論文 The Size Effect on Torsion Strength of Concrete

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ABSTRACT: In this paper, the 3-D size effect was studied based on the analysis of torsion strength of concrete beams. It is shown that the fictitious crack approach can be extended with three orthogonal rod elements, which can successfully predict the size effect on the torsion strength of concrete. The numerical investigation has been carried out by simulation using the program ANACS of three dimensional version. To judge the validity of the results, these were compared with Bazant's size effect law [1].

KEYWORDS: torsion strength, concrete fracture, size effect, finite element method, three dimensional analysis, discrete model, fictitious crack, rod elements

1. INTRODUCTION

Recently, it appears that studying the size effect through three-dimensional analysis has a paramount importance, because significant experimental evidence is reported on the existence of the size effect in concrete structures. However, few numerical investigations have been carried out so far in three-dimensional analysis. Thus, in this research the numerical investigation using the three-dimensional finite element analysis based on nonlinear fracture mechanics and the extended fictitious crack approach have been utilized.

In this analysis, it is demonstrated that the size effect on the nominal strength is controlled by the structural energy-release due to cracking in concrete. If a stable crack growth before reaching peak-load is generated, the size effect may be strongly expected. On the contrary, structure failure promptly at crack initiation does not exhibit any appreciable size effect. This paper represents an energy-based model of the fracture behavior of concrete where the failure surface is simulated by a discrete crack represented by three orthogonal rod elements.

The arc-length solution procedure has been incorporated to detect the post peak behavior, and to recognize the fracture type of ductile or brittle nature. The results of extensive numerical studies for plain concrete beams subjected to torsion are shown and discussed on the size effect. The study is carried out for a broad size range, and the numerical results are compared with Bazant's size effect law.

The current approach of nonlinear fracture mechanics which deals with a single crack is important from the theoretical point of view and useful for determining the material fracture properties. However, in most large concrete structures the single crack growth assumption is unrealistic, as well as the proportionality assumption of the crack length at the peak load [2].

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When considering the damage in the three-dimensional analysis, it leads to the fact that in quasibrittle materials such as concrete the single macro crack is only the final of the complete damage process where the energy is dissipated in the material volume rather than at the crack surface. By carrying out three-dimensional analysis, one should be able to distinguish two different failure modes –brittle failure (at crack initiation) and ductile failure (stable crack propagation), and make the approach more realistic from the mechanical point of view.

Plain concrete beams with or without longitudinal reinforcement subjected to torsion do not have practical importance since in the practice the longitudinal reinforcement is always provided together with transverse reinforcement. Therefore, the size effect study of unreinforced concrete beams subjected to torsion is interesting at present from the theoretical point of view. So far there are only a few experimental and numerical studies related to the size effect on torsional strength of concrete beams. Systematic experimental studies have been carried out on small concrete beams with the maximal size of the cross-section d=150mm. These experiments show strong size effect on the beam sizes up to d=150mm. According to our knowledge only one numerical study has been carried out for the same beams [2], and they confirms the experimental results.

2. FINITE ELEMENT MODELING FOR THREE DIMENTIONAL PROBLEMS

Three dimensional problems have space nodes subjected to three dimensional boundary or restriction conditions. Since the material points have u (X-axis), v (Y-axis) and w (Z-axis) displacement components, a program based on the 8 noded cubic elements is applicable. Based on 3-D constitutive model of (6×6) matrix, all strain components (ε_x , ε_y , ε_z , γ_{xy} , γ_{yz} , γ_{zx}) are considered in calculation of stresses (σ_x , σ_y , σ_z , τ_{xy} , τ_{yz} , τ_{zx}).

The results of the deformation measurements and visual crack observations lead to the conclusion that the nonlinear behavior of concrete is concentrated in a discrete crack [3], or a crack band with small width [4]. Therefore, the concrete elements around the crack path are assumed to follow an elastic stress-strain relation in tension. Accordingly, the torsion failure surface can be easily localized based on the fictitious crack approach.

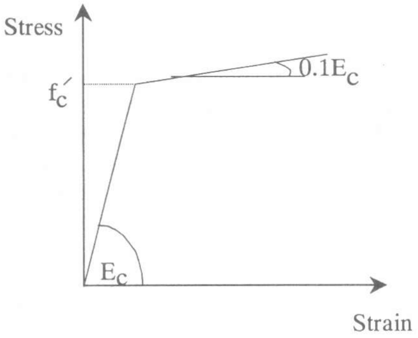

Fig.1 Concrete model in compression

Concrete elements in compression are modeled by bilinear stress strain curve as shown in **Fig.1**. According to the numerical analysis, the observed compression strains adjacent to failure surface were too small to initiate compression failure, which reflected that the nucleation of the failure and the failure load are governed by concrete cracking and aggregate interlock, not by concrete compression failure. Since the cases studied in this paper are concerned with the tensile torsion failure of concrete, the compressive stresses in concrete are confirmed to remain within the elastic range.

The **ANACS** program of its 3-D form has 8 noded isoparametric cubical elements. The program is developed to reduce the bulky input material by incorporating two dimensional generation for the finite element mesh and nodal coordinates. Also, the program involves the generation of mesh geometry, and deformed shapes through a Graphical package. These facilities are especially useful for detecting the failure modes.

3. FICTITIOUS CRACK SIMULATION FOR CONCRETE

The fracture zone is modeled by three orthogonal rod elements (**Fig.2**). The three orthogonal rod elements are placed between coupled nodes of concrete elements along the predefined failure surface, and oriented at some angle θ relative to the global coordinate system. Each rod element can be considered as a virtual surface element with unit length (L=1). The assigned fracture area for each rod element was determined according to its position along the failure surface. In the present model, one of the rod elements is taken perpendicular to the failure surface, and the other two are taken parallel to the failure surface.

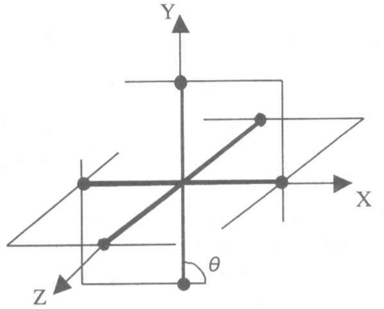

Fig. 2 The rod elements for the 3-D failure surface simulation

The rod element perpendicular to failure surface exhibits nonlinear stress-strain behavior of concrete by using the 1/4th softening curve (**Fig.3**). If the tensile stress reaches the concrete tensile strength, a crack will occur and the stress will begin to drop as the crack opens. For this rod element, it is assumed that the tensile fracture energy remains constant and is equal to 100N/m as a standard recommendation of the fracture energy [5]. The rod element length of unity was utilized to convert the σ-w relationship (the relative displacement along the failure surface versus transmitted stress) to the σ-ε relationship. Therefore, the strain can be calculated which is equivalent to the crack width, then the corresponding stress can be obtained.

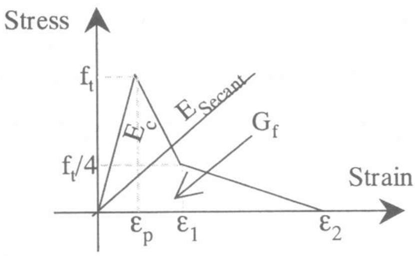

Fig. 3 The stress-strain model for the rod element perpendicular to failure cone surface

In the present analysis, the concrete element size is taken proportional to the specimen size. Since the fracture energy model is implemented into the program and the fracture energy G_F is kept constant for all concrete blocks of different sizes, the mesh sensitivity is considered to be insignificant. In other words, by keeping the fracture energy constant, the relationship between the crack length and the released fracture energy is considered to be the same for any specimen size and mesh discretization.

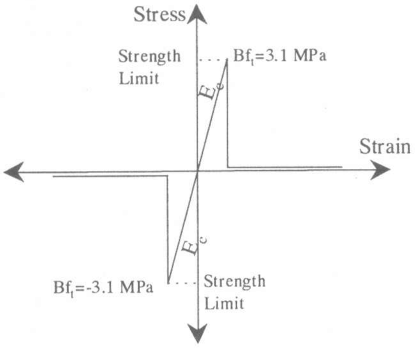

Fig. 4 The stress-strain model for the two rod elements parallel to failure surface

Although many investigations are currently in progress, as summarized by Karihaloo [6], unfortunately the mode II fracture properties of concrete are not yet well established. Therefore, a very simple model has been assigned to the other two rod elements which are parallel to crack surface as shown in **Fig.4**.

The stress-strain relation for the two rod elements parallel to failure surface is taken as linear elastic (**Fig.4**) till the tensile stress in the perpendicular rod element reaches the tensile strength of concrete (**Fig.3**). Thus, when the crack starts at a certain rod element which is perpendicular to failure surface, the resistance of corresponding two rod elements which are parallel to failure surface vanishes.

The value of the ultimate stress chosen for the two rod element which are parallel to failure surface is equal to 3.1MPa. This value is taken according to the strength limit mentioned by Ozbolt and Eligehausen [2] (Strength Limit = Bf_t where B is a constant to be determined experimentally = 1.16 [2] and f_t is tensile strength of concrete = 2.7MPa). This large value reflects that the crack formation, propagation and the ultimate strength mainly depend on the tensile fracture energy stored in the rod element perpendicular to the failure surface. Also, it is better to mention that, in the case of considering a sharp failure surface inclination, the stresses in the rod elements which are parallel to the failure surface may exceed the ultimate range of 3.1MPa. In this case, the strength of the three orthogonal rod elements was considered to vanish, i.e. all the rod elements lose all their stiffness. However, this phenomenon has rarely happened, except for the case of the failure surface inclination of 75°. Moreover unless the stress of one rod element perpendicular to the crack surface exceeds the tensile strength of concrete, the phenomenon can not happen.

Since the fictitious crack is incorporated into the numerical model from the beginning of the load application, i.e. in the elastic range prior to cracking and Young's modulus E_c is used for concrete elements, Young's modulus E_c should be used in all orthogonal rod elements along the crack path, to ensure homogenous media throughout the analysis.

4. THE RESULTS OF THREE DIMENTIONAL TORSION STRENGTH ANALYSIS

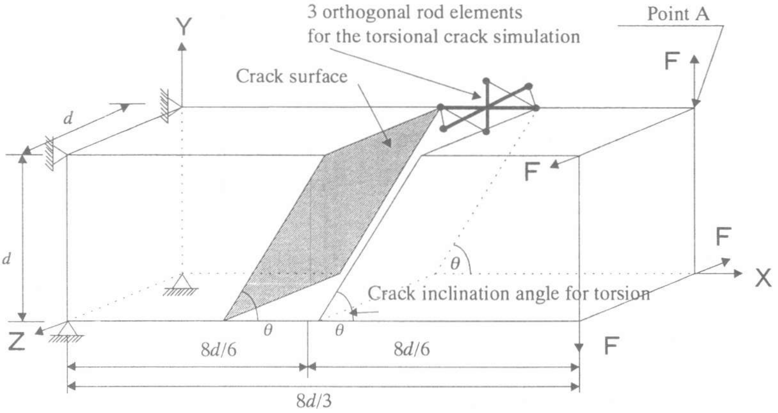

Fig. 5 Schematic diagram to illustrate the failure surface and the boundary conditions

The beam geometry used in the numerical study is shown by the schematic diagram in **Fig. 5**. The geometrical sizes of the analyzed model are taken the same as those tested by Bazant et al. (1988) [7]. **Fig. 5** illustrates the chosen crack, the load application and the boundary conditions.

In order to compare the numerical results with the Bazant's size effect equation which fits the test results, the relevant fracture material parameters are determined to be the same as in the experiments [7]. Concrete fracture properties are identical for all seven concrete beams. These are tensile strength f_t=2.7MPa, fracture energy G_F=100.0N/m, and Young's modulus E_C=35000.0MPa. In comparison to the experiments, where d = 150 mm, the numerical analysis for seven geometrically similar beams with d = 37.5, 75, 150, 200, 300, 750, and 900 mm is carried out.

Based on the extensive parametric study, the inclination of the diagonal failure surface has been investigated. The failure surface is assumed to be oriented at the angle between (40°-75°).

Within this range, 7 inclination angles were selected to perform this study. Results are shown in **Fig. 6**. It has been found that the inclination of the failure surface to give the minimum torsion strength ($M_t = 2Fd$)) is about 62° for a beam depth of 150mm.

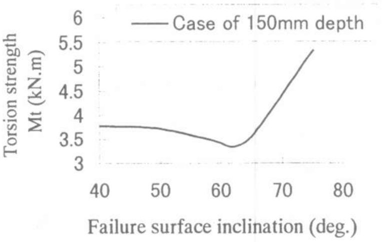

Fig.6 Variation of the torsion strength w.r.t the failure inclination angle θ

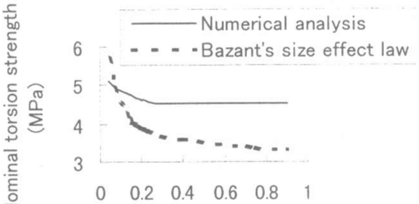

Fig.7 Comparison between the nominal torsion strength from the numerical analysis and Bazant's size effect law [1]

For the seek of the size effect on torsion strength of concrete beams, the same inclination angle of 62° which gives the minimum strength in the case of 150mm beam depth is adopted for all studied depths. In **Fig.7** the numerical analysis is plotted versus the beam depth. The numerical results were transformed to nominal torsion according to the elastic theory [2] illustrated in Eq.(1),

$$\tau = M_t / 0.208 d^3 \tag{1}$$

where τ is the nominal torsion strength, M_t is the torsion strength based on the numerical results at each considered depth from 37.5~900mm, and d is the crossponding beam depth.

The numerical results are compared with Bazant's size effect law [1] in which the relation between the ultimate torsion strength τ_u and the structure size d is presented as follows;

$$\tau_u = Bf_t \left(1 + d_o / d\right)^{0.5} \tag{2}$$

where Bf_t is the strength limit and d_o is constant to be determined experimentally as equals to 0.11m [2].

Fig.7 shows the tendency of the nominal torsion strength to decrease with the increase in the beam depth. On the other hand, the size effect moderates for large depths and the nominal torsion strength of the concrete beams tends to be bounded with a certain limit. **Fig.7** proves that the proposed analytical model can predict the size effect of the torsion strength of concrete beams. **Fig.7** shows that the presented results have a rather good agreement with Bazant's size effect law for depths up to 150mm.

Figs.8 and 9 show the load-displacement diagrams for the case of beam depths 37.5mm and 900mm, respectively. By using the arc-length method, the full torsional moment versus displacement diagram is obtained. The convergence criterion is maintained at all load levels before and after the peak load. In the both figures the vertical displacements are determined at the point A **(Fig.5)**.

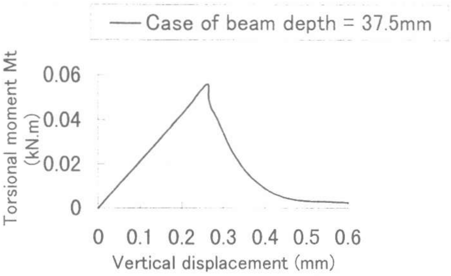

Fig.8 Torsional moment versus displacement diagram

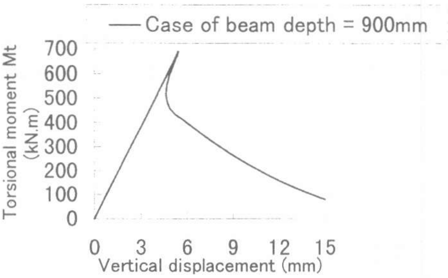

Fig.9 Torsional moment versus displacement diagram

In the case of small beam depths as d=37.5mm, **Fig.8** shows that the failure mode is ductile and the snap back phenomenon does not dominantly take place. On the other hand, for large beam depths as d=900mm, **Fig.9** shows a post peak snap back response, which reflects the brittle behavior of the large beams. The snap back occurs, as a result of a bifurcation process which leads to a sudden drop in both load and deflection. It can be concluded that the torsion failure of concrete beams is significantly affected by the beam depth. The failure mode changes from ductile to brittle as the size of concrete beams increases. In other words, fracture of concrete leads to brittle failures due to the size effect of decreasing nominal strength with increasing structural size.

5. CONCLUSIONS

It is possible to study the influence of different variables on torsion strength of concrete beams numerically by means of nonlinear fracture mechanics. In particular, nonlinear fracture mechanics offer a possibility to explain the size effect in torsion strength. It has been observed that for small depths the torsion capacity of concrete beams is profoundly affected by the size effect. On the other hand, for large concrete beams, the numerical predictions showed that the size effect becomes insignificant. Also, it is found that the inclination of torsional failure surface with angle 62^{o} gives the minimum torsional strength. The previous conclusion indicates that the commonly adapted method assuming 45^{o} torsional failure surface yields exaggerated resisting torsional strength. The torsion strength is mainly dependent on mode I fracture.

REFERENCES

1. Bazant, Z.P. “Size Effect in Blunt Fracture: Concrete, Rock, Metal,” Journal of Engineering Mechanics, ASCE, 110(4), 1984, pp. 518-535.
2. Ozbolt, J and Eligehausen, R. “Size Effect in Concrete and RC Structures Diagonal Shear and Torsion,” Studi E RICERCHE, Politecnico di Milano, Italy, Vol. 17, 1996, pp. 37-67.
3. Hillerborg, A., Modeer, M. and Peterson, P.E., “Analysis of Crack Formation and Crack Growth in Concrete by Means of Fracture Mechanics and Finite Element,” Cement and Concrete Research. Vol. 6, 1976, pp. 773-782.
4. Bazant, Z.P. and Oh, B.H., “Crack Band Theory for Fracture of Concrete,” Materials and Structures, Vol. 16, No. 93, 1983, pp. 155-177.
5. RILEM TC 90-FMA, “Fracture Mechanics of Concrete - Applications, Round Robin Analysis of Anchor Bolts,” Preliminary Report, 2nd ed., May 1991.
6. Karihaloo, B. L. “Fracture Mechanics and Structural Concrete,” Longman Scientific & Technical, Harlow, England, 1995.
7. Bazant, Z.P., Sener, S. and Prat, P. “Size Effect Tests of Torsional Failure of Plain and Reinforced Concrete Beams,” Materials and Structures, Vol. 21, 1988, pp. 425-430.