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論文 SECONDARY MOMENT AND MOMENT REDISTRIBUTION IN A TWO-SPAN CONTINUOUS PC BEAM WITH LARGE ECCENTRICITY

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ABSTRACT: The relationship between secondary moment and moment redistribution of a two-span continuous beam prestressed with large eccentric external tendons is discussed in this paper by using the experimental and numerical results. It was found that such moments are both related in the same manner to the relative rotation of the section at center support. As such, the higher secondary moment would lead to the larger amount of redistributed moment at the ultimate limit state. In addition, it was found that the secondary moment shall not be fully included in the calculation of ultimate moment capacity because it tends to be almost constant after the occurrence of cracking at center support.

KEYWORDS: secondary moment, moment redistribution, continuous beam, prestressed concrete, large eccentricity

1. INTRODUCTION

In a continuous prestressed concrete (hereafter PC) beam, it is generally known that the secondary moment due to prestress, M_{sec}, and moment redistribution, M_{red}, occur due to its continuity at the intermediate support, where the relative rotation is not free to deform unlike in a simply supported beam. These phenomena can have a marked influence on the flexural behavior, particularly in the post-elastic region. Recently, the concept of prestressing with large eccentric external tendons has been adopted for the construction of continuous PC bridges. Owing to the large eccentric tendon, the secondary moment due to prestress is rather high (for a non-concordant tendon profile), compared to that with ordinary eccentric tendon and may significantly affect the moment redistribution, which is an important issue in the design of a continuous beam. As such, the objective of this paper is to investigate the relationship between secondary moment and moment redistribution by using the experimental and analytical results of two-span continuous beams with large eccentric external tendons.

2. LITERATURE REVIEW

2.1 SECONDARY MOMENT

In a two-span continuous beam prestressed with a non-concordant tendon profile, the moment due to prestress is the sum of the *primary moment* (M_{pri}) and *secondary moment* or *hyperstatic moment* (M_{sec}). The primary moment causes the beam to deflect upwards (see **Fig. 1**). However, if the upward deflection is restrained by an additional support,

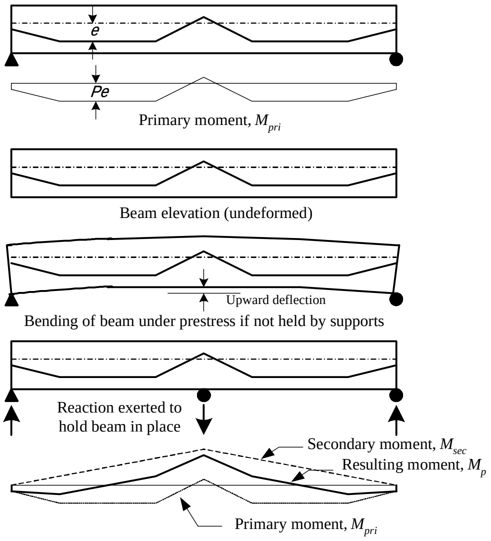

Fig. 1 Secondary moment in a two-span continuous PC beam

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as in a continuous beam, then the center support will exert a reaction on the beam and induce reactions in the other supports. Such reactions cause secondary moments, which are linearly distributed along the entire length of beam (**Fig. 1**).

The treatment of secondary moments in the post-elastic stage has been a subject of much controversy for many years and no general conclusion has yet been made [1]. This also includes whether the amount of secondary moment shall be considered in the calculation of ultimate moment capacity, M_u, of each critical section in a continuous PC beam. The secondary moments are shown by some researchers to remain constant and by others to decrease, increase or disappear at the ultimate limit state. In the ACI 318-71 Code [2], it is prescribed that the effect of secondary moment shall be neglected when calculating the design moments. This was explained that the secondary moments would disappear after the plastic hinge forms at the center support because the structure becomes statically determinate.

However, the treatment of secondary moment was changed later in the ACI 318-95 Code [3], which prescribed that the moment used for computing the required strength shall be the sum of moments due to the factored loads and secondary moment with a load factor of 1. Note that the secondary moments are to be determined using the effective prestressing force. Other researchers [4, 5] have also shown that the secondary moments shall be considered to be present at the ultimate limit state and that they are often beneficial. Further, they also pointed out that the calculated design moment could be nonconservative when the secondary moments are neglected.

Based on the literature review, it is clear that the existence of secondary moment at the ultimate stage depends mainly on the relative rotation capacity of section at intermediate support. In cases when the fixed condition at intermediate support cannot be maintained due to the occurrence of cracking or yielding of reinforcement, the amount of secondary moment is likely not to change with the increase in tendon force. Note that this concept can be applied to a continuous PC beam with either internal bonded or external unbonded tendons.

2.2 MOMENT REDISTRIBUTION

In a continuous PC beam, when the load exceeds the elastic stage, the bending moments in the beam will likely differ from those predicted by a linear analysis. The differences between the actual moments (plastic moment) and elastic ones are referred to as the redistribution of moment. Study by Kodur and Campbell [6] has shown that the extent of moment redistribution in a continuous PC beam with internal bonded tendons depends on a number of factors, such as the stiffness of span, the presence of secondary moments, etc. It has been suggested that the overall structural ductility should be considered in determining the amount of moment redistribution. Mattock [5] has concluded from test results that approximately two-thirds of the moment redistribution was due to the action of the non-concordant tendon

Table 1 Allowable moment redistribution, α, by various design codes

ACI Building Code (318-95) [3]	Canadian Code (A23.3-M84) [7]	British Code (BS8110) [8]
$\alpha \le 20\left[1-\dfrac{\omega_p+\dfrac{d}{d_p}(\omega-\omega')}{0.36\beta_1}\right]\%$	$\alpha \le 30-50\dfrac{c}{d} \le 20\%$	$\alpha \le 50-100\dfrac{c}{d} \le 20\%$

α : allowable percentage of redistribution of support moment calculated by elastic analysis

c/d : neutral axis depth ratio of the section at the ultimate limit state

ω_p, ω, ω' : reinforcement indices for prestressed, tension nonprestressed, and compression nonprestressed reinforcement, respectively

d, d_p : effective depths of the prestressed and nonprestressed reinforcement, respectively

β_1 : equivalent rectangular stress block coefficient (secondary moment) and one-third due to inelastic behavior of the beam. As such, it is concluded that the secondary moments should be considered in the calculation of moment redistribution in a continuous PC beam. However, the effects of high secondary moment in a beam with large eccentric external tendons on moment redistribution have not yet been investigated. This calls for further study on the behavior of moment redistribution in such a beam.

To account for the influence of moment redistribution, many code provisions adopted the concept that the required moment at any section is to be calculated by elastic analysis and may be increased or decreased by the allowable redistributed moment. The recommended values of percentage of redistributed moment, α, adopted in various design codes are summarized in **Table 1**.

3. RELATIONSHIP BETWEEN SECONDARY MOMENT AND MOMENT REDISTRIBUTION

To study the relationship between secondary moment and moment redistribution in a two-span continuous beam with large eccentric external tendons, the experiment carried out by Aravinthan et al. [9] as shown in **Fig. 2** was investigated. Beam B-1 was designed to have a concordant tendon profile, thus the prestress secondary moment was minimal. Beams A-1 and C-1 were designed using the linear transformation concept, therefore, the effect of secondary moments were significant and the ultimate flexural strength was expected to be the same as that of beam B-1. All specimens were applied with two-point static loading at a distance of 1.25 m in each span (symmetrical loading).

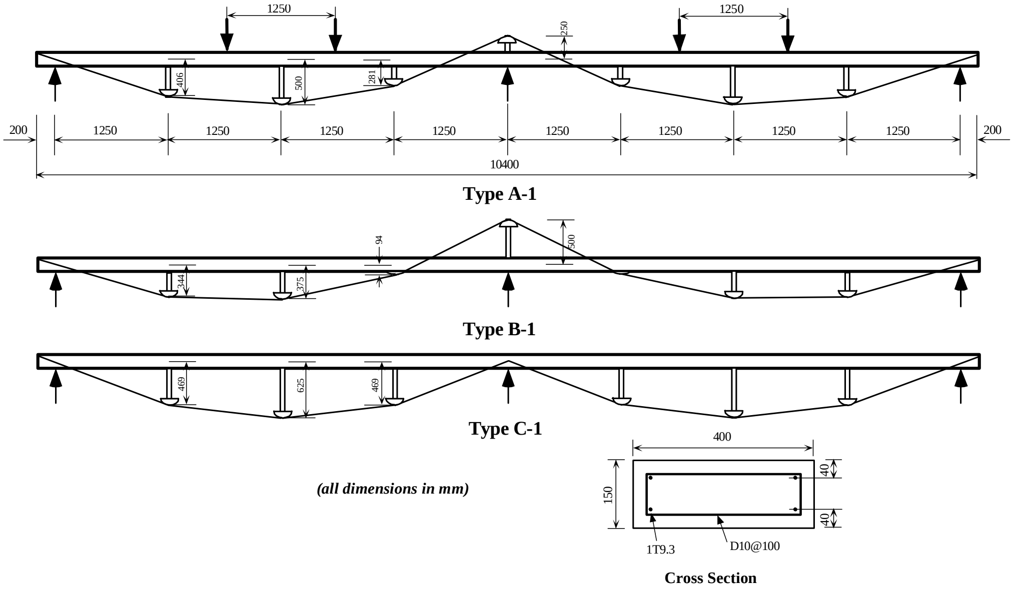

Fig. 2 Layout and details of beams tested by Aravinthan et al. [9]

3.1 ANALYTICAL METHODOLOGY

To analytically investigate the relationship between secondary moment and moment redistribution, a nonlinear analysis was developed based on the *moment-curvature method* together with the *deformation compatibility* to calculate the stress increase in external tendons. The analytical model uses a load incrementing technique and a step-wise linear analysis of secant bending stiffness, *EI*, to trace the nonlinear response of a continuous PC beam with large eccentricity (**Fig. 3**). In the analytical model, the beam is referred to as a concrete beam with only internally bonded reinforcement, thus external tendon is not included in the concrete section. The effects due to external prestressing are considered as the equivalent loads applying on the beam at the end anchorages and intermediate deviators, causing the axial compressive force and bending moment along the beam length. Further details of the analytical methodology can be found in the reference [10].

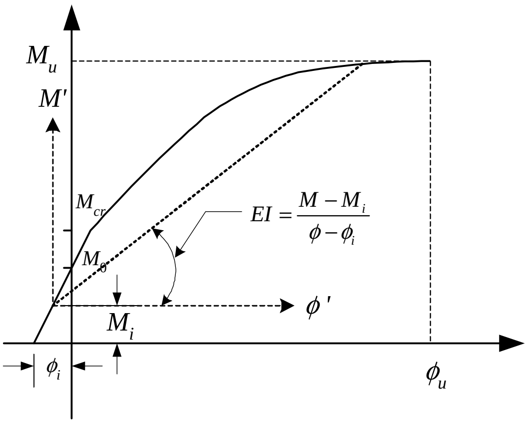

Fig. 3 ***M*-*N*-ϕ relationship of a concrete section of PC beam**

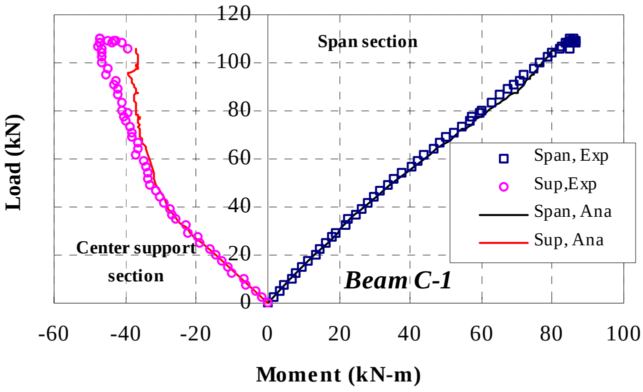

Fig. 4 Comparison of analytical predictions with test results (Beam C-1)

The validity of the analytical methodology was verified by comparing with the experimental results, regarding the load-moment relationship of beam C-1 as shown in **Fig. 4**. Note that, since the test beams were statically indeterminate, the moments at each section were computed from the applied load and support reactions, which were monitored by the loading cells during the test. It can be seen from **Fig. 4** that there is a good agreement between the analytical predictions and test results. As such, it can be concluded that the proposed analytical methodology is capable of predicting the nonlinear behavior with regard to the moment redistribution in continuous PC beams with large eccentric external tendons.

3.2 EFFECT OF SECONDARY MOMENT ON MOMENT REDISTRIBUTION

To study the influence of secondary moment on the amount of redistributed moment in a two-span continuous PC beam with large eccentricity, an analytical beam was selected to be identical to that used in the test by Aravinthan et al. [9] except for the tendon layout, in which the tendon eccentricities were adjusted based on the linear transformation concept to obtain the different levels of secondary moment.

The results of comparison of the percentage of moment redistribution, α, predicted by various design codes (see **Table 1**) with those obtained from the test of beams with symmetrical loading are summarized in **Table 2**. The results from parametric study are also plotted in **Fig. 5**. It can be seen that, beam C-1 having the highest secondary moment compared to other beams, exhibited the largest moment redistribution. Moreover, the amount of redistributed moment decreases with the secondary moment in an almost linear manner. This clearly indicates that the secondary moment and moment redistribution are both related in the same way to the relative rotations of the section at center support. Thus the higher secondary moment causes greater rotational capacity of section at the center support and, subsequently, the larger amount of redistributed moment at the ultimate limit state.

Of greater importance, it can be seen from **Fig. 5** that the results predicted by all design codes are unconservative for beam with $M_{sec}/M_e < 0.3$, that is beam B-1, where the amount of secondary moment is the least. This may be attributed to the large eccentricity of external tendon at the center support in such a beam, thus leading to low *c*/*d* and, consequently, high percentage of moment redistribution, α. In contrast,

Table 2 Comparison of redistributed moment predicted by various design codes with test results

No.	Moment at center support (kNm)				Percentage of moment redistribution, α (%)			
	M_p	M_e	M_{red}	M_{sec}	Exp	ACI	BS	Canada
A-1	68.5	95.2	26.7	37.8	28.0	16.5	20.0	20.0
B-1	88.5	94.6	6.1	11.1	6.4	18.0	20.0	20.0
C-1	48.1	96.9	48.8	63.5	50.4	6.4	20.0	15.6

the moment redistributions predicted by all design codes for the other beams are rather conservative, for beams having $M_{sec}/M_e > 0.3$.

In addition, the relationship of moment redistribution with secondary moment calculated by all design codes is not consistent with the results from experiment and analysis, where beams with high secondary moment show large moment redistribution. This may be attributed to the fact that the parameters included in those design codes are only related to the sectional property of critical section at center support, thus the influence of secondary moment which is structural-dependent (Kodur and Campbell [6]) is not taken into consideration. This clearly indicates that the effect of secondary moment should be considered in the calculation of moment redistribution, particularly in beams with large eccentric external tendon where the amount of secondary moment is rather high compared to beams with ordinary tendon eccentricity.

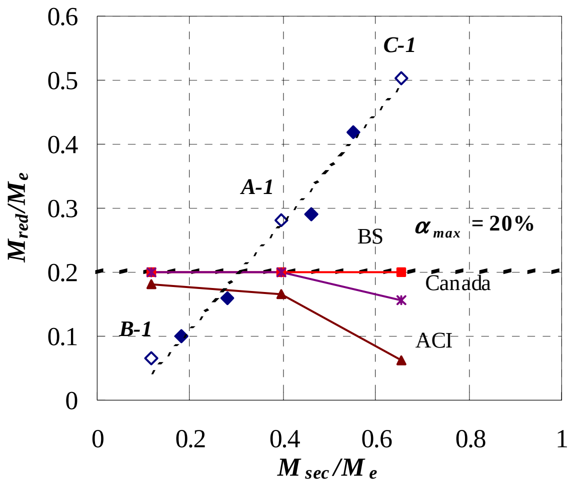

Fig. 5 Moment redistribution vs. secondary moment

4. SECONDARY MOMENT AT ULTIMATE LIMIT STATE

To check the existence of secondary moment at the ultimate limit state, the moment capacities at all critical sections (M_u) were calculated based on the strain compatibility concept and assuming two values of secondary moment: (a) 100% M_{sec} and (b) x% M_{sec}. Note that M_{sec} is the secondary moment obtained from elastic analysis using the ultimate external tendon force. The x value was determined based on the test results and it was found to be 30% for beams used in this study. The results of the calculated moments are compared with those obtained from elastic analysis (M_e) and actual moment observed in the test ($M_{p,exp}$) as shown in **Fig. 6**. It can be seen that the ultimate moments computed by considering 100% M_{sec} show very large discrepancy from that observed in the test ($M_{p,exp}$) by predicting lower moment at midspan region and higher moment at center support. By assuming the secondary moment to be 30% M_{sec} (approximately at the occurrence of crack at center support), however, the calculated moments were in good agreement with the test results, though they were slightly overestimated at the midspan section. The reason may be that, after the occurrence of cracking at the center support, the secondary moment tends to

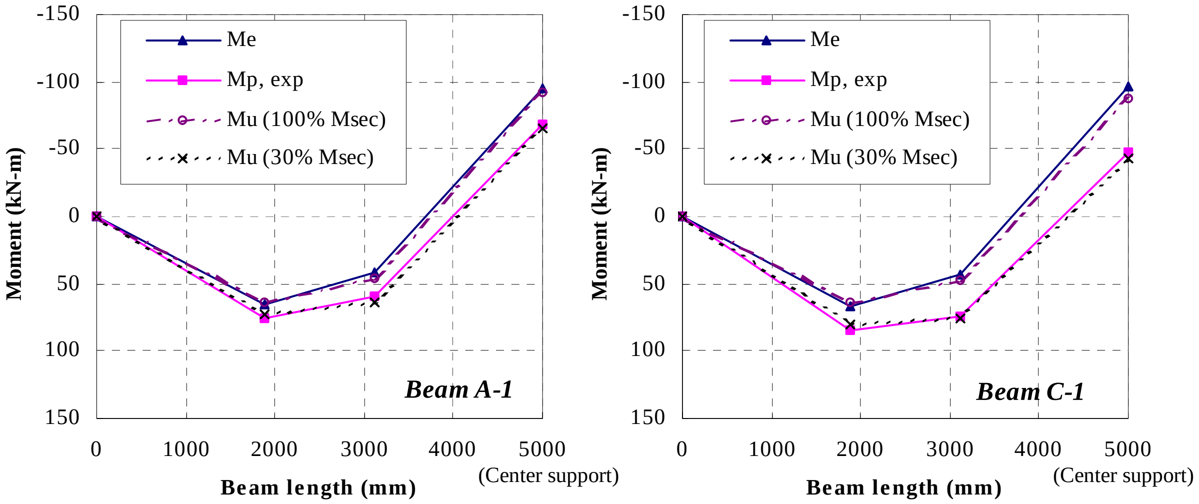

Fig. 6 Distribution of bending moment along span length

be almost constant due to the fact that the restrained relative rotation at the center support does not exist anymore. Based on the above results, it can be concluded that the secondary moment shall not be fully included in the calculation of ultimate moment capacity, especially in beams with large eccentric external tendons. To obtain a better understanding of secondary moment at the ultimate state, it is recommended that further investigation be carried out by considering other influential parameters such as the effective prestress, loading pattern, span-depth ratio (L/d_{ps}).

5. CONCLUSIONS

The relationship between secondary moment and moment redistribution was studied by using the experimental and numerical results of two-span continuous beams with large eccentric external tendons, which were designed based on the linear-transformation concept. It was found that the secondary moment and moment redistribution are both related in the same manner to the relative rotations of the critical section at center support. The higher secondary moment causes the greater rotational capacity of section at center support, thus leading to the larger amount of redistributed moment at the ultimate limit state. Moreover, it was found that the secondary moment at ultimate limit state should not be fully included in the calculation of ultimate moment capacity of beams with large eccentric external tendons. This is due to the fact that, after the occurrence of cracking at the center support, the secondary moment is likely to be constant because the restrained relative rotation at center support does not exist anymore.

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