論文 Improvement of Structural Performance by Optimal Shape Design Using Steel Fiber Reinforced Concrete

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ABSTRACT : According to the state-of-art development of researches on structural optimization related to the civil structures, it is indirectly revealed that a really optimized structure should be not only emphasized on optimizing their structural configuration, topology and geometry (macroscopic approach) but also on strengthening the constituent material (microscopic approach). Thus, this study was motivated to investigate the possibility of the structural optimization by considering the variation on structural shape and the steel fiber volume on SFRC structures. By experimental observations, the possibility and necessity of the structural optimization by the both approaches were verified.
KEYWORDS : Optimal Shape Design, Biological-Growth Strain Method, Steel Fiber Reinforced Concrete, Toughness

1. INTRODUCTION

Because of its excellence on improving the ductility of the design structure, steel fiber reinforced concrete (SFRC) has been accepted as a type of construction material in many practical applications. In accordance with the present development of researches on SFRC, it is understood that the strength performance of structures using SFRC directly depends on fiber geometry , fiber volume, fiber dimension, fiber surface property and fiber type. In meeting with the practical needs, many theoretical and experimental research have been attempted to improve the economic value and mechanical property of SFRC by effectively utilizing the aforementioned material parameters. On the other hand, along with the development of structural optimization, it is gradually clarified that a good structure is not only strengthened by its constituent material (i.e. by high strength material or by composite material at their combination) but also influenced by its structural configuration , topology and geometry. However, for the researches of structural optimization related to the civil structures in the past decades, all of them were emphasized only on optimizing their structural configuration, topology and geometry (macroscopic approach) or on strengthening the constituent material (microscopic approach). Until now, few of them has been conducted from the viewpoint with the care on both approaches.

Hence, this study was motivated to investigate the possibility and the necessity of the structural optimization by the above both approaches. In the following context, the algorithm of the gradientless optimization technique called BGS (Biological Growth Strain Method) and the experimental investigation on structural performance of plain concrete (OPC) and SFRC framed structure with the consideration on shape variation and fiber fraction volume were individually described. Then, the possibility and necessity of simultaneously adopting the above both approaches was discussed.

2. CHARACTERISTICS OF BIOLOGICAL GROWTH STRAIN METHOD

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As stated earlier, for optimally designing the structures at the macroscopic approach, one optimal shape design method proposed by the authors [1] was utilized to design the structures. Compared to other conventional techniques of structural optimization, one of the features of this proposed method was its gradientless calculation. The design objective of this method is to minimize the stress concentration within the design structure, which can be formulated as

$$\mathrm{Min}\left\{\sum_{(x_i,y_i),i=1..n}\left(\bar{\sigma}_i-\sigma_{ref}\right)^2\right\} \tag{1}$$

Here, the design variables are the coordinates (x_i ,y_i) of the design points selected along the design profile Γ and the equivalent stress along Γ is $\bar{\sigma}_i$, where i = 1..n. The reference stress is σ_{ref} . The algorithm of BGS was originated from the simulation on the biological adaptation to their loading environment like trees or bones; i.e. they change their shapes by the growth or trophy of the living tissue near the highly stressed area. Based on this concept, one parameter called biological growth strain $\{\varepsilon_k^B\}$ was defined as follow

$$\{\varepsilon_k^B\}_j=\begin{Bmatrix}\varepsilon_1^B & 0\\ 0 & \varepsilon_2^B\end{Bmatrix}_j$$

$$=\begin{cases}\text{for ductile material: } \dfrac{\sigma_j-\sigma_{mean}}{\sigma_{mean}}\nabla h, \text{ if element } j\in\Gamma^*\\ \text{for brittle mateiral: } \dfrac{\sigma_j-\sigma_{mean}}{\sigma_{mean}}\left|\dfrac{\sigma_k}{f_s}\right|\nabla h, \text{ if element } j\in\Gamma^*\\ \{0\} \quad , \text{if element } j\notin\Gamma^*\end{cases} \tag{2}$$

where j is jth design element within design domain Γ^*; σ_j is equivalent stress within jth design element; σ_{mean} is mean of equivalent stress of all the design elements; σ_k is kth principal stress, k=1..2 ; f_s is uniaxial compressive or tensile strength; ∇h is constant for search step. With the introduction of this parameter, the shape of the design structure can be changed by means of updating vectors of nodes coordinates (i.e. the shrinking or swelling of the design elements) , which is governed by

$$[K]\{u\}=\{\Delta g\} \tag{3}$$

where $[K]$ is the global stiffness matrix and $\{u\}$ is the nodal coordinate updating vectors. The equivalent nodal force vector $\{\Delta g\}$ is derived as

$$\{\Delta g\}=\int_{\Omega_e}[B]^T[D]\{\varepsilon_k^B\}\partial\,\Omega_e \tag{4}$$

where $[B]^T$ is the transpose of nodal displacement-strain matrix and $[D]$ is the elastic stiffness matrix. By substituting eq.(4) into eq.(3), the nodal coordinate updating vectors $\{u\}$ can be attained by solving the global governing equation (i.e. eq.(3)) without calculating the gradient of the objective function or constraint conditions. Therefore, the new shape of the structure is obtained by adding the nodal coordinate updating vectors to the old coordinates of nodes. Then, by iteratively repeating the above procedure, the design process can be regarded at its optimum when the condition in eq.(1) is satisfied.

3. DETAILS ON EXPERIMENTAL INVESTIGATION

For verifying the possibility of the structural optimization with both macroscopic and microscopic approaches, two kinds of material (OPC and SFRC) were used for the design by BGS. To design the structure by BGS, the strength parameters of the material were needed. Besides characterized as its gradientless algorithm, the consideration on the effect of strength ratio (i.e. Fc : Ft) is also the feature of BGS. Based on this feature, for designing two-dimensional structures, the biaxial compressive and tensile strength are necessary to be given. However, the difference between elastic limit of uniaxial and biaxial strength can be practically assumed to be ignored. As a result, all the strength parameters considered in this study were performed on uniaxial tests.

3.1 TEST FOR STRENGTH PARAMETER AND NUMERICAL ANALYSIS

The mix proportions for OPC and SFRC in this study were decided as shown in Table 1, where V_f, W, C, S and G represented fraction of fiber volume, water, Portland cement, fine aggregate and coarse aggregate respectively. The preparation of OPC strength specimen was in accordance with JIS A 1132. As for SFRC, to obtain better workability with the different fraction of fiber volume ranging from 0.5, 1.0 and 2.0% of total batch volume, the placing of SFRC followed the suggested procedure in [2]. The type of steel fiber was STIEBER Deformed fiber with 0.5 × 0.5 × 30 mm, which is the product of Nittetsu Corporation. All the specimen were cured in 20±1°C water for 28 days. As mentioned earlier, the strength parameters needed in the design process of BGS included elastic limit for uniaxial compressive (Fc) and tensile strength (Ft), elastic modulus (E), Poisson's ratio (ν) and specific weight (γ). So far, the test methods for the aforementioned parameters have been standardized except the test method for tensile strength of SFRC. However, based on our experimental project, the tensile strength of SFRC was not necessary. The test procedure to obtain the uniaxial compressive strength followed JIS A 1108. Besides, the tests for elastic modulus and Poisson's ratio were carried out according to ASTM 469-65. Then, by checking the relationship of stress and strain, the elastic limit of compressive strength could be decided. Here, the elastic limit of uniaxial tensile strength ($F_{t,sp}$) was defined by offsetting 60% strength of split tensile strength (JIS A 1113). All the results of these tests were concluded in Table 2. After obtaining the required strength parameters, the design procedure could be undertaken by BGS according to the initial data (the strength parameters, structural configuration and the boundary conditions). The structure was assumed to be in the state of plain stress. The type of loading was two-point concentrated loading and the boundary condition was two-end fixed. The measurement setup of the original shape of specimen for verification was illustrated in Fig.1. In this study, the optimal shape design process was only considered on OPC. Based on this same optimal shape, with the variation on the fraction of fiber volume, the effect of shape and fiber on structural performance could be investigated respectively. With the consideration on the convenience of loading, only the free-loading profile (i.e. the inner

Table 1 Mixing Proportion

V_f	W	C	S	G_{5-13}	G_{13-15}
0.0%	207	414	842	650.5	278.8
0.5%	218	436	836	611	262
1.0%	230	460	976	464.9	199.3
2.0%	251	502	1235.5	196.4	84.2

(unit:kg/m³)

Table 2 List of Strength Parameter

V_f	Fc*	$F_{t,sp}$*	E(x10⁴)*	ν	γ(x10⁻³)**
0.0%	42.05	2.55	3.20	0.2	2.40
0.5%	41.77	***	3.17	0.2	2.42
1.0%	43.37	***	2.89	0.2	2.40
2.0%	40.02	***	2.59	0.2	2.38

(* - unit : Mpa ; ** - unit : kg/m³; *** - lack of data)

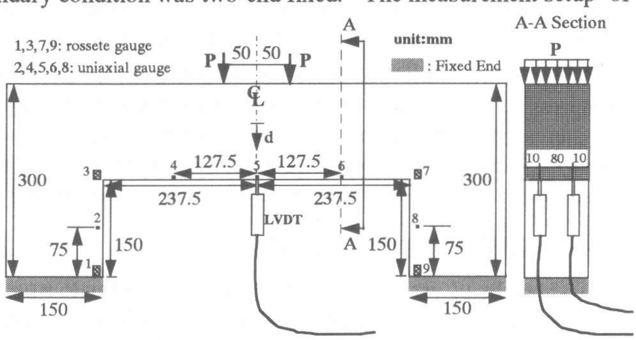

Fig.1 Measurement Setup of the Specimen - Original Shape

rim of the optimized shape) was assigned as the design profile. With the constraint on the framed structure was shown in Fig.2 with the same loading and boundary condition. The converge of objective function and the minimization of stress distribution along the design profile were indicated in Fig.3 (a) and (b) respectively.

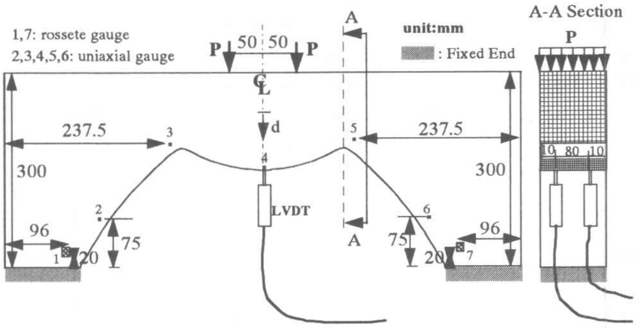

Fig.2 Measurement Setup of the Specimen - Optimized Shape

3.2 TEST FOR EFFECT OF SHAPE VARIATION AND FIBER REINFORCEMENT

In the design process of BGS, the optimal shape of the design structure is found out under a certain loading level within elastic limit. However, the inelastic and postpeak behavior of the optimized structure cannot be clarified at the elastic design stage. As a result, the experimental investigation was utilized to realize the inelastic and postpeak behavior of the design structures. As mentioned earlier, under the same shape of original and optimal structures which was optimally designed according to the strength parameter of OPC, the fiber volume of steel fibers ranged from 0, 0.5, 1.0 and 2.0% of total batch volume. For each fraction of fiber volume, 3 specimen including original and optimized shapes were separately cast. The observation parameters in the tests included (i) load, (ii) central deflection and (iii) strain along the design profile. To remain the design structure in static state, the loading rate was set on 0.5 kN per minute. The setup for measuring central deflection (d) and strain distribution along the design profile (at the specified points) was also schematized in Fig.1 and 2 individually. For realizing the fracture resistance, the flexural toughness of the framed structure was defined as the area under the relationship curve between external load and central deflection up to 3mm. The reason for deciding the measured deflection up to 3mm was to avoid the crush failure of the column feet under excessive compressive load. However, this parameter (flexural toughness) was unavailable for OPC framed structures because the failure of the structures abruptly occurred at the reach of the maximum strength. This situation was represented by asterisks(***) in Table 3. Besides, the two-end fixed condition was achieved by clipping the feet of the framed structure with the auxiliary apparatus.

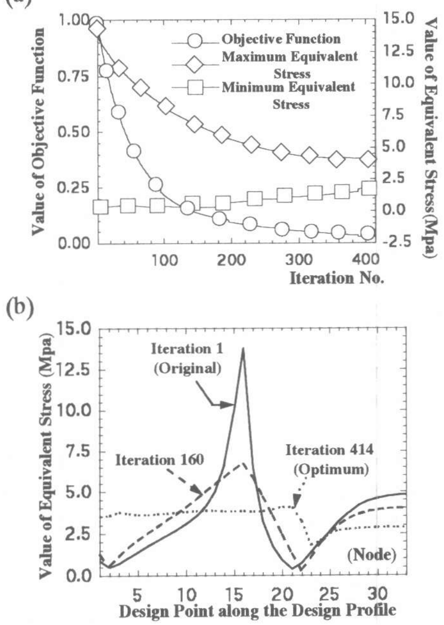

Fig.3 (a) Variation on Objective Function , Maximum and Minimum Equivalent Stress; (b) Minimization of Stress Concentration Along the Design Profile

4. RESULTS AND DISCUSSION

The results of verification tests including (i) ultimate strength (P_{mu}), (ii) elastic limit (P_{me}) and (iii) toughness (T) were concluded in Table 3. In Table 3, Avg means the average value of the results of 3 specimen. Unavailable results are represented by asterisks(***), which resulted from the prefailure of specimen before the loading. To find out the elastic limit of the design structure, the point of elastic limit was assumed to be the point of the initial deviation occurring in the linear load-deflection relationship. From Table 3, the structural performance parameters including elastic limit, ultimate strength and toughness were effectively improved by the variation of structure shape. For example, the average toughness of original SFRC structures with 2% fiber volume was close to that of SFRC structures by varied shape with 0.5% fiber volume. Furthermore, the relationship between loading response and central deflection (d) for each fraction of fiber volume by averaging the available data were depicted in Fig.4. From Fig.4, the significant enhancement on the strength and ductility of the design structure by varied shape could be clearly observed. Therefore, it was experimentally verified that the ultimate strength of the design structure is improved with the improvement of the elastic limit of the design structure by optimal shape design. However, as shown in Fig.4(a), the abrupt failure in the postpeak stage of OPC couldn't offer the ductility even with the optimal shape design.

Table 3 Results of Verification Tests

Original Shape	No.	P_{mu} (kN)	AvgP_{mu} (kN)	P_{me} (kN)	AvgP_{me} (kN)	T (kN-mm)	AvgT (kN-mm)
Vf=0%	1	42.7	40.7	13.2	12.5	****	****
	2	***		***		****	
	3	38.8		11.8		****	
Vf=0.5%	1	42.0	47.2	14.7	15.9	77.3	85.2
	2	48.2		15.4		87.4	
	3	51.5		17.6		91.0	
Vf=1%	1	67.9	64.5	21.4	21.0	144.1	134.4
	2	***		***		***	
	3	61.0		20.7		124.7	
Vf=2%	1	73.5	77.6	25.1	26.4	167.0	186.9
	2	79.2		26.8		196.6	
	3	80.0		27.2		197.1	

Varied Shape	No.	P_{mu} (kN)	AvgP_{mu} (kN)	P_{me} (kN)	AvgP_{me} (kN)	T (kN-mm)	AvgT (kN-mm)
Vf=0% (Opm)	1	71.7	67.5	24.1	21.8	****	****
	2	68.7		21.6		****	
	3	62.2		19.7		****	
Vf=0.5%	1	86.3	76.6	27.1	24.8	189.2	178.2
	2	54.2		***		***	
	3	66.9		22.5		167.1	
Vf=1%	1	103.6	121.7	34.0	40.2	217.0	224.4
	2	137.6		45.6		201.8	
	3	123.8		41.1		254.5	
Vf=2%	1	132.0	147.1	42.5	48.9	244.7	293.5
	2	162.3		55.3		342.2	
	3	***		***		***	

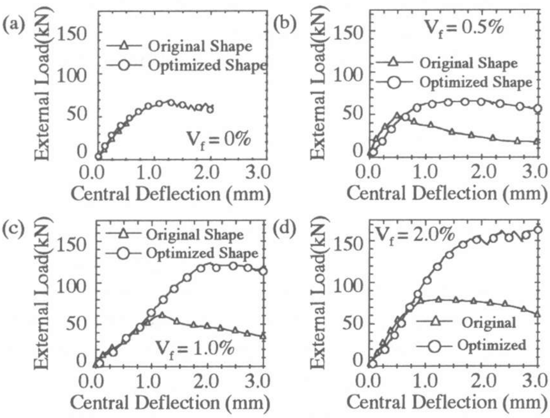

Fig.4 External Load - Central Deflection Relationship

As a result, the enhanced ductility shown in Fig.4(b),(c) and (d) should be considered as the effect of fiber reinforcement in stead of the effect of shape variation. For understanding the effect of fiber reinforcement in the stage of optimal shape design, the relationship between external load within elastic limit of original shape and the standard deviation of the measured strain at the specified points for each fraction of fiber volume were given in Fig.5. By Fig.5(a), the effect of minimization of stress concentration could be observed because the strain deviation of the optimized structure was always smaller than that of the original structure up to the elastic limit of the original shape ; that is , the experimental observation corresponded to the design objective. On the other hand, for SFRC, based on the same optimized shape designed by the strength of OPC, the inconsistency between the experimental observation and the design objective was reflected in Fig.5(b) and (d) while Fig.5(c) indicated the consistency. As known, the role of fiber is used for controlling the propagation of microcracks, which initiate after the first crack. Thus, the inconsistency within elastic limit should not be attributed to the effect of fiber reinforcement because the elastic limit is generally defined as the generation of first crack; namely, the effect of fiber reinforcement couldn't be thought effective in the stage of optimal shape design within elastic limit. Besides, the influence from the effect of fiber reinforcement on the improvement of structural performance was discussed. Here, increasing rate was used for reflecting the improvement of structural performance by defining the value of the strength parameter of the design structure with its varied shape divided by that of the original structure for different fiber volume ,which were tabulated in Table 3. The relationship between increasing rate and fraction of fiber volume was illustrated in Fig.6. From Fig.6, the following facts could be observed : (i) although the absolute value of toughness increased with the increment of fiber volume as listed in Table 3, the increasing rate decreased with the increment of fiber volume. (ii) For OPC, the increasing rate of elastic limit was larger than that of ultimate strength, which agreed with the observation on Fig.5(a) because the optimal shape design was carried out only within elastic limit of OPC (iii) With the inclusion of fibers, the increasing rate of elastic limit was larger than that of ultimate strength at 0.5,2.0% while smaller at 1.0% , which fitted the observation in Fig.5(b), (c) and (d). The reasons could be regarded as (i) ignorance of uniqueness of optimality (ii) improper combination of use of fiber and optimal shape design. It revealed that the structural performance could be obviously improved by either macroscopic approach (i.e. optimal shape design) or the microscopic approach (i.e. the use of fibers) from these observation. In addition, based on these observation , it was found that , without properly incorporating two approaches in the design stage, the optimum of the structural performance could be hurdled.

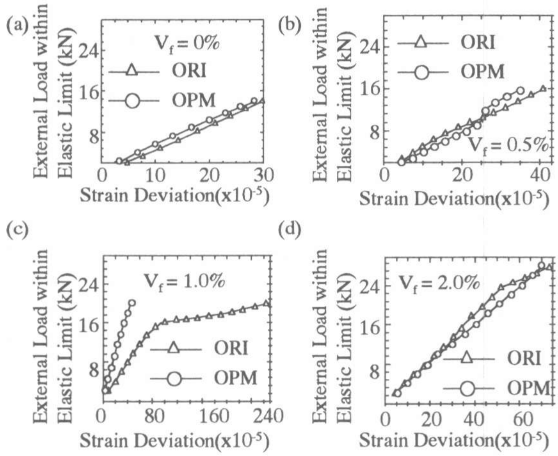

Fig.5 Relationship Between External Load and Strain Deviation at All Specified Points within Elastic Limit of Original Shapes

5. CONCLUDING REMARKS

By the observation and discussion in this experimental investigation, the findings could be summarized as follows : (i) the load-carrying capacity and the ductility for the optimized structures were effectively improved by shape variation (ii) shape optimization and fiber reinforcement should be simultaneously considered in order to achieve the best strength performance in stead of merely the microscopic or macroscopic approach. Based on the above experimental observations, the possibility and necessity of the structural optimization by the both approaches were verified.

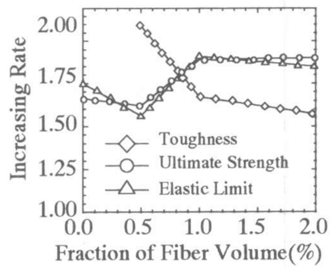

Fig.6 Relationship Between Increasing Rate and Fraction of Fiber Volume

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