論文 Axisymmetric Size Effect Analysis for Pull-Out Strength by the Extended Fictitious Crack Approach

Ahmed Saad Eldin MORGAN*1, Junichiro NIWA*2 and Tada-aki TANABE*3

ABSTRACT: In this paper the size effect on both ultimate strength and failure cone surface formation in pull-out tests under various boundary conditions was studied through precise analysis. This analysis shows that the fictitious crack approach can be extended with two orthogonal rod elements as a new technique, which can successfully predict the size effect on the pull-out strength. The numerical investigation has been carried out by a computer simulation using our original program ANACS. To judge the validity of the results, the numerical results was compared with the empirical equation of Eligehausen and Sawade [1], and test results of RILEM report [2].

KEYWORDS: concrete fracture, size effect, finite element, discrete model, fictitious crack, rod elements, pull-out tests, headed anchors

1. INTRODUCTION

This paper represents an energy-based model of the fracture behavior of concrete where the failure cone is simulated by a discrete crack sewed by two orthogonal rod elements. The experimental results are available only for small concrete blocks with embedded anchorage length up to 150mm. Therefore, the comparison with experimental results is presented only for small embedded length up to 150mm.

The pull-out problem is physically three dimensional, however, it can be considered as axially symmetric if its geometry and material properties are independent of the circumferential coordinate θ. Based on axisymmetric constitutive model of (4×4) matrix, all strains components $(\varepsilon_x, \varepsilon_y, \varepsilon_\theta, \gamma_{xy})$ are considered in calculation of stresses $(\sigma_x, \sigma_y, \sigma_\theta, \tau_{xy})$.

2. FINITE ELEMENT MODELING AND FICTITIOUS CRACK SIMULATION

The concrete finite elements around the crack path are assumed to follow an elastic stress-strain relation in tension. Accordingly, the failure cone surface can be easily localized based on the fictitious crack approach by using the two orthogonal rod elements.

In the present analysis, there is no scope of compression failure. Since the studied cases in this paper are concerned with the tensile cone failure of concrete, thus compressive stresses in concrete are confirmed to remain within the elastic range.

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*1 Department of civil engineering, Nagoya University, Dr., Member of JCI
*2 Department of civil engineering, Nagoya University, Dr., Member of JCI
*3 Department of civil engineering, Nagoya University, Prof., Member of JCI

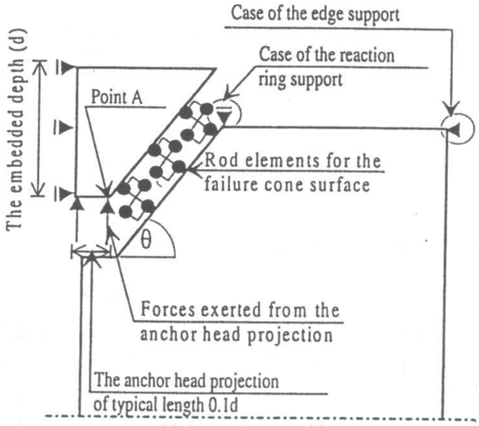

Fig.1 Schematic diagram to illustrate the failure cone surface, and the boundary conditions

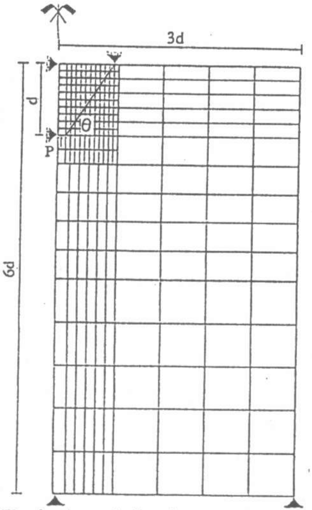

Fig.2 Typical mesh for the concrete block

In the present size effect analysis, the concrete element size is taken proportional to the specimen size. Since the fracture energy model is implemented in the program and the fracture energy G_F is kept constant for all concrete blocks of different sizes, the mesh sensitivity is considered to be insignificant.

The fracture zone is modeled by two orthogonal rod elements (**Fig.1**). Each rod element can be considered as a virtual circular surface element with unit length (L=1). The assigned fracture area for each rod element was considered according to its position along the failure cone surface. The rod element perpendicular to failure cone surface exhibits nonlinear stress-strain behavior of concrete by using the 1/4th softening curve, for more details about the rod elements modeling refer to reference [3].

Although many investigations are currently in progress [4], unfortunately the mode II fracture properties of concrete are not yet well established. Therefore, a very simple model has been assigned to the rod element parallel to failure cone surface [3].

The chosen value of the ultimate stress for the rod element which is parallel to failure cone surface is equal to the proposed compression strength value of concrete. This large chosen value reflects that the crack formation and propagation, and the ultimate strength mainly depend on the tensile fracture energy stored in the rod element perpendicular to failure cone surface.

3. THE PROBLEM ENVIRONMENT

The typical geometry of the concrete cylindrical block and the chosen mesh are illustrated in **Fig.2**. The typical finite element model used for the pull-out analysis consists of 268 elements including 8,7, and 6 noded quadrilateral elements. The number of nodes in this mesh are 858. The chosen geometrical dimensions of the analyzed specimens are determined according to the **RILEM** Round Robin Analysis requirements of Anchor Bolts [2].

In this investigation, three different support conditions were considered. The first support condition case is carried out by considering the reaction ring as inverted roller support on the top surface of the specimen (**Fig.1**), to simulate the standard practice of pull-out tests. The second case is analyzed without ring support, but by considering hinged support at the top edge surface of the specimen (**Fig.1**) to study the failure cone surface behavior without any influence coming from the reaction ring support. The third case is carried out without considering neither the reaction ring support, nor the top edge support, i.e. the top surface of the specimen is considered free from any restrictions. The third case is considered to simulate the headed anchor behavior in a real practice.

4. STUDYING THE SIZE EFFECT ON THE FAILURE CONE SURFACE INCLINATION

Nine different embedded depths are considered, such as d=50, 150, 450, 600, 1000, 2000, 5000, 10000, and 12500 mm. The concrete properties are identical for all nine concrete blocks: 30.0MPa, f_t=3.0MPa, =100N/m, and E_c=30.0GPa.

Based on the extensive parametric study, the inclination of the diagonal failure surface has been investigated. The failure cone surface is assumed to be oriented at angle ranges between (26°-76°), within this range 11 inclination angles were selected. Therefore, 11 finite element meshes are rearranged, for every considered embedded depth in every supporting condition, with respect to the chosen path to cover all crack inclination possibilities. For the sake of studying the size effect on the crack inclination, 99 finite element meshes are prepared for every boundary condition case.

The results are shown in **Figs. 3-11**. It has been found that in the case of considering reaction ring support the inclination of the failure cone surface which gives the minimum pull-out strength is ranging between (60°-53°) for embedded depths up to 5000mm as shown in **Figs.3-9**.

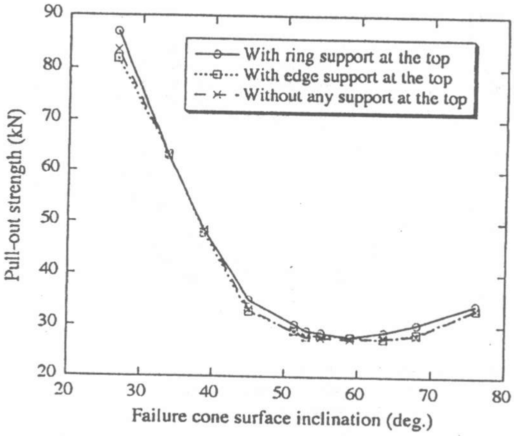

Fig.3 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=50mm)

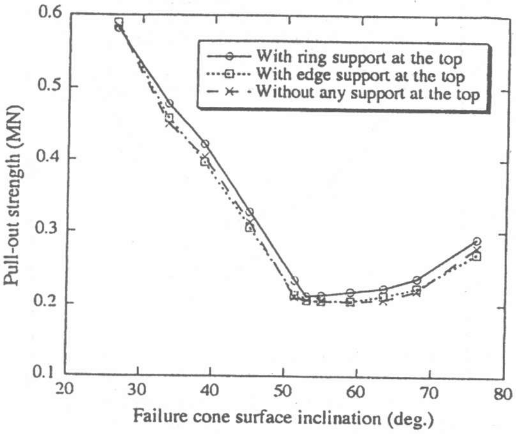

Fig.4 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=150mm)

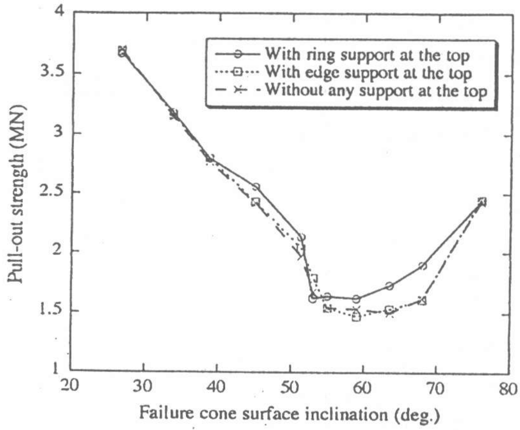

Fig.5 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=450mm)

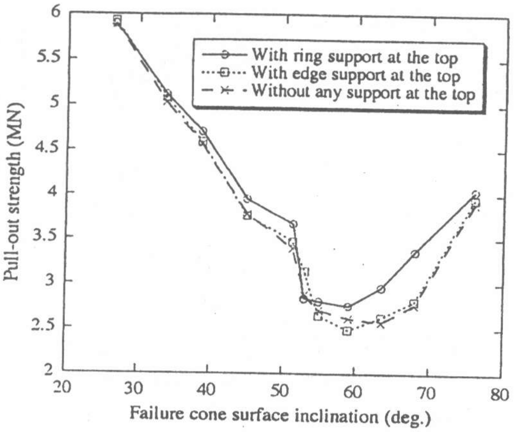

Fig.6 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=600mm)

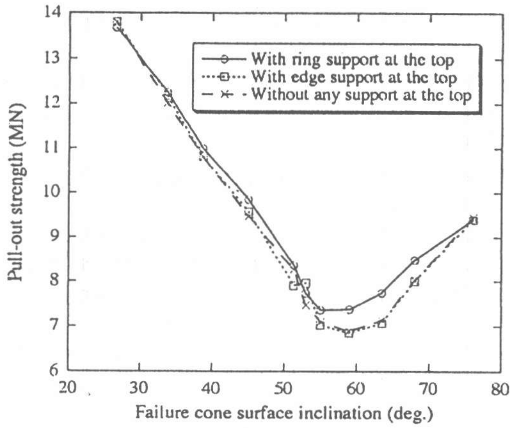

Fig.7 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=1000mm)

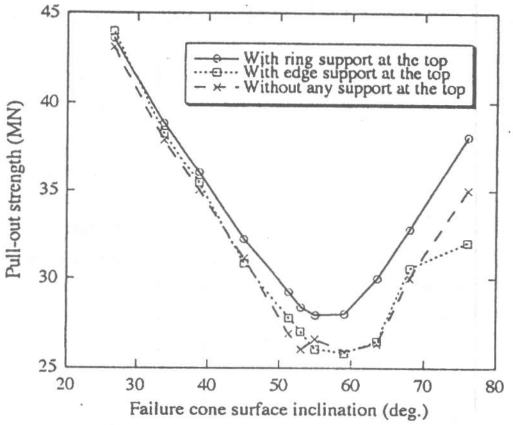

Fig.8 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=2000mm)

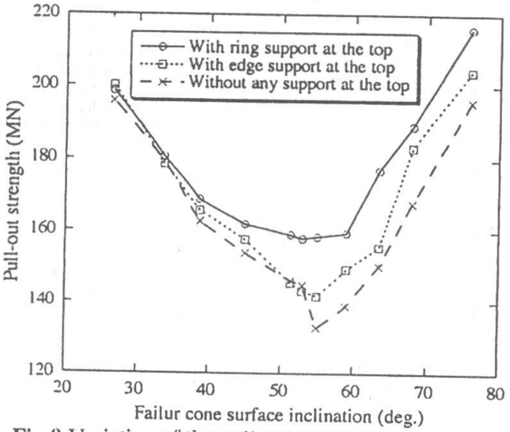

Fig.9 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=5000mm)

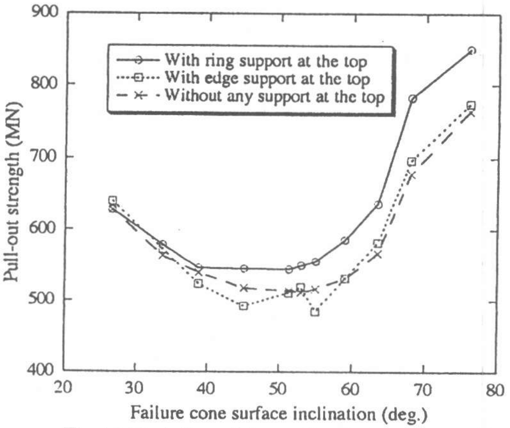

Fig.10 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=10000mm)

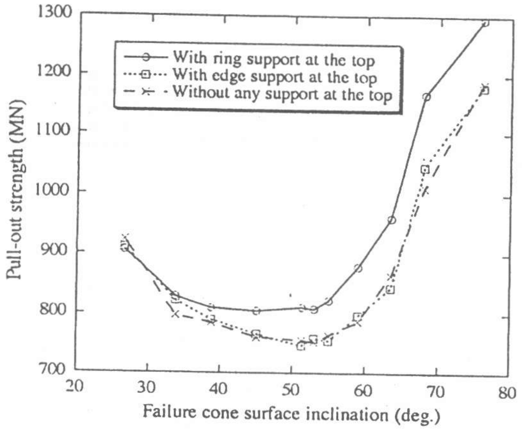

Fig.11 Variation of the pull-out strength w.r.t the cone failure inclination angle θ (d=12500mm)

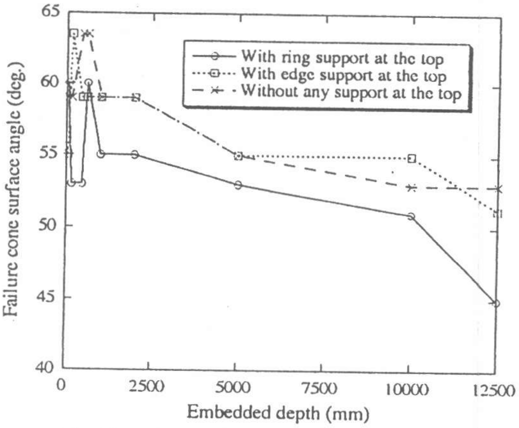

Fig.12 The size effect on the failure cone surface inclination angle θ

On the other hand, for huge embedded depths such as 10000mm or 12500mm, it has been found that the inclination of the crack surface which gives the minimum pull-out strength is ranging between (51°-38°) as shown in **Figs. 10, 11**. Moreover, in the case of considering the edge support, it was found that the inclination of the failure cone surface, which gives the minimum pull-out strength is about 60° for embedded depths up to 2000mm, and it is ranging between (55°-51°) for huge embedded depths such as 5000mm or more. Furthermore, in the case of freeing the specimen top surface from any restriction condition, it was found that the inclination which gives the minimum pull-out strength is ranging between (63.5°-60°) for the embedded depths up to 2000mm, and it is about 53° for huge embedded depths more than 5000mm.

From **Figs. 3-11** it is noticed that the results for the second and third boundary condition cases are almost identical, and they give pull-out strength values less than the case of considering reaction ring support. Also, by comparing the results before and after the minimum pull-out force for different embedded depths, it has been found that there is a significant change in the overall trend of the resulting pull-out strengths.

Fig.12 shows that the size effect on the failure cone surface inclination. **Fig.12** shows that for huge embedded depths such that 10000mm, or 12500mm, the cone failure surface inclination is getting more flatter as compared with small embedded depths. Furthermore, from **Fig.12** it can be noticed that the inclination of the failure cone surface, which gives the minimum pull-out strength in the second and third supporting condition cases is getting more steeper than the case of considering the reaction ring support, i.e. the produced failure cone surface area becomes smaller, consequently the ultimate pull-out strength becomes smaller.

5. SIZE EFFECT ANALYSIS

The resulting minimum pull-out strengths from **Figs.3-11** are utilized for this study. **Fig.13** shows the tendency of the nominal pull-out strength to decrease with the increase in the embedded depth whatever the boundary condition is. This behavior is known as the size effect. For the three boundary condition cases **Fig.13** shows that the size effect moderates for large embedded depths and the nominal pull-out strength of the concrete blocks tends to be bounded with a certain limit. **Fig.13** proves that the proposed analytical model can predict the size effect of the pull-out strength.

The nominal pull-out strength in **Fig.13** is calculated by dividing the resulting minimum pull-out strength by the square of the embedded depth [5] as illustrated in Eq.(1).

$$\sigma = N_u / d^2 \qquad (1)$$

where σ is the nominal pull-out strength, N_u is the pull-out strength, and d is the embedded depth.

The results of Eligehausen and Sawades' empirical equation [1] are illustrated in **Fig.13**. Eligehausen and Sawades' equation gives the relation between the induced pull-out strength N_u and the embedded length h_{ef} as follows

$$N_u = 2.1 (E\, G_F)^{0.5} h_{ef}^{\ 1.5} \qquad (2)$$

where: E is the Young's modulus of concrete, G_F is the fracture energy of concrete and h_{ef} is the embedded depth of the anchor.

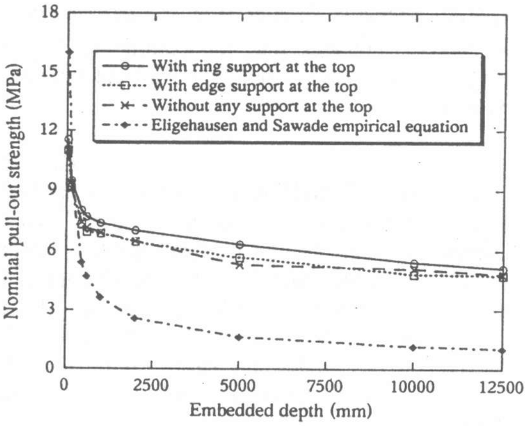

Fig.13 Comparison between the nominal pull-out strength from the numerical analysis and the empirical Eq.(2) [1]

Table 1 Comparison of the present analysis with the experimental and analytical results presented on the RILEM report [2]

Analyzer/ Tester	**Embedded depth=50mm** **The crack inclination θ=45°**	**Embedded depth=150mm** **The crack inclination θ=45°**
1. Barr and Tokatly, tests	**22.5**	**64-220**
2. Ozbolt, analysis		**191**
3. Palm and Gylltoft, analysis		**227**
ELIGEHAUSEN and SAWADE empirical equation	**40**	**211.3**
The present analysis	**27.6 at the crack inclination θ=60°**	**212.5 at the crack inclination θ=53°**

It is better to mention that Eligehausen and Sawades' empirical equation was verified up to 450 mm. Moreover, **Fig.13** shows that the presented results have a rather good agreement with Eligehausen and Sawades' empirical equation for embedded depths up to 450mm.

Furthermore, the results of the numerical analysis are compared with the analytical and experimental results which appeared in the RILEM report [2] as shown in **Table 1**. From **Table 1**, it can be noticed that the present analytical results have a rather good agreement with the previous analytical and experimental results [2]. The ultimate strength in **Table 1** is in kN.

6. CONCLUSIONS

It is possible to study the influence of different variables on pull-out strength of headed anchors embedded in concrete blocks numerically by means of nonlinear fracture mechanics. In particular, nonlinear fracture mechanics offer a possibility to explain the size effect in pull-out strength. It has been observed that for small embedded depths the pull-out capacity of headed anchors embedded in concrete blocks is profoundly affected by the size effect. On the other hand, for large embedded headed anchors, the numerical predictions showed that the size effect becomes insignificant. Also, it is found that in the case of considering reaction ring support, the inclination of failure cone surface with range (60°-53°) gives the minimum pull-out strength for a wide range of embedded depths up to 5000mm, on the other hand for huge embedded depths such as 10000mm, or 12500mm the inclination of the failure cone surface with range (51°-38°) gives the minimum pull-out strength. Moreover, in the case of considering edge support or in the case of considering top free surface, the inclination of the failure cone getting more steeper, consequently the ultimate pull-out strength becomes smaller than the case of considering the reaction ring support. The previous conclusion indicates that the commonly adapted method assuming 45^o failure surface yields exaggerated resisting load. The pull-out strength of cone failure is mainly dependent on mode I fracture.

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