論文 Model for Predicting 28 Day Compressive Strength of Fly Ash-Concretes

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ABSTRACT: A mathematical model is presented to predict 28 day strength of fly ash-concretes. The model takes into account the chemical composition and fineness of both cement and fly ash. The strength of fly ash-concrete is found resulting of the two mechanisms interacting with each other. First of these is the hydration of two silicates, C_3S, and C_2S, and second is the pozzolanic reaction. It is postulated that whole of the strength of fly ash-concrete is resulted from the formation of tricalcium-disilicate-hydrate, $C_3S_2H_3$. The model predictions agree well with the data published by several researchers.

KEY WORDS: Chemical Composition, Fineness, Hydration, Pozzolanic Reaction, Compressive Strength, Volume of CSH Gel.

1. INTRODUCTION

The strength development of Portland cement-concretes is a well understood phenomena. It is generally agreed that strength comes from CSH gel which is formed due to the hydration of C_2S and C_3S. It is further established that at early ages strength is mostly due to the hydration of C_3S and it is only at late ages that C_2S contributes significantly to the strength of concrete. The role of the other two compounds C_4AF and C_3A in strength development is of a secondary nature and these two compounds indirectly affect the rate of strength contribution of C_2S and C_3S.

As far as strength development of fly ash concretes is concerned it is believed that it is resulted of the two mechanisms. First of the mechanisms is hydration of cement and second is pozzolanic reaction of silica from fly ash and calcium hydrate released due to the hydration of silicates from cement. These two reactions are not independent of each other. In this paper it is tried to relate the twenty eight day strength of fly ash concretes to the characteristics of mixture composition and chemical composition and fineness of both of cement and fly ash.

2. EXPERIMENTAL DETAILS

Total of 135 concrete mixtures were cast using same set of aggregate. Three fly ashes of wide ranged chemical composition and fineness were incorporated. A range of water-binder and fly ash-binder ratios were combined to cast concrete mixtures with binder contents of 350, 400 and 450 Kg/m^3. Cylinder specimens of 100x200 mm size were tested after 3, 7, 28, 90, 180 and 365 days of curing. About 150 concrete mixtures were taken from other researchers (3-11) to calibrate and verify the model.

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3. FORMULATION OF THE MODEL

In this section derivation of the model is presented. Chemical compositions of cement and fly ash along with fineness of both the materials are taken into account.

3.1 BASIS OF THE MODEL

Feret [1] presented the following law to relate strength and other characteristics of the cement paste:

$$f'_c = A.\left[\frac{V_c}{V_c + V_w + V_v}\right]^2 \tag{1}$$

where A is a constant and V_c, V_w and V_v are volumes of cement, water, and voids respectively in mortar. Larrard [2] enriched the Feret's Law with Maximum Paste Thickness, (MPT) as in the following to predict 28 day strength of Portland cement concrete:

$$f'_c = A\ MPT^B.\left[\frac{V_c}{V_c + V_w + V_v}\right]^2 \tag{2a}$$

where MPT is given by the following expression [2]:

$$MPT = D_{max}\left[\sqrt{\frac{g^*}{g}} - 1\right] \tag{2b}$$

in which D_{max} is the Maximum Size of Aggregate, g* the packing density of aggregate and g the volume of aggregate per unit volume of concrete. In present investigation it is postulated that strength of fly ash concrete is totally dependent on volume of CSH gel. The 28 day compressive strength is expressed as:

$$f'_c = 283\ MPT^{-0.126}.\left[\frac{V_h}{V_c + V_w + V_v}\right]^2 \tag{3}$$

where V_h is volume of solid fraction of CSH gel, which is given by the following expression:

$$V_h = \sum_{i=2}^{3} HydV_i + \sum_{i=1}^{3} PozV_i + a.r_w + \sum_{i=1}^{3} b_i.r_F^{\ i} \tag{4}$$

where $HydV_2$ and $HydV_3$ are solid fractions of volume of CSH gel resulting from the hydration of C_2S and C_3S respectively, $PozV_1$ to $PozV_3$ are solid fractions of volume of CSH gel resulting from the pozzolanic reaction of silica and calcium hydrate released due to hydration of CaO in fly ash and C_2S and C_3S in cement respectively, r_w and r_F are respectively water-binder ratio and fly ash-binder ratio both by volume.

3.2 VOLUME OF CSH GEL

In the following a summary of hydration and pozzolanic reactions is given in cement chemist's notation:

$$3\ C_3S + 6\ H \rightarrow C_3S_2H_3 + 3\ CH \tag{5}$$

$$2\ C_2S + 4\ H \rightarrow C_3S_2H_3 + CH \tag{6}$$

$$C + H \rightarrow CH \tag{7}$$

$$3\ CH + 2\ S \rightarrow C_3S_2H_3 \tag{8}$$

Eqn. 7 gives combination of CaO in fly ash and mixing water. The CH produced from the hydration reactions described in Eqns. 5-7, combines chemically with the reactive S in fly ash at room temperature to form $C_3S_2H_3$. It can be verified by substituting the molecular weights of various substances in Eqns 4 to 8 that 1 gram of C_3S requires 0.24 grams of water for complete hydration during which it forms 0.75 grams of $C_3S_2H_3$ and releases 0.49 grams of CH which in turn requires 0.26 grams of reactive silica to form 0.75 grams of $C_3S_2H_3$. Similarly it can be shown that 1 gram of C_2S requires 0.21 grams of water for complete hydration during which it forms 1 gram of $C_3S_2H_3$ and releases 0.21 grams of CH. This requires 0.12 grams of reactive silica to form 0.33 grams of $C_3S_2H_3$. Thus silica demand of C_3S and C_2S is 0.26 and 0.12 times of their own weights respectively. It can be noted that as a result of complete hydration 1 gram of C_3S produces 0.75 grams of $C_3S_2H_3$ and C_2S produces 1 gram of $C_3S_2H_3$. Similarly complete pozzolanic reaction results in 0.75 and 0.33 grams of $C_3S_2H_3$ for each gram of C_3S and C_2S respectively. By substituting molecular weight in Eqns 7 and 8, it may be shown that each gram of C in fly ash requires 0.32 grams of water to release 1.32 grams of CH which in turn reacts with 0.71 grams of reactive silica to form 2.03 grams of $C_3S_2H_3$. Thus silica demand of CaO in fly ash is 0.32 times of its weight where as its strength imparting capacity is 2.03 times of its weight. Total silica demand, S_d, in a concrete mixture on the basis of complete hydration, can be written as in the following:

$$S_d = 0.26\ C_3S + 0.12\ C_2S + 0.71\ C_f \tag{9}$$

where C_3S and C_2S are weights of tricalcium and dicalcium silicates in Portland cement and C_f is weight of CaO in fly ash. The amount of actual silica present, S_p, in a mixture is equal to S in fly ash. The pozzolanic activities played by C_3S, C_2S, and C_f depend upon the availability of S for individual compounds. Let us denote the ratio $\left(\frac{S_p}{S_d}\right)^n$, by Silica Coefficient, S_{co}, where n is a power depending upon the reactivity of silica. The pozzolanic activity coefficients of C_3S, C_2S and C_f, denoted by a_3, a_2 and a_1 respectively may be computed as:

$$a_3 = \frac{0.26\ C_3S}{S_d} S_{co} \tag{10a}$$

$$a_2 = \frac{0.12\ C_2S}{S_d} S_{co} \tag{10b}$$

$$a_1 = \frac{0.71\ CaO}{S_d} S_{co} \tag{10c}$$

The solid fractions of volume of CHS gel produced due to hydration of C_3S and C_2S are proportional to 75% and 100% of their weights respectively and are computed in the following

$$HydV_3 = \beta_3\ \Omega_c\ (0.75\ C_3S)/G_C \tag{11a}$$

$$HydV_2 = \beta_2\ \Omega_c\ (1.00\ C_2S)/G_C \tag{11b}$$

where β_3 and β_2 are constants of regression and Ω_c is fineness index of cement defined by $\sqrt{\frac{F_c}{F_{co}}}$, with F_c being specific surface area, SSA, of cement under consideration and F_{co} being an arbitrary SSA. Similarly strength imparting indices of C_3S, C_2S and C due to pozzolanic reaction of S and CH are proportional to the product of their respective pozzolanic activity indices and available silica S. It may be shown that 1 part by weight of S can produce 2.85 parts by weight of $C_3S_2H_3$. Thus solid fractions of volume of CSH gel produced by three pozzolanic reactions may be written as:

$$PozV_3 = \gamma_3\, \Omega_F\,(2.85\, a_3\, S)/G_{CF} \tag{12a}$$

$$PozV_2 = \gamma_2\, \Omega_F\,(2.85\, a_2\, S)/G_{CF} \tag{12b}$$

$$PozV_1 = \gamma_1\, \Omega_F\,(2.85\, a_1\, S)/G_F \tag{12c}$$

where γ_1-γ_3 are constants of regression. Ω_F is fineness index of fly ash under consideration which is given by $\sqrt{\frac{F_F}{F_{Fo}}}$ where F_F is SSA of fly ash and F_{FO} is SSA of an arbitrary fly ash taken equal to 300 m^2/Kg. The terms appearing in the denominators of Eqn 12, G_C and G_F , are unit weights of cement and fly ash respectively and G_{CF} is unit weight of fly ash-cement binder calculated as in the following:

$$G_{CF}=0.65\ G_C+0.35\ G_F \tag{13}$$

It can be observed that G_{CF} is unit weight of the material combining chemically to produce CSH gel. This gel is composed of 65% of CH released from hydration of cement and 35% of S from fly ash. The volume of CSH gel is composed of $\sum HydV_i$ and $\sum PozV_i$ in addition to contributions from water-binder ratio by volume, r_w, and fly ash-cement ratio by volume, r_F, which are computed as in the following:

$$r_w = \frac{V_w}{V_c + V_{fa}} \tag{14a}$$

$$r_F = \frac{V_{fa}}{V_c} \tag{14b}$$

where V_c and V_{fa} are respectively the volumes of cement and fly ash per unit volume of concrete. All other symbols are already defined.

Table 1: Range of various parameters in concrete mixtures

Parameter	Range
Cement Content	53-480 Kg/m^3
Blaine Fineness of Cement	280-458 m^2/Kg
Cement: C_2S content	7-30 %
Cement: C_3S content	44-65 %
Fly Ash Content	24-276 Kg/m^3
Blaine Fineness of Fly Ash	289-760 m^2/Kg
Fly Ash: SiO_2 content	10-55 %
Fly Ash CaO content	1-35 %
Density of Fly Ash	1960-2800 Kg/ m^3
Water-Binder Ratio	0.27-0.60
Fly Ash-Binder Ratio	0.10-0.75
Maximum Size of Aggregate	9.00-25 mm

3.3 CALIBRATION OF THE MODEL

The model was calibrated using part of the data from several researchers [3-11]. About 150 concretes were selected from the above cited literature in addition to the 135 concrete mixtures cast for this research. The model of Eqn. 4 was operated on the remaining data to predict 28 day compressive strength of fly ash-concretes. The range of various mixture parameters is reported in Table 1. Following values of the constants in Eqns. 4, 11 and 12 were found by non-linear multiple regression analysis:

Constant	β_2	β_3	γ_1	γ_2	γ_3	a	b_1	b_2	b_3
Value	1.905	1.524	0.00735	0.2984	-0.0238	-0.00257	0.0097	-.0033	.00027

4. PERFORMANCE OF THE MODEL

Within the range of mixture composition described in Table 1, the model prediction agrees well with experimental strengths. The plot of predicted versus experimental strengths is shown in Fig.1. It can be observed that more than 93% of the data lies within ±20 percent range with most of the data lying on zero error line..

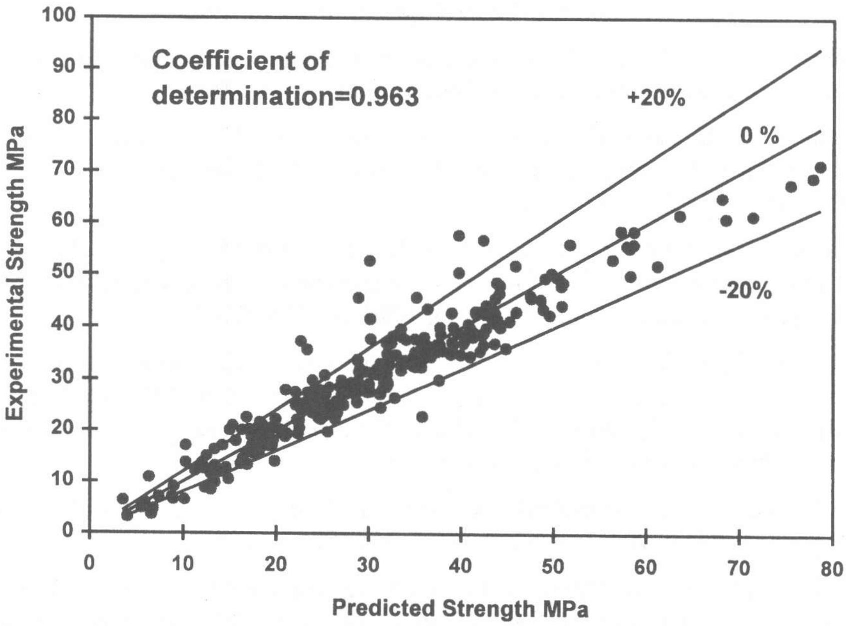

Figure 1. Comparison of Predicted and Experimental Strengths.

5. CONCLUSIONS

The model is presented for 28 days strength prediction of fly ash-concretes based on chemical composition and fineness of fly ash and cement. The model is in line with the present state of knowledge about hydration and pozzolanic reaction. The model predictions agree well with experimental results. The errors in predicted versus experimental values may be attributed to two main factors. First of the factors is discrepancy in modeling and second is error in experimentation. There is no doubt that both of the factors affect the model for strength prediction. Most of the concretes selected from recent literature were cast using super-plasticizers, whose effect on strength development is ignored in modeling. However, extremely accurate strength prediction is not possible due to the fact that several errors are induced during concrete making and testing. It is a rare occasion to achieve a high correlation and small skewness as are obtained in this investigation. It is expected that out come of this investigation will provide a basic tool for determining the strength contribution of both of cement and fly ash to concrete.

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