論文 Necessity of Development of Multiple Damage Model for Analysis of Reinforced Concrete Structures

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ABSTRACT: Gupta et. al.[1,2] had proposed unified concrete plasticity model that can simulate stress-strain in both tensile and compressive region appropriately. It is realized that this type of model, where classical plasticity approach with *one damage* parameter is implemented, cannot simulate stress-strain under cyclic conditions. In this paper, it is proposed to analyze the possibility of use of multiple damage parameter to simulate stress-strain under such cyclic conditions. Two cases of uniaxial and biaxial cyclic cases are studied to demonstrate the possibility of the development of multiple damage model.

KEYWORDS: cyclic loading, multiple damage, concrete, plasticity, 3-D, classical plasticity, unified concrete plasticity model

1. INTRODUCTION

Development of analytical models for analysis of reinforced concrete structures is a very complicated subject. Different researchers have attempted to simulate the behavior of reinforced concrete member using one dimensional models, like beam theory, etc. to two or three dimensional models. In the case of two dimensional analysis, various researchers have adopted discrete crack approach. This approach is very successful in simulating RC members failing in shear mode. However this model has its own limitation of the requirement of defining the crack beforehand and is rarely adopted in the three dimensional analysis. Except in the one-dimensional models and some simplified two and three dimensional analysis, most of the researchers have restricted their research to the simulation of RC members under monotonic loading conditions.

Gupta et. al.[1,2] had presented unified concrete plasticity model that can simulate behavior of concrete in three dimension condition. This model was basically a classical plasticity model where Drucker-Prager model was modified such that we have a more triangular cross-section in tensile zone and a more circular cross-section in compression zone. In this model, the parameters of cohesion C and friction angle ϕ are the most important parameters. By controlling the variation of these parameters appropriately in tensile and compressive zone, this model could simulate satisfactorily simulate stress-strain of condition in all biaxial conditions without changing the model parameters in tensile and compression zone. However, limitation of this model was realized when attempting to simulate the stress-strain behavior of concrete under cyclic condition. It was realized that even though it is possible to simulate stress-strain of concrete in a unified manner without changing the adopted parameters, it is impossible to simulate the cyclic stress-strain relationship with a model with one damage parameter only.

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In this paper, the necessity of development of a model with multiple damage parameters is presented. The main problems faced in the development of this type multiple damage are presented to initiate a debate in this line. Though it is realized that implementation of multiple damage parameter adopting classical plasticity is a big problem, it was thought that it might be worth to investigate the number of damage parameters that is required and the inter-relationship of the different damage parameters. Most important problem in the development of appropriate model simulating stress-strain of concrete under cyclic condition is the question of what should be the stress-strain in such conditions. Very few experimental results exist in these conditions because it is extremely difficult to carry out such experiments and requires special experimental setup. Interpretation of these experimental results is also an important question.

In this paper, different basic stress-strain situations that show the relationship between different damage parameters are adopted, the experimental or numerical models that exist for such conditions are summarized and finally attempt is made to simulate such behavior by the unified concrete plasticity model adopting inter-dependent multiple damage parameter.

This paper shows that the initial stages of the development of the multiple damage parameter model and shows the requirement of additional experimental work that might be necessary to fully understand the inter-relationship between these damage parameters.

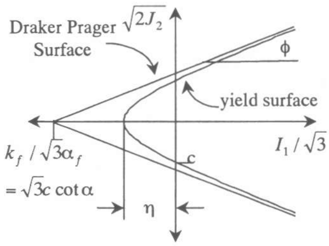


Fig. 1: The Unified Concrete Plasticity Model

2. THE UNIFIED CONCRETE PLASTICITY MODEL AND ITS LIMITATIONS

Gupta et. al.[1,2] presented the unified concrete plasticity model for the simulation of concrete stress-strain in three dimensional condition. This model adopted modified Draker-Prager model as shown in Fig. 1 with appropriate variation of cohesion C and friction angle ϕ in tensile and compressive zone(Eq. 1). $X(=I_1/\sqrt{3J_2})$ is adopted to define the variation in the transition zone.

$$C=\gamma C_0\exp\left[(-m_1\omega)p_1(X)+(-m_2^2\omega^2)p_2(X)\right]+(1-\gamma)C_0$$

$$\phi=\begin{cases}\phi_0+(\phi_f-\phi_0)\sqrt{(\omega+k)(2-\omega-k)}\,p_2(X) & \omega\le 1\\ \phi_0+(\phi_f-\phi_0)p_2(X) & \omega>1\end{cases}\qquad(1)$$

This model could satisfactorily simulate stress-strain in different biaxial conditions without adopting different set of parameters in tensile and compressive zone. Fig. 2 shows comparison of peak stresses in proportional biaxial conditions showing good match with Kupfer's[3] experimental results.

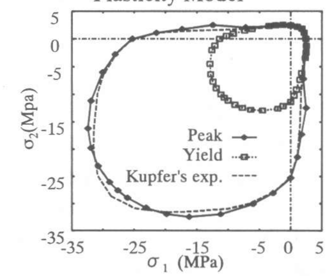
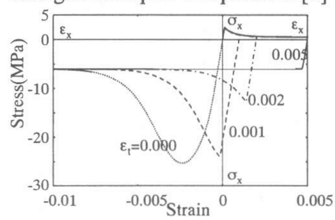
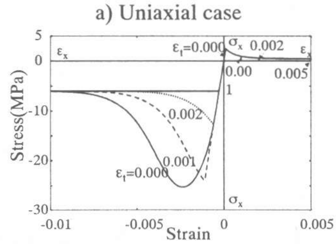


Fig. 2: Comparison with peak strength of Kupfer's experiment[3]

Fig. 3 shows the limitation of this model in simulation of the cyclic stress-strain conditions. This is because this type of approach adopts a single damage parameter for the calculations. Once damage accumulates in a particular path, this value remains in memory. If we unload and load is some other loading path, this model would reflect the damage accumulated in previous path. This is contrary to the experimentally observed facts. For example, if we take a cracked RC specimen and load it compressive loading



a) Uniaxial case



b) Biaxial case

Fig. 3: Stress-strain using one damage parameter

perpendicular to the crack orientations, we do not expect much reduction of compressive stress. This condition is similar to case where tensile stress is applied followed by compressive stress in same direction. Experiments with cyclic tensile stress and compressive stress in perpendicular direction, shows that the damage are interrelated. Maekawa et. al.[4] have performed experiments of plain concrete and Hsu and his research group[5,6] and other researchers have performed such experiments on RC specimen which show that damages in this two directions are interrelated. This is because of the fact that both cases produce cracks in similar direction. Though Gupta et. al.[1,2] have argued that unified concrete plasticity model can simulate stress-strain under various proportional loading conditions from tensile to compressive region without adopting different set of parameters, it can be realized that model adopting single damage parameter can not satisfactorily simulate stress-strain under cyclic conditions satisfactorily.

3. DEVELOPMENT OF MULTIPLE DAMAGE PARAMETER MODEL

In previous section, it was clear that it is important to develop a strategy and model that can take care of multiple damage parameters. It is assumed that we need ***6 damage parameters, one each in tensile and compressive region in all the three directions***. Now as explained in previous sections, it is expected that these damage parameters will be different, however interrelated. This development has two sets of problems: a) Development of a strategy to implement multiple damage parameter, b) determine the relation between different damage parameters. Though the first is important, it is possibly practical to pursue the later in the initial stages. After the relationship between different damage parameters are clear, we can possible think of the strategy to integrate the 6 parameters in a logical manner.

One more severe problem exists in this development. Experimental data are very rare in this field. Hence whatever data exists in this field is very important and we have to interpret the data carefully. For example, we have experiments of RC and plain concrete members. Whereas experiments on plain concrete end at peak strength at cracking or crushing point, experiments on RC specimens go much beyond the peak. Hence experimental results have to be carefully considered in developing this multiple damage parameter model.

This paper presents two case studies, which show clearly that it might be fruitful trying to develop this type model. Further experimental and careful analytical consideration is necessary before the full establishment of this type of model.

4. DITERMINATION OF RELATION BETWEEN VARIOUS DAMAGE PARAMETERS

Two case studies in determining the possible relation of damage parameters are present here. In classical plasticity, only one damage parameter can be implement. Hence, we switched the damage parameters when we adopted a different stress path. The other parameters adopted in this analysis are: C_0=28.25, ϕ_0=5, ϕ_f=22, f'_c=25.2N/mm^2, f_t=2.52N/mm^2, E_c=21700 N/mm^2, μ=0.22, k=35, ω_1=2.5, ω_2=1.0, β=0.82[1,2]. This study intends to check the feasibility of implementation of multiple damage parameters. Hence, empirical formulas derived may not be general.

4.1 CYCLIC TENSILE AND COMPRESSIVE LOADING IN SAME DIRECTION

Experimental studies exist that shows the possible stress-strain relationship of concrete in both uniaxial tension and compression. While post peak tensile behavior is said to depend of fracture criteria or tension stiffness effect depending on the concrete is part of plain concrete or reinforced concrete member or zone, post peak behavior of concrete under compression is assumed to undergo gradual softening. However, what should be the exact nature of the post-peak softening is still a matter of further research. But whatever may be the softening slope in either of these cases, Gupta et. al.[1,2] have shown that the unified concrete plasticity model can simulate them to the satisfaction of the user by changing the rate of change of Cohesion C and Friction angle ϕ of Eq. 1.

There are various experimental work and analytical models about the unloading branches of concrete under uniaxial tension and compression[7-11]. It has been shown that unloading stiffness gradually undergoes degradation both in uniaxial tension and compression. There is hysteretic loop in unloading and reloading also in both the cases. Though experiments exist showing the cyclic stress-strain in individual case of uniaxial tension and uniaxial compression, experiments are not available combining the two situations.

In this paper, well-known focal point model[7,8] is adopted as reference. In this model, stress unloads toward a focal point, ($-f_c$,$-f_c/E_c$) and ($-f_t$,$-f_t/E_t$) in tension and compression respectively. The stiffness degradation could be simulated quite easily based on the following assumptions.

a) Damage parameters ω_{1c} in compression and ω_{1t} in tension are independent parameters.
b) Stiffness degradation is achieved the simple formula [E]= α[D], where α is given in Eq. 2 to match the expected results of focal point model(Fig. 4). Eq. 2 implies that degradation is a direct function of the damage in respective condition.

$$\alpha_c = 0.97e^{-3.5\omega} + 0.03(1-0.06\omega) \qquad (2.a)$$

$$\beta = f_t[0.18 + 0.82\exp(-m_1 m_2(\varepsilon - \varepsilon_t))]$$
$$\alpha_t = (\beta + f_t)/(\omega / m_2 + 2\varepsilon_t) \quad m_1 = 2.5, m_2 = 603 \qquad (2.b)$$

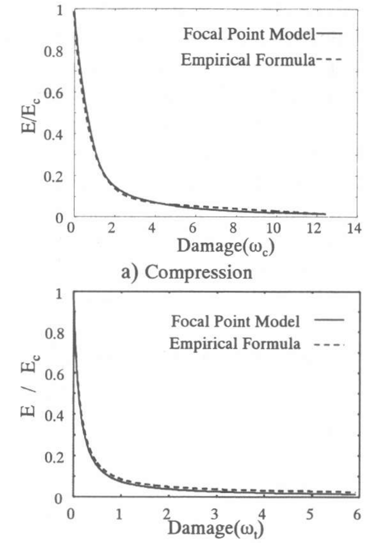


Fig. 4: Stiffness degradation

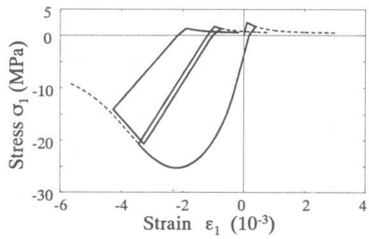


Fig. 5: Cyclic stress-strain in uniaxial condition

It was possible to determine exact relation for uniaxial tension (purposefully shown little differently in the figure), where as the empirical formula uniaxial compression is an approximate equation. Fig. 5 shows the stress-strain under cyclic stress conditions. The dotted line show the stress strain if the particular stress path is followed in place of the reverse path in cyclic loading. The adoption of independent damage parameters is well justified at least in the initial stage because crack produced by the tensile stress does not create weakness for the compressive stress in same direction. In this analytical experiment, it is not yet possible to simulate the hysteretic loop.

4.2 CYCLIC TENSILE AND COMPRESSIVE LOADING IN PERPENICULAR DIRECTION

When compressive stress is applied, micro cracks and at later stages visible cracks appear in orthogonal direction. This is the same direction in which crack would appear if tensile stress is applied in the perpendicular direction. Hence it can be expected that damage parameters for tension and compression in perpendicular directions be interrelated.

In the experiment of tensile load applied by compressive load of RC member, Hsu and his research group[5,6] have shown through experiments of RC members that the relation depends on sequential or proportional loading. They have also shown that the case of sequential load where compressive loading is applied without unloading, the tensile load yields results comparable to the case of proportional loading. In case of proportional loading, softening of both peak stress and peak strain was observed, whereas only softening of peak stress was observed in case of sequential loading (where tensile load is unloaded more then 90% level). In this case of numerical experiment, the case of sequential loading after full unloading on initial loading path is considered.

(1) Simulation of compressive loading after by tensile load is unloaded

Maekawa et. al.[4] had performed experiment on plain concrete member. This set of experimental results exists for various level of compressive load, where no experimental results exist for the post peak region (beyond 1.09 ε_0). Two important observations in this experiments are : a) Softening of the stiffness or slope of stress-strain curve $E_{1t\text{-}M}$, b) Softening of peak stress $\sigma_{1t\text{-}M}$, where subscript $_M$ represent experimental result by Maekawa. et. al.[4].

The present unified concrete plasticity model implements tension stiffening effect for reinforced concrete member. In uniaxial tension, a particular point(σ_{1t},ε_{1t}) on the softening curve represent a particular value of damage ω_{1t}. The unloading slope E_{1t} depends on the ω_{1t} as shown in Eq. 2b. Hence if we assume ω_{1t} and ω_{2c} are interrelated, i.e. ω_{1t} develops due to development of ω_{2c} in compression, then the stress strain curve will start parallel to the unloading slope E_{1t} and the peak stress σ_{1t} depending on damage ω_{1t}. Hence we can understand that it is impossible to match both stiffness and peak stress and would depend on the assumed softening slope of the uniaxial tension curve.

To correlate ω_{1t} and ω_{2c}, we assume E_{1t}= $E_{1t\text{-}M}$. Fig. 6 shows the flow chart for the calculation and plotted in Fig. 7. This relation is quite linear and can be written as

$$\omega_{1t} = \alpha\, \omega_{2c} \quad \text{where } \alpha = 0.58 \qquad \text{Eq. 3}$$

Fig. 8 shows the stress strain behavior under such cyclic loading condition.

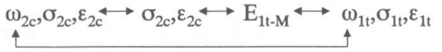


Fig.7 Flow Chart to relate ω_{1t} and ω_{2c}

(2) Simulation of tensile loading followed by compressive load is unloaded

We assume same relationship between ω_{1t} and ω_{2c} determined in previous section in Eq. 3 is also valid here. Fig. 9 shows the stress strain behavior under such condition. Fig. 10 shows the peak stress softening in comparison the experimentally derived relationship by Hsu et. al. Though it does not match properly, both show downward trend. In this case we are trying to match results for RC specimen based on relationship derived from experimental results of plain concrete specimen. Hence more experimental study and in depth consideration is required.

This clearly shows that quite logical results can be simulated using multiple damage model.

5. CONCLUSION

To overcome the limitation of approaches using classical plasticity with only one damage parameter in describing the cyclic stress strain relationship, this research attempts to implement multiple damage parameters. Unified concrete plasticity model proposed by Gupta et. al.[1,2], a model that can simulate stress-strain properly in most proportional case is adopted in this analysis. Though it is important to find a methodology to implement the 6 (one each in tension and compression in all the

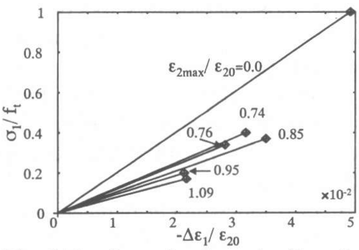


Fig. 6. Tensile strain after unloading of compressive stress[4]

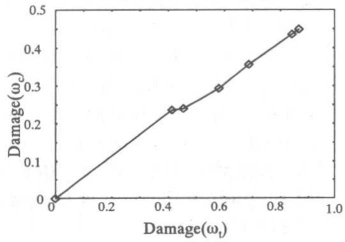


Fig.8: Relation of damage parameters

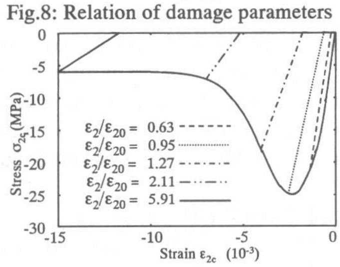


Fig. 9: First stage of compressive loading

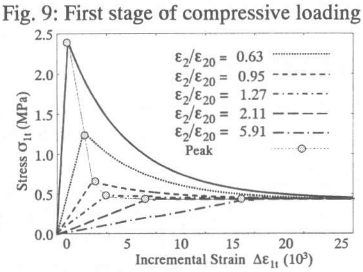


Fig. 10: Second stage of tensile loading

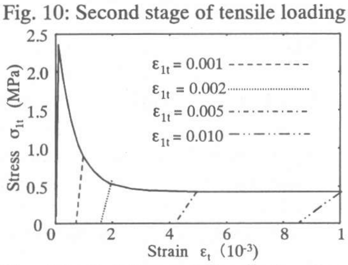


Fig. 11: First Stage of Tensile Loading

three axis) parameters together, it was decided that it is more important to check the possible relations between the damage parameters.

In this paper, two case studies of cyclic loading of tension compression in uniaxial and biaxial condition are considered. The following conclusions were drawn from the case studies:

a) Two independent damage parameters for uniaxial case were assumed. After implementation of appropriate proportional softening of unloading stiffness, it was possible to simulate the stress-strain relation similar to the popular *focal point model*.
b) In biaxial case, direct relationship between the two damage parameters were derived from experiment of plain concrete specimen by Maekawa et. al.[4]. Logical stress-strain analysis was observed, after implementation of this model and the softened stiffness for unloading derived in the uniaxial case.

From the above analysis it was understood that it might be worth trying to implement the multiple damage model. The most important problem faced in this research is the scarcity of experimental results. Authors are at present looking for experimental work on plain or reinforced concrete members due to sequential biaxial compressive loading to understand the possible relationship between damages in compression in different directions.

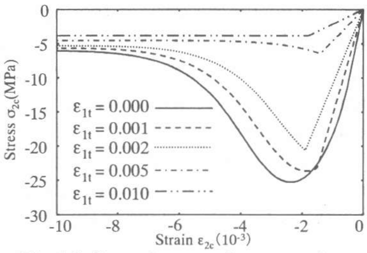


Fig. 12: Second stage of compressive loading

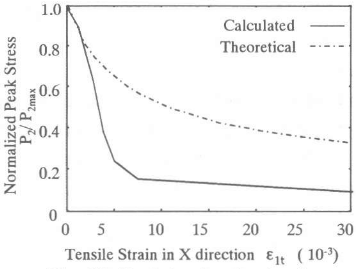


Fig. 13: Peak load softening in comparison to experimental results [5,6]

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