

# 論文 Flexural Behavior of Externally Prestressed Beams with Large Eccentricities

Thirugnanasuntharan ARAVINTHAN <sup>\*1</sup>  
Tadayoshi NIITSU <sup>\*1</sup>

Hiroshi MUTSUYOSHI <sup>\*2</sup>  
An CHEN <sup>\*3</sup>

**ABSTRACT:** The ultimate flexural strength of externally prestressed beams is comparatively smaller than the one with bonded beams. One possibility of enhancing the strength is to provide tendons with large eccentricities, i.e., below the depth of the girder. This paper describes the experimental investigation carried out to study the behavior of such beams. It was found that substantial increase in flexural strength can be obtained, compared to ordinary externally prestressed beams. The experimental results agree well with analytical predictions carried out by two methods, one a rigorous analysis and the other using a simplified prediction equation.

**KEYWORDS:** external prestressing, flexural strength, large eccentricities, prestressed concrete, ultimate tendon stress

## 1. INTRODUCTION

The development of external prestressing has been one of the major trends in construction over the past decade. External prestressing may be defined as prestress introduced by tendons placed outside the structure over the greater part of their length. This type of prestressing could be applied to both new structures and those being strengthened. Moreover, a significant number of segmental concrete box girder bridges with external tendons have already been constructed. Substantial economical and construction time savings have been indicated for this type of construction. However, previous investigations have shown that the ultimate strength of such beams is comparatively lesser than the internal bonded beams [1, 2].

One of the possible methods of enhancing the flexural strength of externally prestressed beams is to place the tendons with large eccentricities. This kind of construction is possible only when external prestressing is used, since the tendons need not be within the concrete section. By this methodology, either improvement in strength or reduction in the amount of prestressing can be achieved, leading to economical structures. However, there may be limitations on the amount of effective prestressing which may tend to produce cracks in the top fibers of the beam at the prestressing stage. To obtain an insight of the ultimate flexural behavior of such beams, an experimental program was conducted on monolithically cast simply supported beams with large eccentricities. The results of this investigation are presented in this paper, with emphasis on the influence of tendon position on ultimate strength and effective prestressing. The experimental results are compared with the predicted ones based on a non-linear analytical program developed for the analysis of beams with external prestressing having large eccentricities. Finally, using a simplified prediction equation, the ultimate flexural strength is evaluated.

<sup>\*1</sup> Graduate School of Engineering, Saitama University, Student Member of JCI

<sup>\*2</sup> Department of Civil Engineering, Saitama University, Member of JCI

<sup>\*3</sup> Graduate Student, Dalian University of Technology, China  
( former Research Student, Saitama University)

## 2. EXPERIMENTAL METHODOLOGY

### 2.1 TEST VARIABLES AND SETUP

A total of four specimens having a T-shaped section were cast. The test variables are shown in Table 1. Two of these were of the normal type of beams with the depth of tendon,  $d_{ps}$  being 250 mm (Series-A). The other two were provided with large eccentricity,  $d_{ps}$  being 375 mm (Series - B). The span-to-effective depth ratios were 20.8 and 13.9 for Series A and B respectively. Fig. 1 shows the layout of test specimens. Specimen A-1 and B-1 were designed to achieve the same ultimate strength, but with different prestressing force, so that the reduction in prestress required can be compared. Specimen A-2 and B-2 were designed to compare the increase in strength when the same amount of initial prestress is introduced. In Series -B, the deviators were cast separately and fixed to the beam using epoxy resin and bolts.

Strain gages were attached to all the reinforcements (main, stirrups and confinement) at critical locations before concreting. Strain gages were also attached to the concrete surface of the cast specimen along the depth of the beam at desired points. To measure the strain variation in the tendons, gages were fixed at various locations along the length. Strain gages in the form of  $\pi$ -gages were mounted at critical locations where cracks were expected. Load cells were placed at the anchorage ends of the external cables to measure the cable force. They were also set below the beam at each support to obtain the support reactions. The applied load was measured by the load cells put below the jacks. Displacement transducers were placed at the mid span and deviators to measure the vertical deformation of the beam.

### 2.2 TEST PROCEDURE

For specimen A-1, two steel cables of type SWPR7A with 15.2 mm diameter were used as external tendons with a total design prestressing force of 265 kN which was the maximum allowable value without allowing for cracking. However, for specimen B-1, which was designed to achieve the same strength, the cable size was reduced to 10.8 mm and the applied prestress was only 118 kN. For specimens A-2 and B-2, two cables of the same type with 12.4 mm diameter were used and the initial prestress was 177. In specimen B-2, cracking was expected at the top fibers of the deviator section at the time of prestressing. This was mainly caused by the self weight of the girder, which was not sufficient to resist the moments due to prestressing due to smaller cross-section. To prevent this, one cable with 10.8 mm diameter was placed in the top compression flange as internal bonded tendon with a prestressing force of 59 kN. However, in practical situation this is not necessary since the dead loads on the beam including the weight of deck etc. are generally sufficient to prevent cracking. The applied prestress was in the range of 50 - 55% of the ultimate tensile strength of the tendons. Teflon sheets were inserted between the tendons and the deviators to eliminate friction. Two point static monotonic loading was applied symmetrically at a distance of 0.90 m as shown in Fig. 1. The loading was stopped when crushing of concrete occurred at the critical section.

Table 1. Test variables and materials used

No.	Description of specimens	Tendon depth $d_{ps}$ (mm)	Reinforcement		Type of tendon (nos. - size)		Introduced prestress force (kN)		Concrete strength (MPa)
			top/bot.	stirrup	bottom	top	bottom	top	
A-1	Normal external PS	250	4-D6/ 3-D10	D6@100	2-15.2 mm	-	265	35	
A-2		2-12.4 mm			177				
B-1	Large eccentricity	375			2-10.8 mm		118		
B-2	external PS				2-12.4 mm	1-10.8 mm	177		59

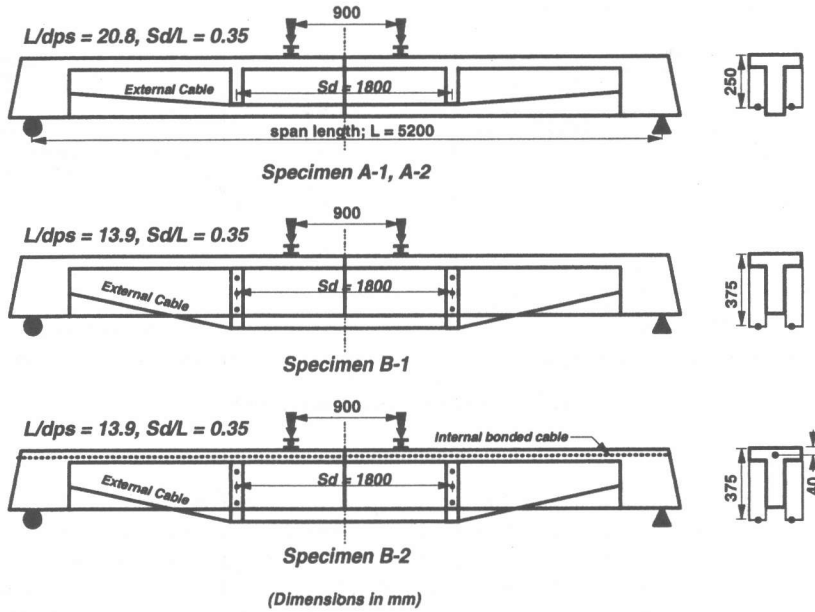


Figure 1. Layout diagram

### 3. TEST RESULTS AND DISCUSSION

The experimental results of important parameters are summarized in Table 2. The failure mode was due to crushing of concrete at the midspan region for all the specimens. There was no yielding of external tendons in specimens of Series-A. However, in the specimens with large eccentricities (Series -B) yielding was observed in the external tendons.

#### 3.1 LOAD-DISPLACEMENT CHARACTERISTICS

The observed load-displacement characteristics of the specimens A-1 and B-1 are compared in Fig. 2(a). The behavior of these specimens was nearly the same up to the occurrence of first cracking. Cracking took place when the applied load was about 54 kN and 44 kN respectively. The maximum load carrying capacity of the specimens was nearly the same about 90 kN. However, the applied prestress for specimen B-1 was reduced by 55%. The ultimate deflection (in this case, when the crushing of concrete in the critical section), of specimen B-1 was higher than that of A-1 by about 40%. This increase could be mainly attributed to the lower prestress force applied in specimen B-1 and the yielding of the tendons. From this it can be deduced that by providing tendons at large eccentricities, the same ultimate strength can be achieved even with substantial reduction in the applied prestress, thus economizing on the prestressing and the amount of tendons.

Table 2. Summary of experimental results

No.	Description of specimens	Cracking load (kN)	Max. load (kN)	Ultimate deflection (mm)	Stress increase in ext. tendon (MPa)	failure mode
A-1	Normal external PS	54.0	91.2	72.8	357	crushing of concrete
A-2	Normal external PS	34.3	70.3	88.1	417	
B-1	Large eccentricity external PS	44.1	88.6	101.3	698	crushing, yielding of tendon
B-2	Large eccentricity external PS	54.0	109.6	78.3	552	

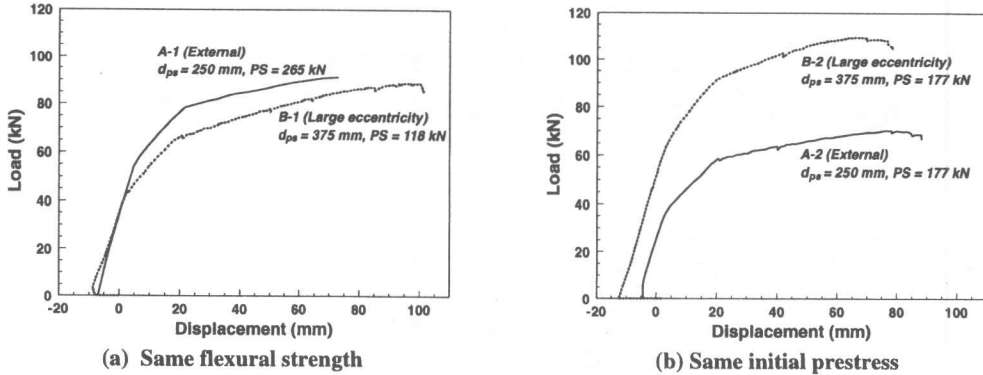


Fig. 2 Load-displacement characteristics

The load-displacement behavior for specimens A-2 and B-2 is given in Fig. 2(b). Both the cracking load and the maximum applied load of specimen B-2 that had large eccentricity was about 56% higher than that of the ordinary specimen A-2. However, the ultimate deflection of the specimen B-2 was smaller than that of A-2 by about 11%. This could be attributed to the reduction in curvature in specimen B-2 at the mid-span section due to large eccentricity, compared to A-2. It should be noted that the initial camber at the time of prestressing was considerably large in specimen B-2. This is too attributed to the eccentricity of the tendons and the small self weight of the beam. From the above discussion, it can be clearly seen that there is significant increase in the ultimate and service loads when the large eccentric external prestressing is used.

### 3.2 STRESS INCREASE IN EXTERNAL TENDONS

The ultimate tendon stress in the external prestressing is another important factor that influences the ultimate strength of such beams. Increase in external tendon stress with mid-span displacement is given in Figure 3. It was observed that the external tendons did not yield in specimens of test Series -A. However, yielding of external tendon was observed in test Series-B, that had tendons at large eccentricities. This is attributed to span-to-effective depth ratio, which was 20.8 and 13.9 for Series A and B respectively. The above observation confirms the finding of the previous research that the span-to-depth ratio is one of the most important factor influencing the ultimate tendon stress and as this ratio increase, the ultimate tendon stress is significantly reduced [3].

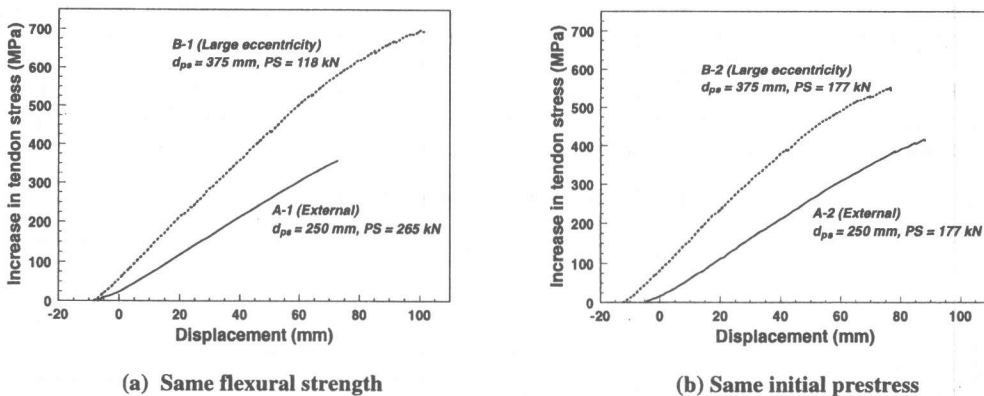
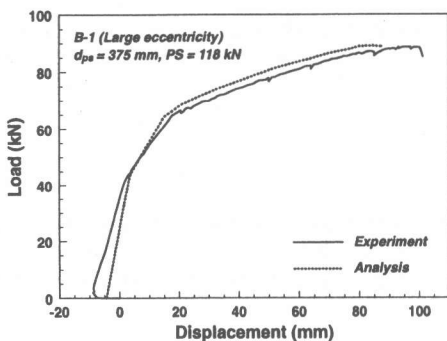
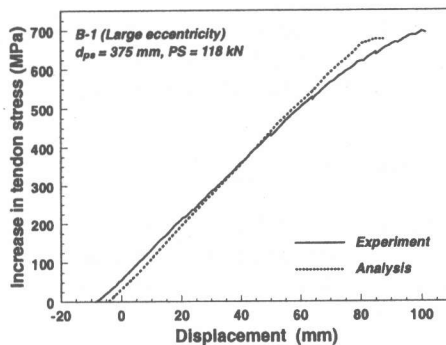


Fig. 3 Variation of stress in external tendons



(a) Load-displacement characteristics



(b) Variation of force in external tendons

Fig. 4 Comparison of experimental results with analytical prediction

## 4. COMPARISON WITH ANALYTICAL RESULTS

### 4.1 USING RIGOROUS ANALYTICAL METHODOLOGY

A non-linear analytical approach was adopted to predict the flexural behavior of precast segmental continuous beams with external prestressing based on the existing methodology [4,5]. This consists of a multi-level iterative method taking into account both material and structural non linearities. The ultimate failure is when either concrete strain of top fiber reaches the strain of 0.0035 or breaking of cable whichever is reached first. This methodology was applied to specimen with large eccentricities, assuming they behave similar to the normal external PC beams. The comparisons of experimental to analytical results are given in Table 3. The load-displacement characteristics and variation of tendon force are shown in Fig.4(a) and (b) for specimen B-1. There is very good agreement between the analytical and experimental results. Similar agreement was obtained for the other three specimens. The error in the prediction of increase in tendon stress is within 5% while for maximum strength it is less than 2%. As such, it can be concluded that the flexural behavior of such beams could be predicted using the above analytical methodology.

### 4.2 USING PREDICTION EQUATION

The methodology discussed in the previous section is fairly complex and for practical situations, a simplified design equation was proposed by Mutsuyoshi, et.al. [4]. This equation simplifies the member analysis to simplified sectional analysis by incorporating a strain reduction coefficient,  $\Omega_u$  proposed by Naaman [6]. In addition, the change in tendon eccentricity is taken care by introducing the depth reduction factor,  $R_d$ . The proposed equation was modified incorporating other parameters as well in a later study [3]. The applicability of this equation for girders with large eccentricities was verified in this study. The results are summarized in Table 3. In this case, it can be seen that the error in the prediction of the stress increase in tendon is larger than the rigorous analysis, but still

Table 3. comparison of experimental results with analysis and prediction equation

No.	Description of specimens	Maximum Strength (kN)			Stress increase in ext. tendon (MPa)		
		Experiment	Analysis	Equation	Experiment	Analysis	Equation
A-1	Normal external PS	91.2	91.5	91.9	357	348	372
A-2	Normal external PS	70.3	70.3	70.6	417	405	447
B-1	Large eccentricity external PS	88.6	89.1	86.3	698	677	620
B-2	Large eccentricity external PS	109.6	111.2	107.8	552	524	517

within 10%. However, it should be noted that the ultimate flexural strength is predicted with less than 5% error. From this it is concluded that the proposed prediction equations are suitable for the use of structures with large eccentricities.

## 5. CONCLUSIONS

The experimental investigation on flexural behavior of externally prestressed beams with large eccentricities was carried out using monolithically cast simply supported beams. The effect of large eccentricities on strength and amount of prestressing was verified. The followings can be concluded from this study.

- The ultimate flexural strength can be substantially enhanced by providing the external tendons at large eccentricities, utilizing the tendons effectively, resulting in economical structures.
- The flexural behavior of externally prestressed beams with large eccentricities can be predicted using the non-linear analytical methodology as there is very good agreement between the experimental and analytical results.
- The proposed design equation gives a simplified method to estimate the ultimate tendon stress and flexural strength, which agree well with the experimental results.
- It is proposed that further investigation is carried out on continuous girders with large eccentricities. This may lead to tendons extending above the beam flange at the supports, resembling the extradosed type of bridges.

## ACKNOWLEDGMENTS

Sincere gratitude is expressed to Mr. Yoshimasa Takeda, undergraduate student of Saitama University for his assistance in conducting the experiment. Also sincere thanks to Sumitomo Electrical Industries and Sho-Bond Corporation for their cooperation.

## REFERENCES

1. Matupayont, S. and et.al., "Loss of Tendon's Eccentricity in External Prestressed Concrete Beam," Proceedings of JCI, Vol.16, No.2, June 1994, pp. 1033-1038.
2. Tsuchida, K. and et.al., "Flexural Behavior of External PC Beams," Proceedings of JCI, Vol.16, No.2, June 1994, pp. 1009-1014.
3. Aravinthan, T. and et.al., "Prediction of the Ultimate Flexural Strength of Externally Prestressed PC Beams," Proceedings of JCI, Vol.19, No.2, June 1997, pp. 1233-1238.
4. Mutsuyoshi, H., et.al., "Flexural Behavior and Proposal of Design Equation for Flexural Strength of Externally PC Members," Proceedings of JSCE, Vol.508/V(26), August 1995, pp.
5. Alkhari, F. M. and Naaman, A. E., "Analysis of Beams Prestressed with unbonded Internal or External Tendons," Journal of Structural Eng., ASCE, Vol.119, No.9, September 1993, pp. 2680-2700.
6. Naaman, A. E. and Alkhari, F. M. "Stress at Ultimate in Unbonded Post-Tensioned Tendons: Part 2-Proposed Methodology," ACI Structural Journal, Vol.88, No.6, Nov-Dec. 1991, pp. 683-692.