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論文 Fatigue Life Analysis of Reinforced Steel-Fiber-Concrete Beams

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ABSTRACT: A procedure for predicting the fatigue life of reinforced concrete members is proposed. An analytical model for reinforced concrete (RC) and reinforced steel-fiber-concrete (RSFC) beams without stirrups is developed based on FEM with discrete crack concept. The mechanical degradation of materials under fatigue load is considered as a major factor for fatigue crack propagation and subsequent fracture. The results from the analysis show that fibers can improve fatigue resistance of RC beams by extending fatigue life and changing the failure mode from shear to flexure. It also reveals that fiber-concrete could be applied for the repair of structures subjected to fatigue load.

KEYWORDS: fatigue life, steel-fiber-concrete, shear failure, flexural failure, FEM analysis, material degradation

1. INTRODUCTION

Steel-Fiber-Concrete (SFC) is widely introduced for the repair of structural members. According to its improved fatigue resistance and crack control capacity, it is increasingly applied for repairing concrete structures subjected to fatigue load, such as reinforced concrete bridge decks. Now a day, bridge decks, which are expected to directly resist millions of cycles of repeated axle loads from vehicles, deteriorate significantly due to the continuous increase in traffic amount. The repair of the deteriorated bridge decks is urgent for extending service life before their failure. Among several repair methods, one promising and cost-effective method is the use of SFC.

The examination of fatigue life is necessary process for developing a suitable repair method. Experiments under moving wheel-load conditions have been used in order to predict the fatigue life of bridge decks. Although this approach can establish a fatigue load-life relation of bridge decks, a large number of specimens have to be tested for more than one million cycles of loading to obtain a complete fatigue life relation, which makes this approach time-consuming. Moreover, it lacks theoretical support and the consideration of failure mechanisms. Therefore, experimental approach is not suitable for the fatigue life prediction of existing structures that concerns multiple governing parameters. An analytical model that is based on the governing mechanisms and governing parameters is one preferable way for the prediction of fatigue life. However, at present, there is no analytical model based on theoretical study developed.

The objectives of this study are to propose such an analytical model and to demonstrate the improvement of fatigue resistance resulting from the addition of steel-fibers in RC structures. An analytical model of slender beams without web reinforcement is developed using 2D FEM analysis as an elementary step towards further development.

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2. FATIGUE LIFE ANALYSIS

The analytical scheme for predicting fatigue life of structures consists of two components: analytical model and material models as shown in Fig. 1. An FE analytical model is developed based on fatigue failure mechanisms of structures. In reinforced concrete structures, the propagation of a crack leads to the failure of structural members. The deterioration of materials under fatigue load promotes the propagation of cracks, and it leads to the failure of the structures. Material test is conducted in order to obtain the material models of concrete or SFC and a reinforcing bar. With this scheme, the fatigue properties of structures, such as fatigue life and crack propagation, can be predicted for given materials' properties and given geometries.

2.1 ANALYTICAL MODEL OF RC BEAMS

Slender RC beams without web reinforcement generally fail in two failure modes: flexural failure or diagonal tension failure. Mode of failure depends mainly on reinforcement ratio and shear span to effective depth ratio. Failure of an RC slender beam is caused by one localized crack, either the flexural crack or the shear crack. For the flexural crack, the mode I cracking criterion can be clearly applied, since no shear stress or shear slip occurs along the crack path. For the shear crack called a diagonal tension crack, although this crack is pointed out to be in a mixed mode between mode I and mode II, it has been reported that this crack can be treated as a mode I crack if the location of the crack is properly selected [1]. Therefore, the same crack properties as flexural cracks are assumed for diagonal tension cracks in this study.

To represent the localized cracks, which cause the failure of beams, the concept of a fictitious crack originally proposed by Hillerborg [2] is adopted in this study. Interface elements are used for representing either shear or flexural cracks. A fictitious flexural crack path is selected a priori at the midspan which experiences maximum moment. A critical shear crack path can be determined by varying the angle and position of a crack in the monotonic analysis. The path that provides the minimum ultimate load is the critical crack path. By taking advantage of symmetry with respect to the centerline, only half of a beam is analyzed as shown in Fig 2.

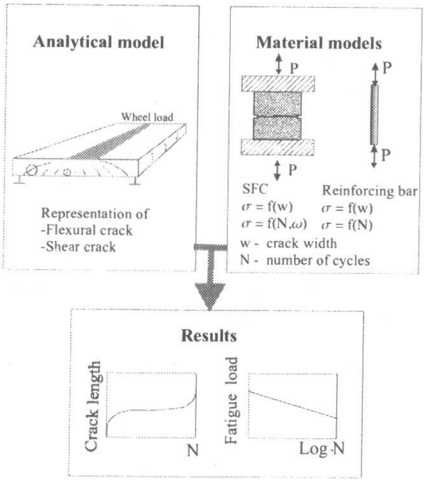


Fig. 1 Components of fatigue life analysis

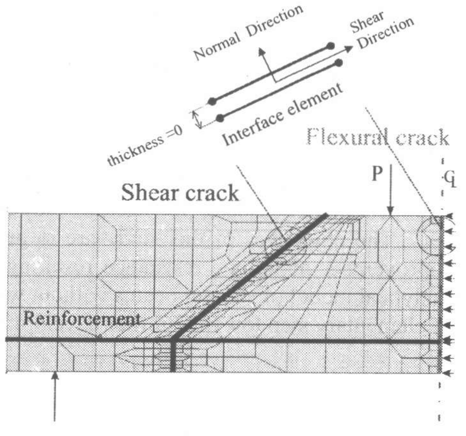


Fig. 2 FE representation of RC beams

2.2 MATERIAL MODELS

Materials treated in the analysis are concrete or SFC and reinforcing bars. Four-node rectangular elastic elements with linear elastic stress-strain relation are used for representing concrete, SFC, and reinforcing bars, while interface elements with non-linear stress-displacement relations are assigned for representing the stress transmission of cracks and reinforcing bars on the crack planes. Young's Modulus and Poison's ratio are assigned for linear stress-strain relation of elastic elements. For interface elements, non-linear stress-displacement relations are used to describe the bridging relation of concrete or SFC and pull out relation of a reinforcing bar. Explanations of the non-linear relations follow below.

(1) Bridging Relation of Concrete or SFC

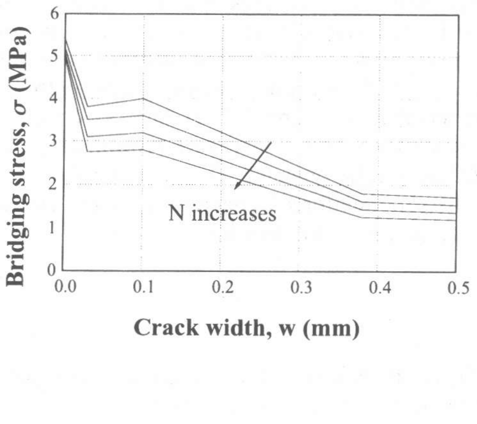
A theoretical model for the bridging stress-crack width (σ-w) relation based on micromechanical viewpoints was proposed by Li et al. [3, 4]. The bridging relation of SFC under monotonic load can be obtained by uniaxial tensile test. The bridging stress degradation under fatigue load is governed by the cracking of concrete and the debonding of fibers. Bridging stress decreases with increase of crack width because stress transfers across crack decrease. As a result, material degradation law of concrete or SFC is a function of maximum crack width, and number of cycles, N, and it can be expressed as

$$\frac{\sigma_N}{\sigma_1} = f(w, N)$$

where f(w,N) is a bridging degradation factor, which is less than 1. σ_N and σ_1 are the bridging stress at N-th cycle and first cycle, respectively. Fig. 3 shows the bridging relation of straight steel fiber concrete under fatigue load by which the bridging stress decreases with increasing number of cycles. The relation is based on the study of Zhang et al. [5].

(2) Pull Out Relation of a Reinforcing Bar

For the representation of bond between concrete and reinforcing bars in the FE model, instead of including multiple local bond elements in the FE model as in the conventional FE representations, these local bond components are integrated into only one element, which is a pull out interface element on a crack plane.



$$\frac{\sigma_N}{\sigma_1} = \begin{bmatrix} 1 + k_1 \log(N) & 1 \le N \le 10 \\ 1 + (k_1 - k_2) + k_2 \log(N) & 10 < N \le 10^4 \\ 1 + (k_1 - k_2) + 4(k_2 - k_3) - k_3 \log(N) & 10^4 < N \le 10^5 \end{bmatrix}$$

$$k_1 = \begin{bmatrix} B_1 w & 0 \le w \le w_{01} \\ (B_1 - B_2) w_{01} + B_2 w & w_{01} \le w \le 0.5 \end{bmatrix}$$

B_1= -1.43 mm^{-1}, B_2= 0.339 mm^{-1}, and w_{01}= 0.166 mm

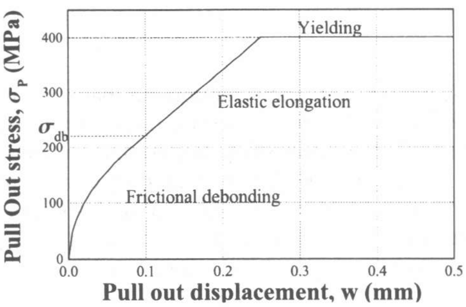
$$k_2 = \begin{bmatrix} C_1 w & 0 \le w \le w_{02} \\ (C_1 - C_2) w_{02} + C_2 w & w_{02} \le w \le 0.5 \end{bmatrix}$$

C_1= -0.9 mm^{-1}, C_2= 0.0982 mm^{-1}, and w_{02}= 0.07 mm

$$k_3 = \begin{bmatrix} D_1 w + A_3 & 0 \le w \le w_{03} \\ (D_1 - D_2) w_{01} + A_3 + D_2 w & w_{03} \le w \le 0.5 \end{bmatrix}$$

D_1=-0.777mm^{-1}, D_2=0.0704 mm^{-1}, w_{03}= 0.109mm, A_3=-0.0432

Fig. 3 Bridging relation of SFC under fatigue load



$$\sigma_P = \begin{bmatrix} \frac{\pi}{2A_S}\sqrt{E_S d_S^3 \tau_b w} & 0 \le \sigma_p < \sigma_{db} \\ \frac{E_S w}{l_b} & \sigma_{db} \le \sigma_p < \sigma_y \\ \sigma_y & \sigma_y \le \sigma_p \end{bmatrix}$$

$$\sigma_{db} = f_t * A_c \text{ and } l_b = \frac{f_t * A_c}{\tau_b u}$$

Fig. 4 Pull out relation of a reinforcing bar

The pull out stress and pull out displacement relation can be divided into three regions: 1) frictional debonding, 2) elastic elongation, and 3) yielding as shown in Fig. 4. First, the interface between concrete and a reinforcing bar undergoes a frictional debonding procedure. The debonding length gradually increases with the increase of load, so the stiffness of pull out relation in the first zone gradually decreases. When pull out stress reaches full debonding stress, σ_{db}, debonding length extends to all the interface between the concrete and the reinforcing bar, the bar elongates elastically until it reaches the yield strength. In this model, perfect plasticity is assumed so that the pull out stress keeps constant after yielding. In Fig. 4, A_s, d_s and Es are cross section area, diameter and Young's Modulus of the reinforcing bar. τ_b is the interfacial bond stress and u is the perimeter of the reinforcing bar. A_c is the area of the concrete surrounding the reinforcing bar and f_t is the tensile strength of the concrete. σ_y is the yielding stress of the reinforcing bar.

Fatigue relation or S-N relation of the reinforcing bar is a commonly used function which depends on only number of cycles, N. Fatigue life for 1,000 cycles is attained at 90% of the ultimate strength, and endurance limit for 1,000,000 cycles at 50%, as shown in Fig. 5 [6]. $\sigma_{p,N}$ and $\sigma_{p,1}$ refer to the steel stress at N-th cycles and first cycle, respectively.

2.3 PROCEDURE OF FATIGUE LIFE ANALYSIS

The procedure of fatigue life analysis is shown by a simple example of a simply supported plain SFC beam subjected to a fatigue load (Fig. 6). The constant fatigue load, P_k, which is below the ultimate load, is applied in fatigue life analysis. At the first cycle, the crack propagates by the length, a_0, with bridging stress distribution along the crack shown by dash line. When P_k at the second cycle is applied, the crack opens again, and the bridging stress reduces in the cracked area according to the material degradation laws. As a result, the internal resistance cannot balance with P_k. Accordingly, newly cracked area with additional length, Δa_1, has to be developed in order to balance with P_k. At the Nth cycle, the cracked area is divided into N sections with different degree of degradation from 1 to N, and newly cracked area with new additional length, Δa_{N-1}, opens. This process continues repeatedly until the equilibrium for this beam cannot be satisfied with further crack propagation. This moment defines the fatigue life, and the cycles to failure can be obtained. By including the stress degradation of reinforcing bars, the fatigue life analysis of reinforced concrete members can be executed for both shear and flexural failures by applying the proposed procedure.

3. FATIGUE LIFE COMPUTATION

The effect of fibers on the increase of fatigue life of RC beams without stirrup is investigated. Fatigue life of RSFC and RC beams is examined by using the proposed analytical model. For material properties, Young's Modulus and Possion's ratio of either concrete or SFC are 30 GPa and

0.17, respectively. The tensile strength of concrete and SFC with 1% fiber volume fraction are 3.0 MPa and 3.2 MPa respectively. The bridging relations of SFC under fatigue load as shown in Fig. 3 are adopted. Young's Modulus, Possion's ratio and diameter of the reinforcing bar are 200 GPa , 0.2, and 16 mm, respectively. The interfacial bond stress is equal to 6.2 MPa.

3.1 SINGLE CRACK FATIGUE LIFE ANALYSIS

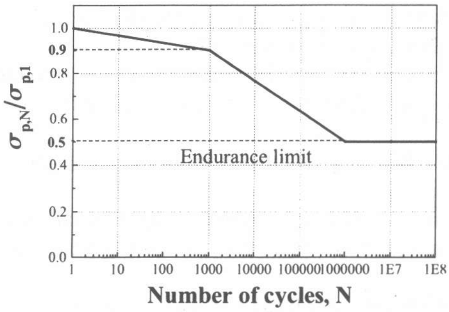
In single crack fatigue life analysis, shear and flexure failures are separately considered by including only one kind of fictitious crack in a beam model. Fig. 7 shows the relationships between fatigue load and cycles to failure (the fatigue load-life relations) of both RC and RSFC beams. The results show that RSFC beams have a longer shear fatigue life than flexural one. Thus, this beam is expected to fail in flexure. On the other hand, RC beams show a shorter shear fatigue life than flexural one, so shear failure is expected.

In addition, it can be noticed that the flexural fatigue life of RC and RSFC beams show nearly the same curves, and these curves are similar to a fatigue fracture relation of a steel reinforcing bar. This is caused by the fact that flexural fatigue failure is governed by the fatigue failure of a reinforcing bar. In contrast, the shear fatigue life of RSFC beams is much longer than that of RC beams. This is because shear fatigue life is controlled by the fatigue failure of concrete matrix. The presence of fibers in concrete slows down the rate of bridging stress degradation of concrete, and it efficiently contributes to crack tip shielding of a fatigue crack. It can be concluded that SFC under fatigue loading performs much better fatigue resistance than normal concrete.

3.2 DOUBLE CRACK FATIGUE LIFE ANALYSIS

In double crack fatigue life analysis, both of a flexural crack and a shear crack are put simultaneously in the analytical model. The dominant failure mode and the fatigue load-life relations of both beams are examined and shown in Fig. 8. All points on the fatigue load-life relations of RC beams have shear failure, while all points of RSFC beams have flexural failure. These failures conform to the prediction from the single crack analysis.

It is found that the RSFC beam provides a longer fatigue life, and it exhibits flexural failure that is ductile. The addition of fibers in concrete leads to the improvement in fatigue resistance and significantly increases shear capacity of the beams. This assists in the change of failure mode from shear to flexure. It can be deduced that the addition of fibers in concrete leads to the improvement in fatigue resistance and the change of failure mode.



$$\frac{\sigma_{p,N}}{\sigma_{p,1}} = \begin{bmatrix} 1-[\log(N/1000)]/30 & 1 \le N \le 10^3 \\ 1.62 * N^{-0.085} & 10^3 < N \le 10^6 \\ 0.5 & 10^6 < N \end{bmatrix}$$

Fig. 5 Fatigue relation of a reinforcing bar

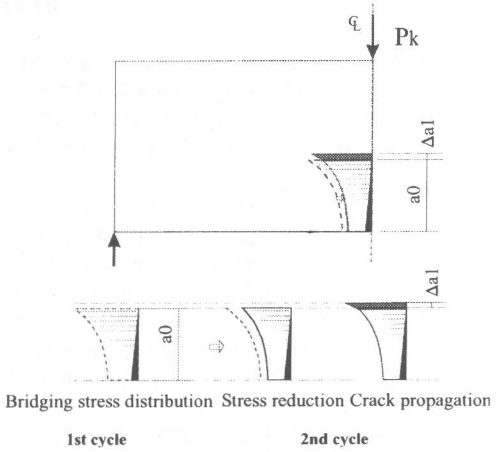


Fig. 6 Procedure for fatigue life analysis

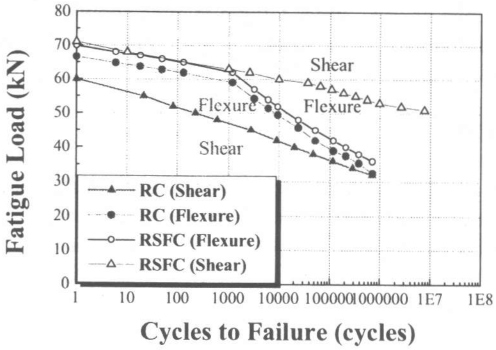


Fig. 7 Flexural and shear fatigue life of RSFC and RC beams

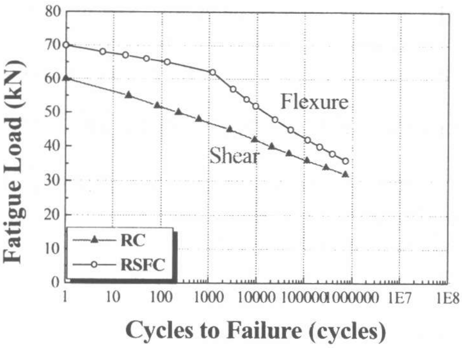


Fig. 8 Fatigue life of RSFC and RC beams from double crack analysis

4. CONCLUSIONS

The improvement of fatigue properties of RSFC over RC beam has been investigated. From the analysis, the addition of fibers leads to the improvement in fatigue life of reinforced concrete beams, particularly, the improvement in shear fatigue resistance. Shear fatigue life increases significantly, and thus, the failure mode of RC beams with fibers can be changed from shear which is brittle to flexure which is ductile. This also reveals that fiber-concrete could be used for the repair and improvement of structures subjected to fatigue load, such as bridge decks.

For the further study, in order to verify the proposed model, it will be needed to compare the predicted fatigue life from analysis with results from experiments of RSFC beams or RC beams. Moreover, in the proposed model, only the propagation of localized cracks is considered as a major failure mechanism. Therefore, the effect of minor cracks should be investigated whether they significantly affect the fatigue life of structures of not. Finally, the analysis and development of a bridge deck repair method by using fiber-concrete is one of the interesting topics for the future plan.

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