論文 Analysis of the Flexural Behavior of Externally Prestressed Concrete Beams with Large Eccentricities

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ABSTRACT: Previous researches have shown that the stress increase in external tendon is member-dependent which can be evaluated by considering the compatibility of concrete strain at tendon level. However, the applicability of this method may be limited when using externally PC beam with large eccentricities in which major portion of the tendon is below the concrete section. An attempt is made in this paper to verify such concept by comparing with the experimental results and also with a different methodology based on the geometrical compatibility of external tendon.

KEYWORDS: external prestressing, flexural analysis, compatibility of deformation, geometrical compatibility

1. INTRODUCTION

It has been observed in recent years that the application of external prestressing has been considerably utilized in most bridge structures. This can be attributed to various advantages of the external prestressing system such as ease in construction method, reduction of construction time and cost, and the possibility of replacement or re-tensioning of external tendon. However, because there is no bond between external tendon and concrete structure, the ultimate flexural strength of such beams is comparatively smaller than that of internally bonded ones and the flexural behavior is mainly dependent upon the overall behavior of structure [1].

In order to improve the ultimate flexural strength of externally PC structure, an experimental and analytical study was carried out by Aravinthan et al. [2]. It has been found that the ultimate flexural strength of such beams can be substantially enhanced by providing the external tendon at large eccentricities resulting in an effective utilization of external tendon which is an important advantage of this innovative method. This concept was also applied to the two-span continuous beams by Aravinthan et al. [3] showing an improvement of structural performance compared with the simply supported beams.

As mentioned earlier, the analysis of externally PC structure is mainly dependent upon the overall behavior of structure because there is no bond between external cable and concrete structure. Many researchers have investigated the flexural behavior of externally PC beams by using either conventional flexure analysis taking into account "*the compatibility of deformation*" over the entire beam length or a rigorous finite element method. It can be seen that most of these analytical approaches are based on the assumption for calculating stress in unbonded tendon by considering the total deformation of concrete section at the tendon level is equal to the total elongation of unbonded

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tendon. This concept was verified and considered to be effectively applicable for internal unbonded prestressed concrete beams and subsequently was adapted for using in the analysis of external unbonded tendon. Recently, this method was extended to the flexural analysis of beams with large eccentricities [2] and a good agreement between the analytical results and the test observations was reported. It is important to note that, because the eccentricity of external tendon is placed beyond the concrete section an additional assumption was adapted considering an imaginary concrete strain at tendon level (see Fig.1).

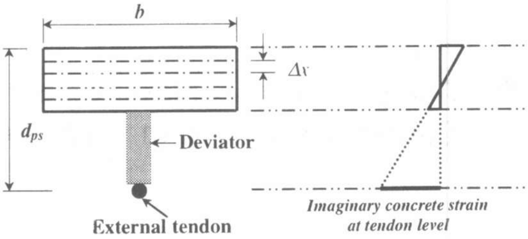


Fig. 1 Imaginary concrete strain at tendon level

Another method to evaluate stress in external unbonded tendon has been proposed by Virlogeux [4]. Due to the rectilinear shape of external tendon between end anchorages, the elongation of tendon between deviations can be calculated assuming the beams remain uncracked and linear elastic. This method is considered to be more realistic than the previous concept because it is based on the change of geometry of external tendon with increasing load and is rather easy to visualize how the stress in external tendon can be increased at any stage of loading. This concept can be referred as "*the geometrical compatibility of external tendon*".

To the best of the authors' knowledge, a verification of the concept of compatibility of deformation have not been yet clarified in two-span precast segmental continuous beam with large eccentricities. As such, an attempt is made to investigate the applicability of such a method by comparing with test observations. In addition, to overcome the assumption of an imaginary concrete strain adapted in previous method, an analytical methodology based on the geometrical compatibility of external tendon is proposed. Finally, a preliminary comparison is made using the predicted results obtained by both methods and the accuracy is verified with experimental observation consisting of simply supported beams with large eccentricities.

2. ANALYTICAL METHODOLOGY

A nonlinear analytical methodology based on the conventional flexural analysis and the compatibility of deformation developed by Matupayont [5] was adapted for predicting the ultimate flexural behavior of precast segmental beam with large eccentricities. Assumptions used in such analytical methodology can be summarized as following: (1) plane section remains plane after bending; (2) consider nonlinear behavior of material and geometry; (3) shear deformations are neglected; (4) friction at deviators and external tendon is neglected. It should be noted that in externally PC beam with large eccentricities the contribution to the flexural strength is mostly attributed to external tendon because of its large eccentricity. Therefore a constitutive model that can accurately predict the overall nonlinear behavior of prestressing steel is considerably essential. In this study, the stress-strain relation of prestressing steel was modified by using the model proposed by Menegotto and Pinto [6] which has a smooth change from elastic to plastic

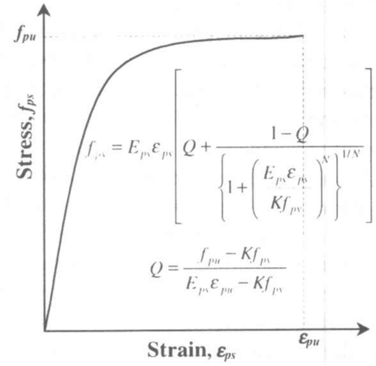


Fig. 2 Constitutive model of prestressing steel

behavior as shown in Fig.2. By using this model, a better prediction can be obtained in cases where the ultimate limit state is reached in the region of yielding of tendon.

3. COMPARISON WITH EXPERIMENTAL RESULTS

The test results of two-span precast segmental beam with large eccentricities conducted at Saitama University is used for verifying the applicability of compatibility of deformation (see Fig.3). Beam was prestressed with both external unbonded and internally post-tension bonded tendon with a magnitude of 25 kN and 200 kN respectively and subjected to symmetrical two-point loading with a distance of 1.25 m in each span. The material properties and the test variables are given in Table 1.

Table 1. Test variable and material properties of beam A-1a

No.	Description	Segment width (mm)	f_c' (MPa)	Reinforcement		Prestressing Tendon	
				Main	Stirrup	Internal (bond)	External
A-1a	Segmental, Epoxied joint	312.5	60.4	4-DB10	DB10@100	SWPR7A 2-T12.4 (2 x 100 kN)	SWPR7A 1-T10.8 (25 kN)

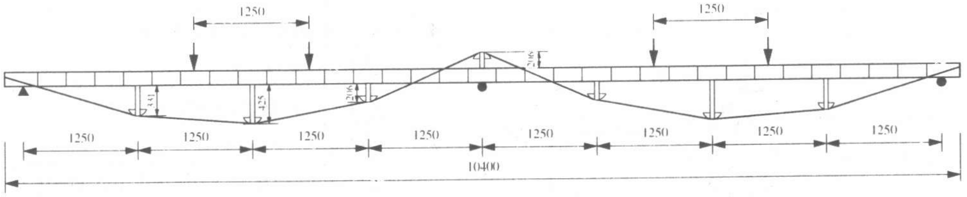


Fig. 3 Layout of two-span continuous beam (A-1a)

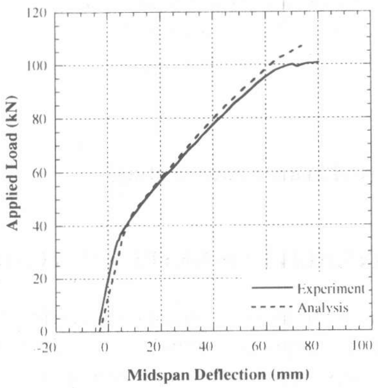


Fig. 4 Load and deflection relationship

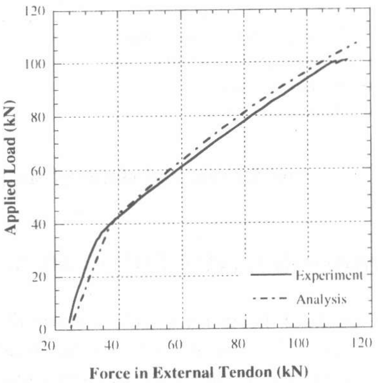
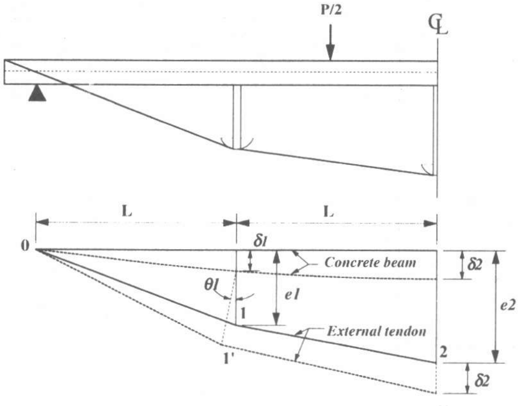


Fig. 5 Force in external tendon

It can be seen from Fig.4 that the analytical results based on compatibility of deformation showed a very good agreement with the test observations up to the ultimate stage. Similarly, it can be seen from Fig.5 that the path of applied load and force in external tendon from analytical method showed nearly the same as that of the test results. As such, it can be concluded that the concept of deformation compatibility can be applied to the flexural analysis of externally PC beam with large eccentricities with a good accuracy provided the assumption of imaginary concrete is assumed to be valid.

4. GEOMETRICAL COMPATIBILITY OF EXTERNAL TENDON

The application of geometrical compatibility of external tendon to the analytical methodology previously mentioned can be described by using the analytical model of simply supported externally PC beam with three deviators (see Fig.6). The assumption adapted for using this concept is that the strut deviator is considered as a rigid member implying that there is no relative deformation at the connection between concrete structure and deviators. In addition, the strain in external tendon is assumed to be uniformly distributed over the entire length of beam. Therefore, we can calculate the elongation of external tendon directly from the known deformations (θ, δ) of two consecutive deviators, the total elongation of tendon along entire beam length (ΔL) and strain increase in external tendon $(\Delta\varepsilon_{ps})$, respectively. This calculation was implemented in the step of calculating force in external unbonded tendons rather than the compatibility of deformation in the analytical program. After obtaining the new value of strain in external tendon, the new deformations of concrete structure at deviators will be used for the next step of iteration until an allowable error is obtained.



L_{ij} : beam length between i and j
e_1, e_2 : eccentricity of tendon
θ_1, θ_2 : rotation of beam
δ_1, δ_2 : deflection of beam

◆ Initial elongation of tendon (L_i)

$$L_{01} = \sqrt{L^2 + e_1^2}$$
$$L_{12} = \sqrt{L^2 + (e_2 - e_1)^2}$$
$$L_i = L_{01} + L_{12}$$

◆ Final elongation of tendon (L_f)

$$L_{01'} = \sqrt{(L_{01'})_x^2 + (L_{01'})_y^2}$$
$$= \sqrt{(L - e_1 \sin\theta_1)^2 + (\delta_1 + e_1 \cos\theta_1)^2}$$
$$L_{1'2'} = \sqrt{(L_{1'2'})_x^2 + (L_{1'2'})_y^2}$$
$$= \sqrt{(L + e_1 \sin\theta_1)^2 + (\delta_2 - \delta_1 - e_1 \cos\theta_1)^2}$$
$$L_f = L_{01'} + L_{1'2'}$$

◆ Strain increment in tendon $(\Delta\varepsilon_{ps})$

$$\Delta L = (L_{01'} + L_{1'2'}) - (L_{01} + L_{12})$$
$$\Delta\varepsilon_{ps} = \frac{\Delta L}{L_i}$$

Fig. 6 Deflected shape of simply supported beam under loading

5. COMPARISON WITH GEOMETRICAL COMPATIBILITY OF EXTERNAL TENDON

An experimental program of the externally PC beam with large eccentricities carried out by Aravinthan et al. [7] was used for a verification of both analytical methods. The tested beams consisted of monolithic and precast segmental simply supported beams as shown in Fig.7. The material properties and test variables are given in Table 2. All tested beams were provided with internal tendons for preventing crack at the extreme fiber of concrete section due to the self weight and initial prestressing moment from the external tendon. In order to reduce the effect of friction force, the teflon sheets were inserted at the surface between deviators and external tendons.

A comparison between the experimental results and the analytical predictions based on both analytical concepts is summarized in Table 3. From the experimental results, all specimens were failed by crushing of concrete at critical section under loading point and the external tendon reached the yielding point.

Table 2. Test variables and materials of simply supported beams (D-1 and D-1a)

No.	Description	f_c' (MPa)	Main Reinforcement	Prestressing Tendons	
				Internal	External
D-1	Monolithic	57.2	-	Bond 4-T9.3 (4 x 50 kN)	1-T10.8 (25 kN)
D-1a	Segmental	70.1	4-DB10	Unbond 2-T12.7 (2 x 100 kN)	1-T10.8 (25 kN)

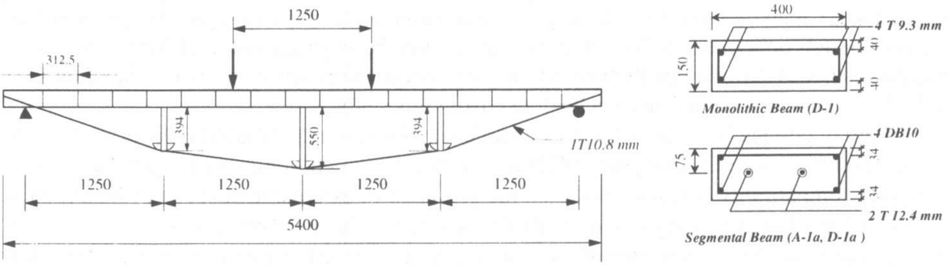


Fig. 7 Layout of simply supported beam (D-1a) and cross section details

Table 3. Summary of experimental results and analytical predictions

No.	Description	Ultimate Load (kN)			Ultimate Deflection (mm)			Ultimate tendon force (kN)		
		Exp	Def	Geo	Exp	Def	Geo	Exp	Def	Geo
D-1	Monolithic	94.5	92.9	93.2	130.3	94.2	96.4	118.1	120.4	120.5
D-1a	Precast segment	86.3	86.7	87.2	100.2	72.8	74.3	114.3	113.5	113.8

Note:*1. Exp: experiment, Def: compatibility of deformation, Geo: geometrical compatibility of external tendon
*2. The ultimate stage is defined as the crushing of concrete at the critical section

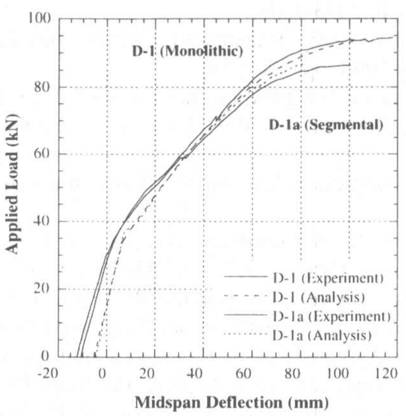


Fig. 8 Load and deflection relationship

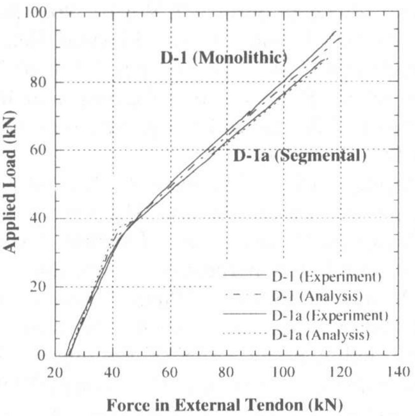


Fig. 9 Force in external tendon

Comparing the experimental results with the analytical predictions based on a nonlinear analytical methodology by using geometrical compatibility of external tendon is shown in Figs.8 and 9. It can be seen that both the flexural strength and ultimate tendon force can be predicted with a good accuracy. Therefore, it can be concluded that such an analytical methodology can be used as a compatibility condition for analyzing the flexural behavior of externally PC beam with large eccentricities with a good accuracy. However, there is a need to extend this methodology for continuous beams as well.

6. CONCLUSIONS

The analytical methodology for ultimate flexural strength of externally PC beams with large eccentricities based on compatibility of deformation has been discussed and verified with the experimental results. A different method based on geometrical compatibility of external tendon is also proposed. The following conclusions can be drawn from this study:

- The concept of compatibility of deformation can be applied for calculating the flexural strength of two-span precast segmental externally PC beams with large eccentricities with a good accuracy.
- A different compatibility condition based on the geometry of external tendon can also be used as a basis in analytical methodology of externally PC beams with large eccentricities.
- The advantage of the second method is that it can be utilized without an assumption of the imaginary concrete strain at tendon level. And this concept is believed to be also applicable for the flexural analysis of two-span continuous externally PC beam with large eccentricities.
- Further study is necessary to extend the proposed methodology for continuous beams with large eccentric external tendons.

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