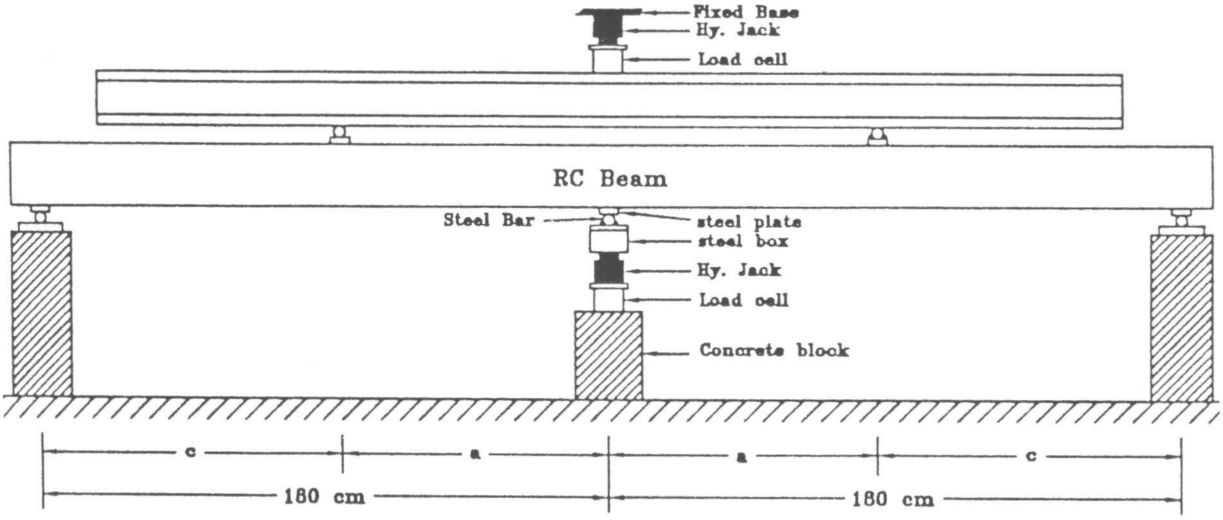


Fig. 1 Loading system

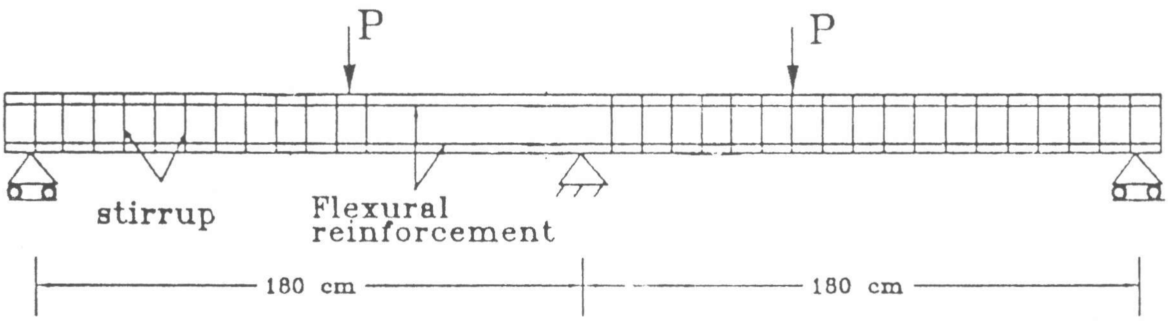


Fig. 2 Details of tested continuous beam

3. EXPERIMENTAL RESULTS

The experimental results showed that all the beams failed in shear. The crack pattern of a typical beam at the failure is shown in Fig. 3. The modes of failure along with the ultimate shear capacities of the indeterminate beams are presented in Table 1. Except the beam with a'/d=1.56 which failed in a tied arch mode, the failure mode of other beams with $a'/d \geq 2.2$ is a diagonal tension mode. As can be seen in Table 1, the large increases of the shear capacities of the indeterminate beams in comparison of that of the determinate beam (the basic simply-supported beam) are caused by the combined effects of inflexure points and indeterminacy. It is noted that the increases of shear capacities in Table 1 were calculated by the ratio of the shear capacities of the indeterminate beams and that of the basic beams with the same shear span [5].

The effects of inflexure points on the increase of ultimate shear capacity can be explained by considering the formation of an imaginary hinge at the position of the inflexure point in the span. Due to this hinge formation, a total unsupported shear span can be divided into two equivalent shear spans, i.e. the distance from the inflexure point to the nearest support or the loading point. Since the beam with a shorter equivalent shear span can resist a higher shear force, shear failure will occur in the larger of the equivalent shear spans [1].

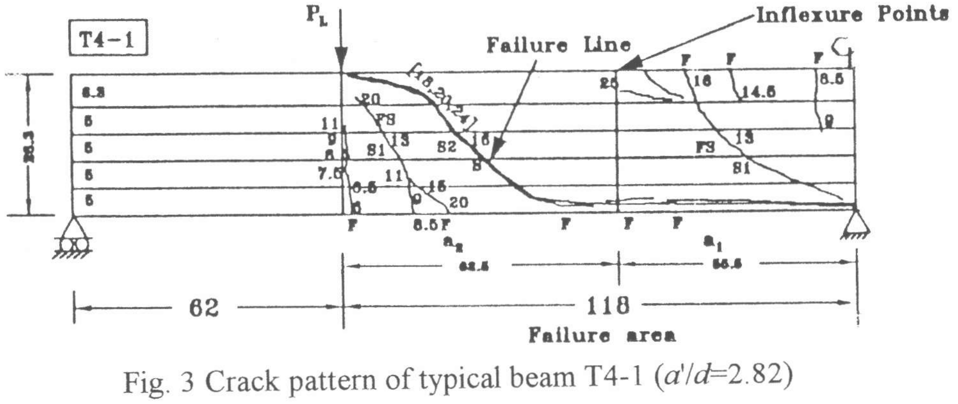
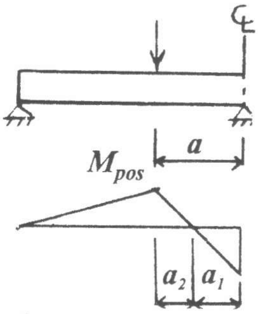


Fig. 3 Crack pattern of typical beam T4-1 (a'/d=2.82)

Table 1 Experimental results of the shear capacities of the indeterminate beams

a'/d	a/d	Ultimate shear capacity (kgf)	Increase of shear capacity	Average increase	Modes of failure
1.56	2.24	14664	137%	137%	tied arch
2.2	4.04	7312	48%	45%	diagonal tension
2.35	4.71	6925	47%		diagonal tension
2.82	5.29	6568	44%		diagonal tension
3.48	6.05	6105	38%		diagonal tension



$a' = \mathbf{max.}(a_1, a_2)$

Note : a' is the maximum equivalent shear span defined as a maximum value between the distance from the nearest loading point to the inflexure point and that from the nearest support point to the inflexure point; a is the shear span, i.e. the distance from the loading point and the mid-support point; d is the depth of beam section.

For the effects of an indeterminacy on the increase of an ultimate shear capacity, it was observed from the experimental results of the indeterminate beams that the sufficient opening and propagation of two major shear cracks in two equivalent shear spans is necessary, and this causes the delay of a shear failure. The effects of the indeterminacy could be explained by another reason. It was observed from the relationship between the applied load and the tensile strain of longitudinal bar within the failure span and another span (left or right span from the middle support as can be seen e.g. in Fig. 3 the failure span is the left one) that, after the formation of the shear cracks the longitudinal tensile strain within the failure span was less than that within another span. This indicated that a release of bending moment occurred within the failure span and resulted in more shear resistance. The above two phenomena indicated the severe redistribution of internal actions after the formation of the shear cracks due to the effects of indeterminacy [5]. A quantitative evaluation of these effects will be studied in the near future.

4. PREDICTION OF SHEAR CAPACITIES

By using the current JSCE formula and considering the fact that the presence of inflexure points artificially divides the shear spans into two equivalent shear spans, Aoyagi et al. proposed the design formula to predict the shear capacities of RC beams without shear reinforcements [1]. The experimental results were compared with the existing different equations (JSCE, ACI, AS3600) [2]~[4] and the proposed equation by Aoyagi, et al. in Fig.4. From the comparison in Fig. 4, it is clear that the proposed equation gives the best fit to the experimental results among the four equations. In case of the determinate beam (a'/d=4.0), the proposed equation by Aoyagi et. al is very close to the experimental result as also indicated in Ref. [1],[6]. However, in the indeterminate beam cases, the proposed equation gives rather conservative results. This might be because of the increase of shear capacities due to the effects of the indeterminacy which was excluded in the proposed equation.

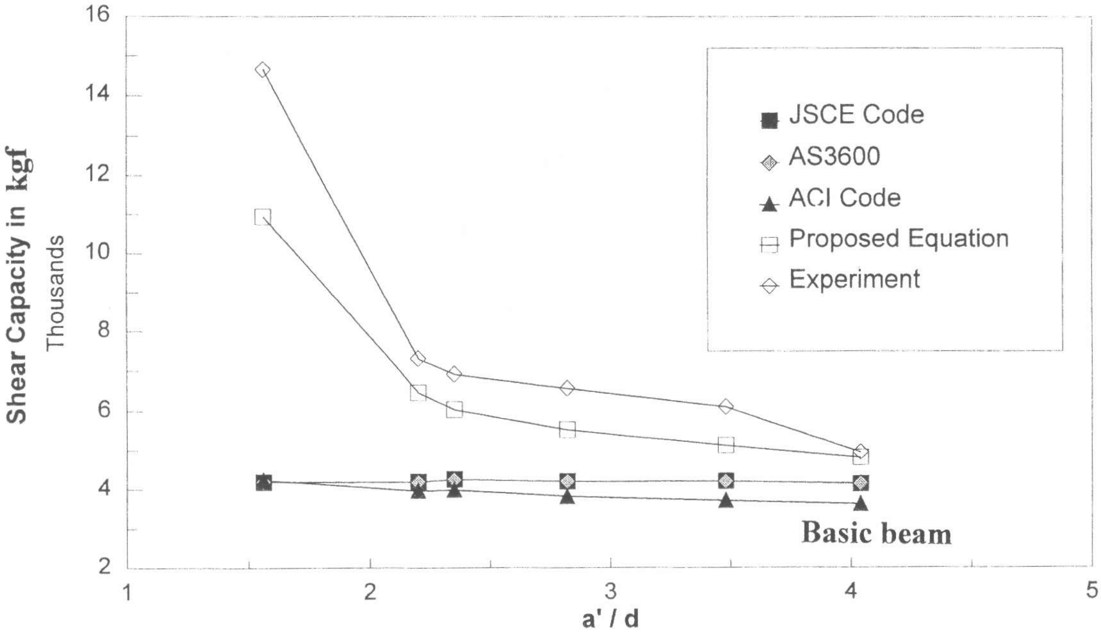


Fig. 4 Comparisons between shear capacities by experiments and different analytical methods

5. CONCLUSIONS

1) The positions of inflexure points have the large effects on the ultimate shear capacity of the reinforced concrete indeterminate beams without shear reinforcement. It can be assumed that at the position of inflexure point in the span an imaginary hinge is generated and divides the shear spans into two equivalent ones. Hence, the beam can take more shear capacity. It is found that, in case of indeterminate beams, as much as 137% average increase of shear capacity occurred in tied arch failure mode, while 45% average increase occurred in diagonal tension failure mode.

2) The current practical design codes (JSCE, ACI, AS) are very much conservative in predicting the ultimate shear capacities of the RC beams with the presence of inflexure points especially in the cases of the indeterminate beams with a shorter equivalent shear span-to-depth ratio.

3) By using the equation proposed by Aoyagi, et al., and considering the effects of inflexure points, the predicted shear capacities in all cases are closest to the experimental results compared with the three design formula selected in this study, i.e., JSCE, ACI and AS. In case of the indeterminate beams, the proposed equation predicts somewhat conservative values.

CONVERSION FACTORS

1 kgf = 9.807 Newton

REFERENCES

1. Aoyagi, Y., Endo, T., "Ultimate Shear Capacity of Continuous Beams Subjected to Distributed Loading", Proceedings of the Fourth East Asia-Pacific Conference on Structural Engineering and Construction, Seoul, Sep.1993, pp. 727-732.
2. Japan Society of Civil Engineers "Standard Specification for Design and Construction of Concrete Structures, Part 1. (Design)", 1986.
3. American Concrete Institute, "Building Code Requirements for Reinforced Concrete (ACI318-89)", ACI Manual of Concrete Practice, 1992.
4. Australian Standard, AS3600-1988, "Concrete Structures by Standard Association of Australia", 1988.
5. Md. Mahmudur, R., "Effects of the Position of the Inflexure Points and Indeterminacy on the Ultimate Shear Capacity of Indeterminate Continuous RC Beams without Shear Reinforcements", Master Thesis, Asian Institute of Technology, April 1995.
6. Aoyagi, Y., Chaisomphob, and Shahidul, I., "Effects of the Position of Inflexure Points on the Ultimate Shear Capacity of Reinforced Concrete Beams", Proceedings of the Japan Concrete Institute, Yokohama, Japan, June 1994, pp. 593 - 598.
7. Chaisomphob T., Aoyagi Y. and Ching-Chiung, C., "Effects of the Position of Inflexure Points on the Ultimate Shear Capacity of Reinforced Concrete Beams with Shear Reinforcements", Proceedings of the Japan Concrete Institute , Hiroshima, Japan, June 1995, pp. 815 -820.