論文

COMPUTATIONAL MODEL FOR TIME-DEPENDENT COMPRESSIVE BEHAVIOR BASED ON PLASTICITY AND FRACTURING APPROACH

Khaled EL-KASHIF*1, Koichi MAEKAWA*2

ABSTRACT: As post-peak mechanics of concrete structures is highly time dependent in general, constitutive modeling of softened concrete under short-term straining is thought to serve collapse analysis of higher accuracy. The main objective of this research is to propose a time dependent constitutive model for hardened concrete under short-term uniaxial cyclic loadings, which cover both pre- and post-peak regions in compression. Towards better evaluation for structural performances, the coupled elaso-plastic-damaging law in both time and strain paths is presented. Derivation of constitutive laws from experiment is discussed and its verification is performed under arbitral loading paths.
KEYWORDS: softening, time-dependent plasticity, time-dependent damaging and creep rupture.

1. INTRODUCTION

Challenging efforts are being addressed to post-peak analysis of concrete structures and softened nonlinearity of concrete compression is regarded as one of key issues. The short-term time-dependency of concrete gets predominant when stress level exceeds 80% of the uniaxial strength and creep rupture is experienced within a shorter period of stressing [1]. This tendency is much accelerated beyond the peak. Then, it is thought that some consideration of time-dependency shall be made when the collapse analysis is conducted. Okamura et al. took into account this time-dependency of higher stresses by simply factorizing plastic evolution law for concrete in their static/dynamic structural analyses [2], because the strain rate reproduced in laboratory-based static structural experiments much differs from that under dynamic loads. For further generality, however, constitutive modeling of concrete should be explicitly formulated in coupled strain and time histories. The main objective of this research is to propose full-time path-dependent model for hardened concrete. Tabata generated combined creep-relaxation hysteresis in concrete close to the uniaxial strength and investigated short-term plastic and damaging evolutions, and it was concluded that the combined concept of elasto-plastic and fracturing can be applied to the pre- and post-peak time-dependent mechanics [3,4]. Based on this, Song et al. tried some formulation of complexity [5]. The authors try to extend the time dependent plasticity and damaging that is installable in nonlinear structural analyses. Here, discussion is made with strain rate ranging 0.1-1000μ/sec in nonlinear creep domain. The long-term creep under lower stress states is out of scope.

2. EXPERIMENTAL PROGRAM

2.1 MATERIAL

The work described in the following sections is based on the uniaxial compressive loading to concrete cylinders with 10 cm diameter and 20 cm height. The specimens were cured under the standard condition for 28 days and allowed to dry to laboratory ambient conditions during the further period of two weeks. The mix proportion is shown in **Table 1**. The average strain was measured by disp.-transducers.

Table 1- Mix Proportion

Material type	Water /Cement ratio	Cement (kg/m^3)	Sand (kg/m^3)	Coarse aggregate (kg/m^3)	Chemical admixture (mm^3/m^3)
Normal Concrete	0.5	352	866	890	440

*1 Doctor Student, Dept. of Civil Eng., The University of Tokyo, Member of JCI
*2 Professor, Dept. of Civil Eng., The University of Tokyo, Member of JCI

2.2 EXTRACTION OF PLASTIC AND FRACTURING EVOLUTIONS

The authors try to extend the elasto-plastic and fracturing (damage) concept [2] to the time-dependent non-linearity. Concrete is idealized as a set of elasto-plastic components, and the applied total stress is idealized as the summation of all stresses acting on un-damaged components. Damage is conceptually defined as a loss of components. Then, the stiffness degradation K defined as fracture parameter represents the ratio of active components that can sustain stresses as shown in Fig.1. Maekawa et al. formulated the evolution law of plasticity and damaging in terms of elastic strain that represents the intrinsic stress developing in the non-damaged components [2]. In this formulation, time-factor was not explicitly considered. Thus, the key issue is to formulate the plastic and damaging evolution not only by the elastic strain hysteresis but also by time.

In order to obtain the plastic and fracturing evolution by experiments, sustained higher stresses and/or greater strains were actuated in concrete cylinders, and unloading paths were put on the loading programs. Let A and B denote the states of concrete on stress-strain space as shown in **Fig. 1**. Unloading paths bring about increments of plastic strain and fracture parameter or their evolution rates. From A to B, the elastic strain also varies. Here, the average elastic strain between A and B can be assumed to represent the static state of alive components. Thus, we can have a set of ($d\varepsilon_p/dt$, dK/dt, ε_p, K, ε_e). If some generic relations are found among these variables, it may take position in the time-dependent constitutive modeling.

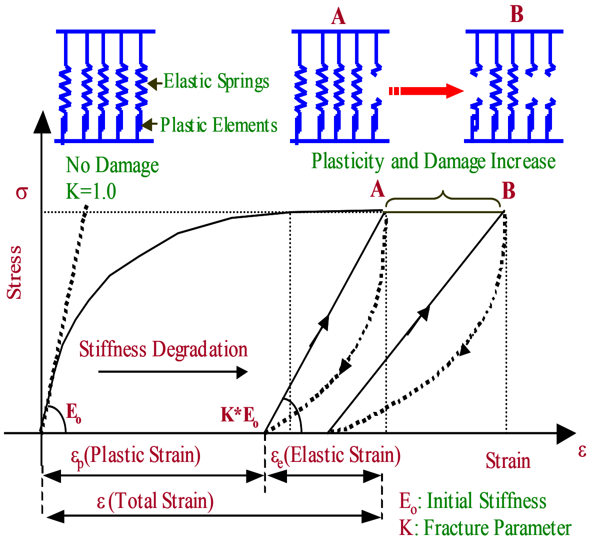
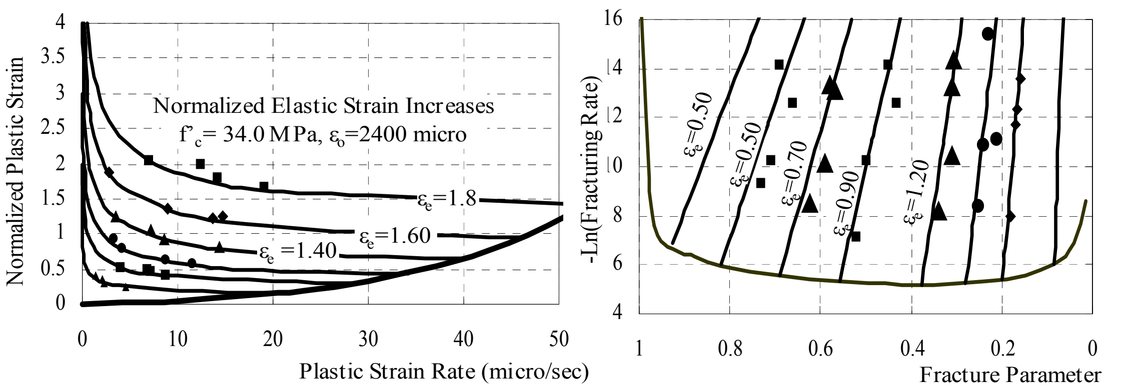


Fig. 1 Experimental Program

Fig. 2 shows the extracted rates of plasticity and fracturing from various loading paths under different stress/strain levels. The authors also included relaxation paths where the total strain was kept unchanged but the stress was degraded. It is clearly shown that the plastic rate under the similar elasticity is sharply reduced when the absolute plastic strain evolves. Just before creep failure, progressive total strain increase was experienced. In this case, the elasticity was simultaneously increased due to evolved fracture. Then, when we direct attention to the rate of plasticity under the similar elasticity level, we can pick up the pure plasticity of the components. This is qualitatively common with the long-term creep under the lower stress states where continuum damaging is rare. The fracturing rate is rapidly reduced when fracture itself proceeds under the similar elasticity that represents the internal stress intensity. With these facts, let us derive the explicit formulation of total stress-strain relation with consistency in the following section.



a- Normalized Plastic Strain versus Plastic Strain Rate   b- Fracturing Rate versus Fracture Parameter

Fig. 2 Extracted Plastic and Fracturing Evolution Rates.

3. MODELING

3.1 POTENTIAL TERM OF NONLINEARITY

The basic constitutive equations to express the elasto-plastic and fracturing (see Fig.1) and the general Taylor's series of state variables (plastic strain and fracture parameter) can be reduced to,

$$\varepsilon = \varepsilon_e + \varepsilon_p \quad , \quad \sigma = E_o \varepsilon_e K \tag{1}$$

$$d\varepsilon_p = \left(\frac{\partial \varepsilon_p}{\partial t}\right) \mathrm{dt} + \left(\frac{\partial \varepsilon_p}{\partial \varepsilon_e}\right) \mathrm{d}\varepsilon_e \quad , \quad dK = \left(\frac{\partial K}{\partial t}\right) \mathrm{dt} + \left(\frac{\partial K}{\partial \varepsilon_e}\right) \mathrm{d}\varepsilon_e \tag{2}$$

Where, K: fracture parameter, ε_e: elastic strain, ε_p: plastic strain, σ: stress, t: time and E_o: initial elastic modulus. For simplicity of formulation, let us define the above stress and strain as the normalized ones by the uniaxial compressive strength and the corresponding peak strain, respectively. Similar to the theory of plasticity, it was experimentally explored that plastic and fracturing potentials (F_p, F_k) exist, on which the derivatives with respect to the elastic strain increment in Eq.2 reveal non-zero. As the nonlinearity also proceeds with dF_p=0 and dF_k=0, we have,

$$\begin{aligned} &\left(\frac{\partial \varepsilon_p}{\partial \varepsilon_e}\right) = 0 \quad \text{when } F_p < 0, \qquad \left(\frac{\partial \varepsilon_p}{\partial \varepsilon_e}\right) = -(\partial F_p / \partial \varepsilon_e)/(\partial F_p / \partial \varepsilon_p) \quad \text{when } F_p = 0 \\ &\left(\frac{\partial K}{\partial \varepsilon_e}\right) = 0 \quad \text{when } F_k > 0, \qquad \left(\frac{\partial K}{\partial \varepsilon_e}\right) = -(\partial F_k / \partial \varepsilon_e)/(\partial F_k / \partial K) \quad \text{when } F_k = 0 \end{aligned} \tag{3}$$

Here, the authors employ the following plastic and fracturing potentials [2], which derived from experiments under higher loading rate, where the time dependent plasticity and fracture are thought to be small as,

$$\begin{aligned} &F_p = \varepsilon_p - 0.038\left(\exp\left(\frac{\varepsilon_e}{0.55} - 1\right)\right) \\ &F_k = K - \exp\{-0.73\beta(1 - \exp(-1.25\beta))\}, \qquad \beta = -\frac{1}{0.35}\left(\ln\left(1 - \frac{7\varepsilon_e}{20}\right)\right) \end{aligned} \tag{4}$$

With this assumption, rates of both plasticity and fracturing get identical. In the following section, formulation of nonlinear derivatives with respect to time will be discussed.

3.2 PLASTIC RATE FUNCTION

Fig. 3(a) schematically shows the instantaneous plasticity envelope (F_p=0) and the rate of plasticity on (ε_e, ε_p) plane. It is natural to assume that the rate of plasticity exhibits maximum on the instantaneous plastic envelope. As shown in **Fig. 3(a)**, the rate of plasticity is decayed when the state of active elasto-plastic components represented by (ε_e, ε_p) leaves away from the envelope. By referring to experimentally obtained short-term rate of plasticity as shown in **Fig. 2(a)**, we have,

$$\frac{\partial \varepsilon_p}{\partial t} = \phi\left(\frac{\partial \varepsilon_p}{\partial t}\right)_b, \quad \left(\frac{\partial \varepsilon_p}{\partial t}\right)_b = 0.034\left(\exp\left(\frac{\varepsilon_e}{4} - 1\right)\right), \quad \phi = \exp\left(-6.0\left(\frac{F_p^{0.6}}{\varepsilon_e^{1.2}}\right)\right) \tag{5}$$

Where, $(\delta\varepsilon_p/\delta t)_b$ means the referential rate defined on the plastic potential envelope and ϕ indicates the reduction factor in terms of plastic evolution.

The computed values by Eq.5 are overlaid on **Fig. 2(a)**. The plastic rate function can be uniquely specified by the elastic strain that represents the intrinsic stress applied to the parallel components. If the total stress would be used for plastic rate function, unique relation of $d\varepsilon_p/dt$ and σ cannot be found since at least two plastic strains may exist in pre- and post-peak regions corresponding to the total stress.

3.3 FRACTURING RATE FUNCTION

Fig. 3(b) indicates the tendency of fracturing rate on (ε_e, K) space. Similar to the case of plasticity, fracturing rate decreases according to the continuum damage evolution under sustained elastic strains. For rational formulation, it will be meaningful to make clear the physical image of continuum fracturing represented by *K*. Song introduced fictitious non-uniformity of parallel elasto-plastic components in terms of local strength of concrete composite [5] and explained the instantaneous evolution of fracturing. This concept is implemented with regard to the potential term in Eq. 3, but it does not explain the delayed fracturing term denoted by d*K*/dt.

Here, let us introduce a probabilistic image of damaging. The delayed fracturing is thought to be associated with micro-crack propagation. The newly created micro-cracking can develop just inside remaining non-damaged volume that is denoted by *K*. Then, even though the probability of delayed fracturing would be common among individual component, the averaged fracturing rate of the assembly of remaining components will be proportional to *K*. In fact, fracturing rate shall be null when the fracture parameter converges to zero as shown in **Fig. 3(c)**. Then, the following formulae based upon this imaginary micro-fracture are introduced as,

$$\frac{\partial K}{\partial t} = \left(\frac{\partial K}{\partial t}\right)_b \exp\left(45\left(\frac{K}{K-F_k} - 1\right)\right)$$

$$\left(\frac{\partial K}{\partial t}\right)_b = \left(\frac{\partial K}{\partial t}\right)_n (K - F_k), \quad \left(\frac{\partial K}{\partial t}\right)_n = 0.015 \cdot Ln(K - F_k) \tag{6}$$

Where, $(dK/dt)_b$ represents the referential fracturing rate on the envelope (F_k=0) on which instantaneous fracturing may occur with respect to the increment of *K*. In order to formulate $(dK/dt)_b$, the intrinsic fracturing rate to indicate the delayed evolution of micro-crack per unit active volume is given with the same manner as plasticity. As stated above, this rate is factored by the fracture parameter (*K*- F_k) in order to consider the remaining volume where new micro-defects can develop. The computed rates are shown with experiments in **Fig. 2(b).**

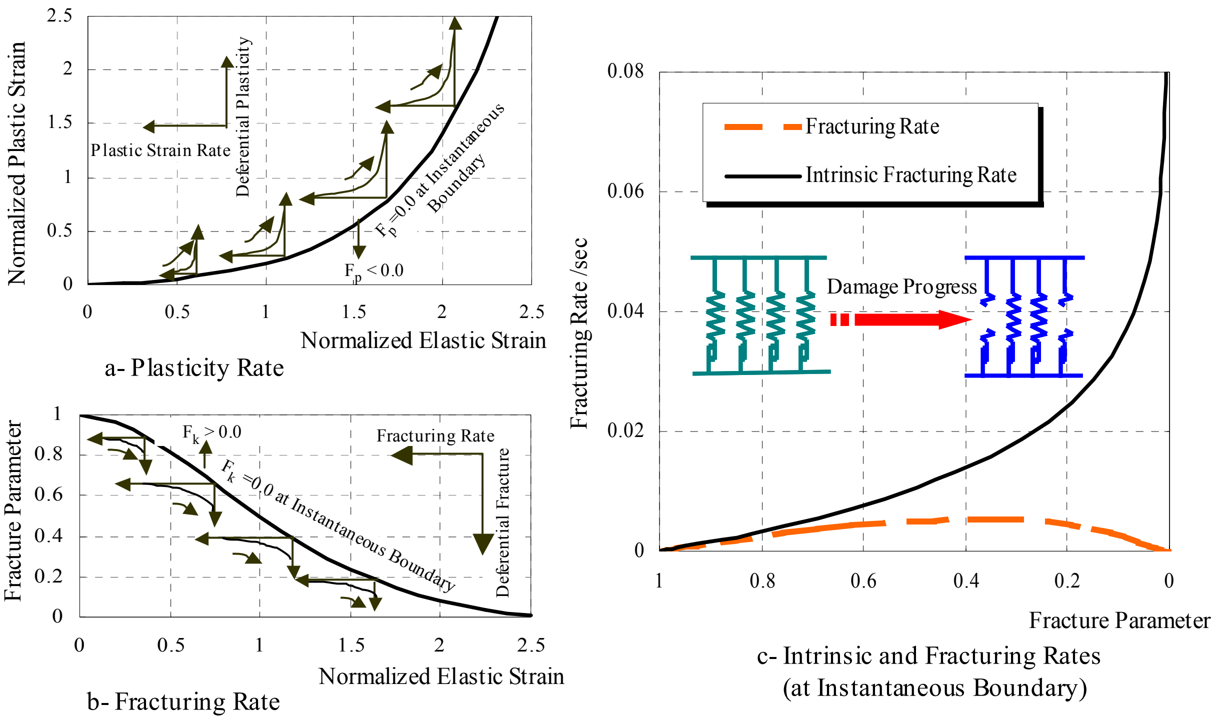


Fig. 3 Fracture and Plasticity Progresses under Compression.

4. VERIFICATION OF THE PROPOSED MODEL

The deferential form of total stress strain relation is derived by simultaneously solving equations (1)-(6), and the total stress/strain can be integrated under any strain/stress history. Here, the experimental verification is performed under different strain or stress histories including cyclic hysteresis as below. **Fig. 4** and **Fig. 5(a)** show the nonlinear creep paths close to and beyond the uniaxial state of capacity. The greater progress of the total strain is seen especially in the post-peak zone but comparatively smaller evolution of plasticity can be observed. The unloading stiffness varies in time and it drastically drops in the softening condition. It must be noted that the time dependent plasticity and fracturing continue to proceed even though the unloading paths are enforced to concrete and it leads to some nonlinear shown in cyclic hysteresis under highly damaged conditions.

Fig. 5(b) shows the relaxation under greater stress and strain states. When the total strain is hold, plasticity and fracturing are still in progress, and the elastic strain is rapidly reduced for compatibility. As the reduction of elasticity also holds back nonlinear evolution, progress of overall nonlinearity is quickly stabilized unlike the creep test cases. As a matter of fact, unstable and rapid increase in total strain occurs in both experiment and analysis. In this case, the elastic strain continues to increase because of the fracturing evolution. Consequently, the rate of plasticity and fracture are accelerated. Fig. 6 shows the nonlinear creep and delayed fracture test of concrete by Rüsch [1]. Provided that the sustained stress is approximately below 70%, long-term creep failure hardly occurs. But, if the stress exceeds the threshold, creep rupture may take place and the loading period till failure is extremely shortened in accordance with the stress level. The proposed constitutive modeling portrays short-term life span of concrete and stress-strain relation under pre- and post-peak states.

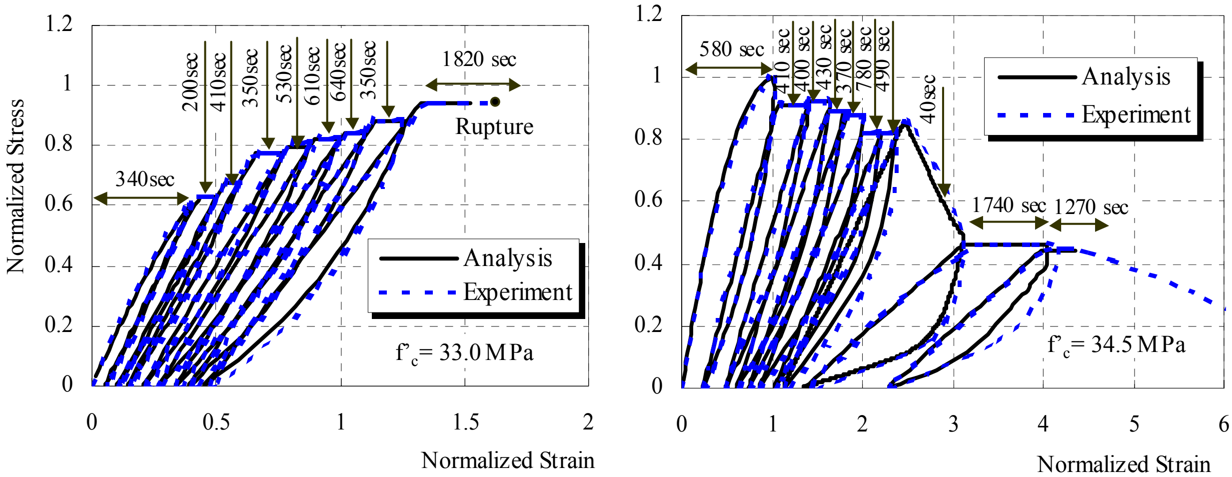
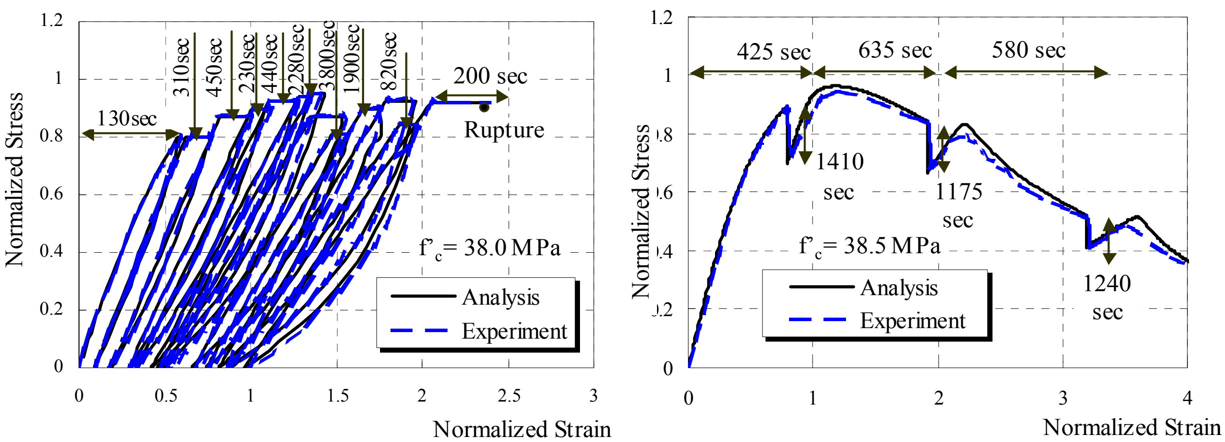


Fig. 4 Comparison between Analytical and Experimental Results under Cyclic Loading.



a- Normalized Stress versus Normalized Strain

b- Normalized Stress versus Normalized Strain

Fig. 5 Comparison between Analytical and Experimental Results – Relaxation and Cyclic Paths.

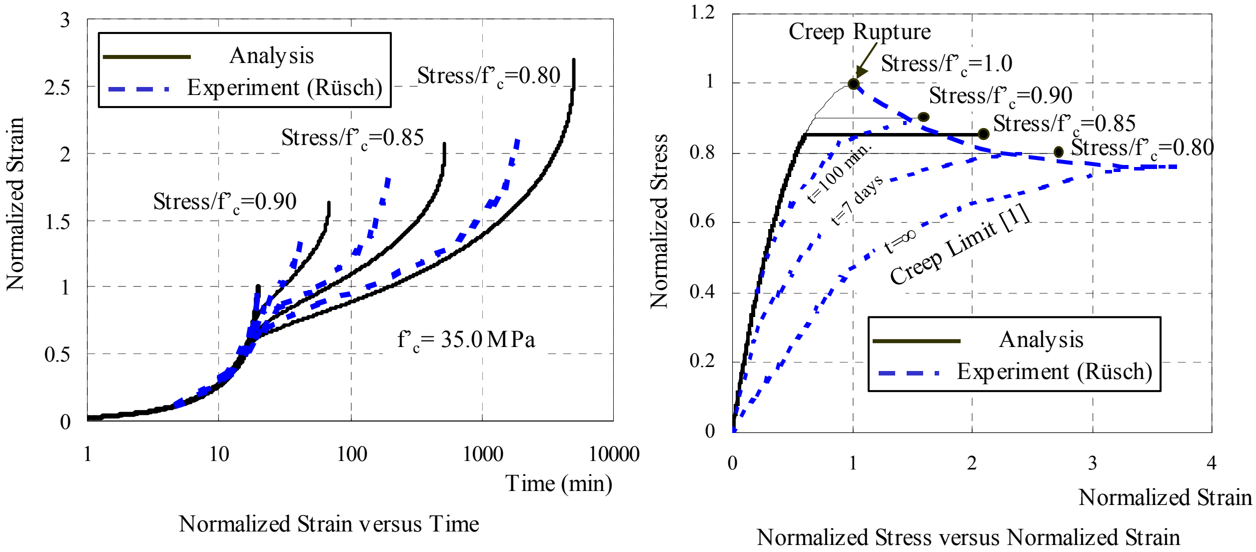


Fig. 6 Comparison between Analytical and Experimental Results – Creep Failure [1].

5. CONCLUSIONS

A short-term time dependent constitutive modeling applicable to cyclic loads was proposed under pre- and post peak states for collapse analysis of concrete structures and the following conclusions are earned.
(1) The plastic and fracturing rates of concrete were successfully extracted from the compression tests of cyclic loadings.
(2) The plastic evolution law was formulated in terms of updated elastic strain and the accumulated plasticity as demonstrated by the elasto-plastic and fracturing concept.
(3) Similar to plasticity, the continuum fracturing rate was proposed as a function of updated elastic strain and the accumulated fracture parameter to indicate the reduction rate of unloading/reloading stiffness. Here, the delayed fracture proportional to the static volume of concrete was newly conceptualized.
(4) Integrated modeling leads to total stress-strain relation by differential form and the modeling was verified in use of specimen-based experiments under uni-axial conditions.
(5) The installation of this explicit type modeling into the structural analysis program has been completed and numerical stability has been examined.

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