- Technical Paper -

THERMAL CRACKING ANALYSIS OF CONCRETE WALL STRUCTURE USING RBSM

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ABSTRACT

The numerical method to evaluate the cracking behavior induced by hydration reaction in mass concrete was developed. The method is combined with structural and heat transfer analysis, in which the Rigid-Body-Spring Model (RBSM) is used as a basis of a structural model and truss network model is utilized as heat transfer model. Moreover, a hardening behavior model of concrete (stress-strain relationship) at early age was proposed and the solidification concept depending on time is applied to the constitutive model of concrete before and after cracking. For the application model (wall structure), the structural behaviors reasonably correspond with real structure.

Keywords: Mass transfer, Rigid-Body-Spring Model, Truss network model, Solidification

1. INTRODUCTION

The problem of thermal stress cracking in mass concrete (for instance on wall, dam, etc.) has been a major problem depending on the temperature history during hardening of concrete. The important parameters affecting to the durability are crack width, spacing and length. In order to avoid undesirable effects on the durability, crack propagation and crack widths need to be evaluated at the first stage.

The first objective in this paper is to develop a computationally efficient procedure of thermal stress analysis based on Rigid-Body-Spring Model (RBSM) and truss network model. The second is to propose a stress-strain relationship including after cracking during hardening of concrete based on a solidification concept.

For the 2-dimensional thermal stress analysis, the results from RBSM developed in this paper are verified in comparison with the results from Finite Element Method (FEM). Then, before and after cracking, the deformation and behavior of wall structure are analyzed which the effect of the strength at the interface is considered.

2. STRUCTURAL ANALYSIS MODEL

2.1 Structural model

Discrete model has been applied to model for material discontinuity for the crack occurrence in concrete because it is the discontinuous material. A Rigid-Body-Spring Model (RBSM) developed by Kawai[1] is one of the discrete models used to analyze the fracture behavior of concrete besides Finite Element Method (FEM) and discrete model method.

According to RBSM by Kawai, concrete is modeled as an assemblage of rigid particles interconnected by zero-size springs. The springs have 3 components in the direction of normal, tangent and rotation as shown in Fig.1. Fig.1 shows the structural model which k_n, k_s and k_θ are stiffness of normal, shear (tangential) and rotational spring. Random geometry using Voronoi diagrams, J.E. Bolander Jr. [2], is applied to partition the material onto an assemblage of rigid particle. Random geometry of the network does not represent any structural feature within the concrete material, but rather is used to reduce mesh bias on potential crack directions.

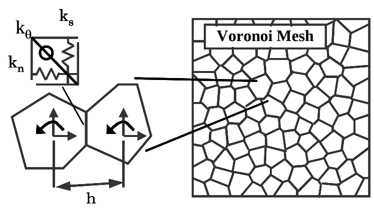

Fig.1 RBSM modeling of concrete

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2.2 Concrete material model

The lack of the information on the time dependent characteristics of mechanical properties at the early age concrete is the main problem existing in the study of the behavior of massive concrete. Before cracking, elastic incremental method has been used to analyze the stress-strain behavior. In contrast, the variation after cracking has not been defined. Therefore, in order to define the variation of concrete properties at the early age, solidification concept that is revised concept of solidification theory proposed by Z.P Bazant(1989),[3] is applied in the constitutive model. From the analysis of the solidification process, a history integral should be used to express the rate, rather than the total value, of the strain component.

In the concept of solidification, volume function of concrete related to the cement hydration is absolutely required. At each time step, the change of concrete strength depends on the incremental volume, $dv(t_i)$ of concrete. Cement hydration during each time step makes solidification volume, $dv(t_i)$. Material properties depend on the volume and local stress-strain relationship is provided from the global strain, $\varepsilon(t_i)$ corresponding to the time i. Then, global stress, $\sigma_g(t)$ is given by the superposition of all local stress $d\sigma_g(t)$ as shown in Eq.1 where t is total time and τ is time at each step. For example, total stress at t_3 as shown in Fig.2 is calculated by Eq.2. The concept is simple and applicable for strain history depending on time before and after cracking. Before cracking, it shows the elastic incremental method.

$$\sigma_g(t)=\int \sigma(\varepsilon(t)-\varepsilon(\tau))dv(\tau) \tag{1}$$

$$d\sigma_g(t)=dv(t_1)\sigma(\varepsilon(t))+dv(t_2)\sigma(\varepsilon(t)-\varepsilon(t_1)) + dv(t_3)\sigma(\varepsilon(t)-\varepsilon(t_2)) \tag{2}$$

2.3 Constitutive model

The stress-strain relationships of hardened concrete for tensile, compressive and shear stress, which are used to consider solidification volume at each time, are defined by tension softening behavior, parabolic curve as shown in Fig.3 and Mohr-Coulomb type criteria, S. Saito [4].

2.4 Mass transfer model

Truss model is utilized for mass transfer analysis and combined with structural model. Each of Voronoi elements is linked to truss network at the mother point (center of boundary). In truss model, one dimensional diffusion equation is used

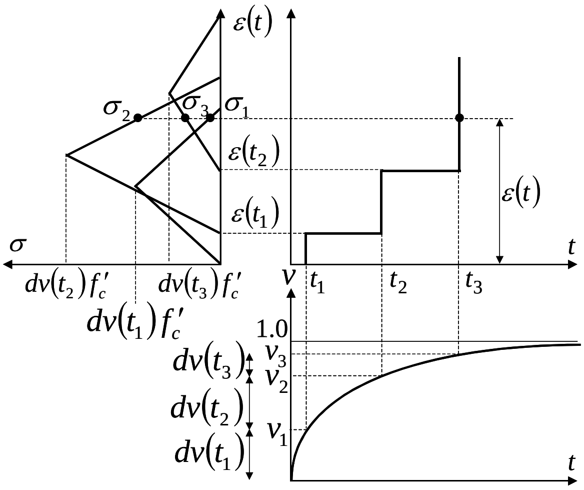


Fig. 2 Solidification concept

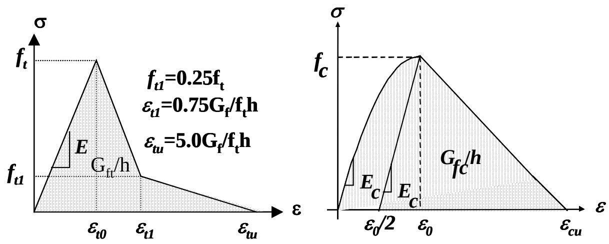


Fig.3 Tension and compression model

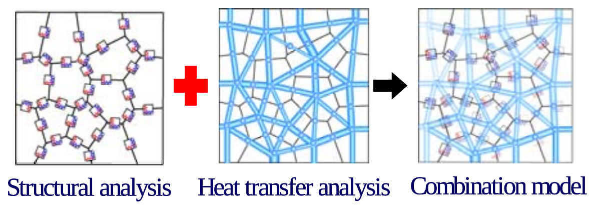


Fig.4 Concept of combination model

to solve the problem of heat transfer. Based on the governing equation of the transient heat conduction on a one-dimensional system, therefore, diffusion equation for each truss element is represented.

$$\frac{Ak_x}{L}\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix}+\frac{1}{\omega}\frac{AL}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix}+hA\begin{Bmatrix}0\\ T_2-T_\infty\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \quad \ldots(3)$$

where T_1, T_2 are temperature at both ends of truss element, T_∞ is the surrounding temperature, k_x is thermal conductivity, h is heat transfer coefficient, A is truss element area and L is the length of truss element. In this study, ω in the second term is added into Eq.3 for converting the Voronoi volume to truss volume due to the effect of overlapping volume of adjacent truss element.

$$\omega=\frac{\sum_{i=1}^{n} A_i\cdot L_i}{V_{real}} \tag{4}$$

where n is total number of elements, A_i is cross-sectional area of each truss element, L_i is length of each truss element and V_{real} is volume of the structure. For 2 dimensional analysis, ω is equal to 2.

3. 2-DIMENSIONAL ANALYSIS MODEL

Two-dimensional model shown in Fig.5 was analyzed by proposed program and FEM program (JCMAC) in the elastic state. The change of elastic modulus is considered based on solidification concept. Then, the reliability of the proposed method was verified and compared with both results.

$$T = 45 \times (1 - e^{-0.8t}) \tag{5}$$

Atmosphere temperature is set to 25°C and adiabatic temperature rise is shown in Eq.5 where T is temperature and t is time (day). Hydration temperatures generated inside the structure can diffuse to the outside of the specimen only on the adiabatic boundary. The volume function, v(t) of concrete is shown in Eq.6 which Young`s modulus is equal to 28000 MPa at 28 day. Fig.6 and Fig.7 show the comparison results between proposed and FEM method at point A and B.

$$v(t) = \frac{30t}{28(2+t)} \tag{6}$$

where t is time (day). From the temperature distribution, the results from truss model correspond very well with the FEM results, when the number of Voronoi elements for proposed analysis is more than FEM analysis because of the difference of shape function between truss model and FEM model.

As shown in Fig.7, element stresses are almost the same with the FEM method. Therefore, the combined method of RBSM and truss model based on the solidification concept is appropriate to analyze the behavior at the early age concrete.

4. APPLICATION ANALYSIS MODEL

The massive concrete (wall structure) as shown in Fig.8 and Fig.9, with 0.3m and 0.95m wall thickness, 15.0m lengths and 2.0m heights was analyzed in this paper to investigate the behavior before and after crack occurrence.

The newly concrete is assumed to become totally restrained by the ground slab while between the ground slab (hardened concrete) and underground are restrained. The stiffness of the

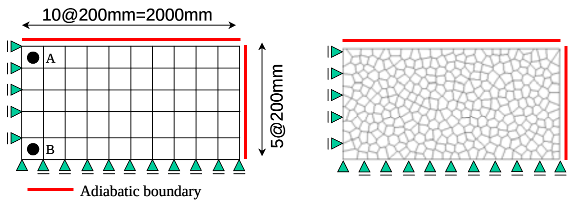


Fig.5 Two-dimensional model

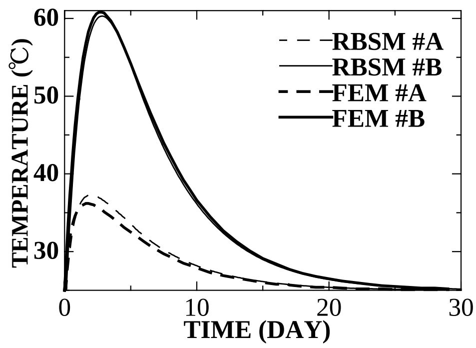


Fig. 6 Temperature distribution

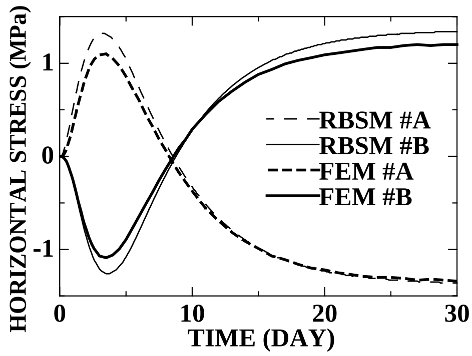


Fig. 7 Horizontal stress distribution

Table1. Thermal properties of concrete

Thermal properties	newly	hardened
specific heat, c (kcal/kg °C)	0.23	0.23
thermal conductivity, k (kcal/mh°C)	2.5	5.0
concrete density, (kg/m^3)	2400	2400
heat transfer coefficient, h (kcal/m^2h°C)	80.0	15.0
initial temperature (°C)	30.0	30.0
outside temperature (°C)	25.0	25.0

wall concrete will have an influence on a possible crack formation during the hardening period.

Thermal stress in concrete due to non-uniform temperature distribution which was derived with the consideration of the time dependent adiabatic temperature rise formula, Eq.7 was obtained. Thermal concrete properties are shown in table 1.

$$T = 50 \times (1 - e^{-1.7t}) \tag{7}$$

where T is temperature and t is time (day). The coefficient of linear expansion is set to

0.00001/°C. For the hardened concrete, tensile and compressive strength at 28 day are equal to 2.5 and 25 MPa, respectively. For newly concrete, strain of the maximum compressive strength at 28 day is used to define compressive strength equation. In this paper, it equals 0.002. Eq.8 represent the volume function, v(t) of concrete that Young`s modulus is equal to 25000 MPa at 28 day.

$$v(t) = \frac{100 \times t}{25(4.577 + 3.799t)} \qquad (8)$$

where t is time (day). Three points at the crack path in newly concrete are the observation point in this paper. Upper point is located at 10 cm under the wall surface. Middle point is at the center of newly concrete and lower point is above the interface 10 cm.

Case 1: elastic analysis

In this case, elastic analysis was applied to analyze wall structure. Temperature distribution and stress history show in Fig.10 and Fig.11. From Fig.11, crack will occur on the 4th day because the tensile stress is higher than tensile strength at that time. But at the upper and lower part of newly concrete has no cracking.

Case 2: no slip at the interface

The strength at the interface (spring stiffness) is set to be very strong by setting the same value for tensile and compressive strength in order to require no slip at the interface. However, the results of nonlinear analysis are provided in this case. Fig.12 and Fig.13 show horizontal stress distribution, deformation and crack behavior at each time step.

It can be clearly seen from Fig.13, surface cracks occurred on the 1st day due to the big difference of temperature at early age as shown in Fig.10. Compressive stress occurred inside the newly and hardened concretes on the 1st day and gradually decrease to tensile stress. Due to this effect, microcracks occurred inside the newly concrete can be observed from the stress distribution on the 3rd and 5th day. The stress at the element that microcrack occurred is lower than the surrounding element in the newly concrete. On the 6th day, crack occurred near the middle span of specimen and element stress that are adjacent to this crack decreased. Crack propagated in the upward and downward direction at the same time (7th day) then through crack occurred on the 7.5th day and element stress suddenly decreases. For crack propagation, it is clearly shown in Fig.14 where the crack width at main crack is presented.

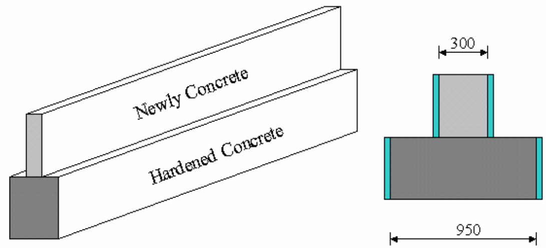
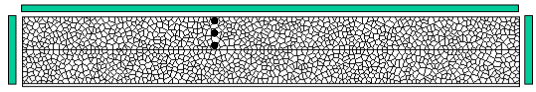


Fig.8 Wall structure



Voronoi : 1500 elements

adiabatic heat boundary

Observation point (upper, middle, lower)

Fig.9 Numerical model

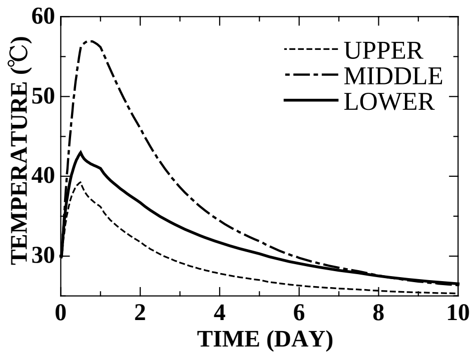


Fig.10 temperature distribution

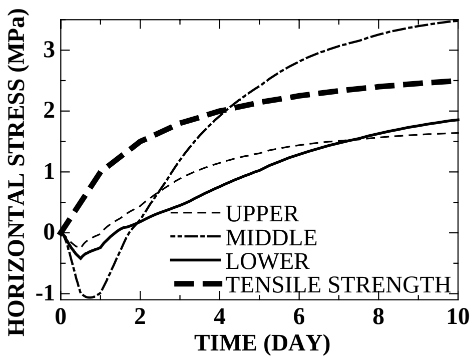


Fig.11 stress distribution

From Fig.14, the main crack occurred at the middle part of newly concrete on the 4th day but no crack occurred at the upper and lower part (near surface and interface, respectively). Crack width gradually increased and suddenly spread on the 7.5th day. At that time, crack at the upper and lower part abruptly occurred.

Fig.15 shows the comparison results

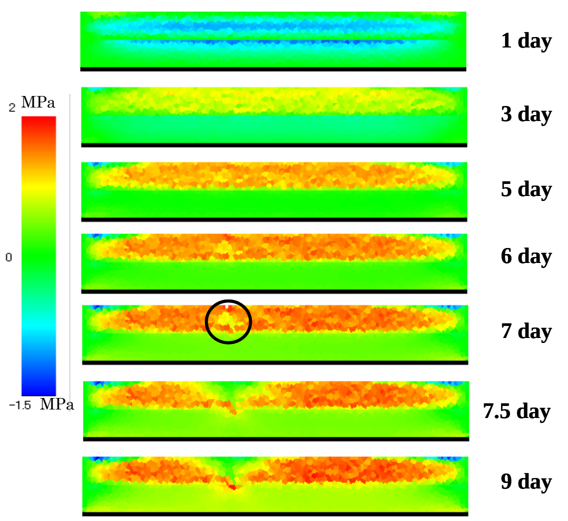


Fig.12 Horizontal stress distribution

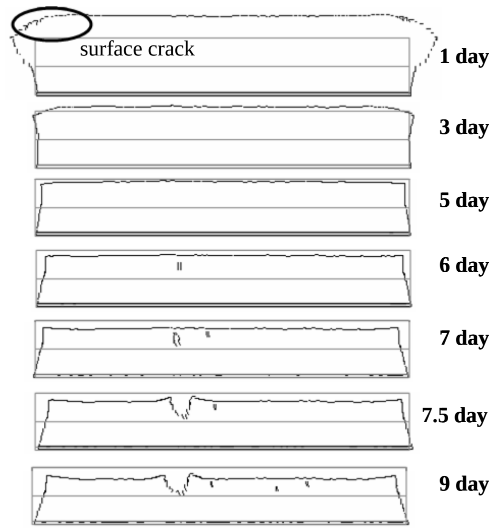


Fig.13 Deformation and Crack pattern

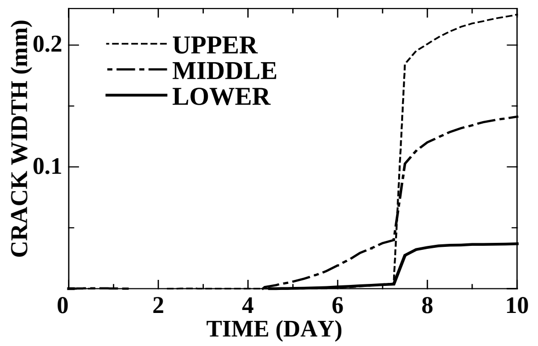


Fig.14 Crack width

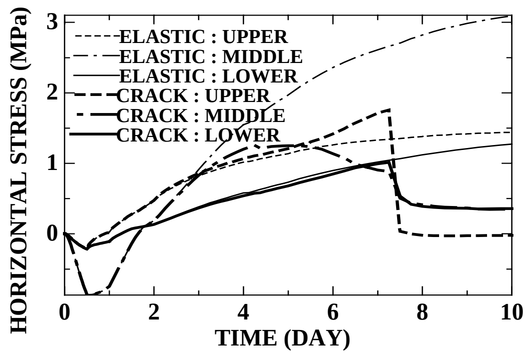


Fig.15 Horizontal stress

between elastic analysis and no slip at the interface analysis. The difference between two results comes from the effect of micro and macro crack inside the newly concrete as mentioned. At the upper zone, the effect of re-distributed stress behavior observed after cracking because elastic stress are lower than crack stress.

Case 3: interface behavior considered

The mechanism and property of concrete at the interface are very complex and difficult to define the appropriated value. To investigate the crack propagation and the effect of the interface behavior, the tensile strength at the interface was set to 1/100 of compressive strength. Fig.16 and Fig.17 show horizontal stress distribution, deformation and crack pattern at each time step.

As shown in Fig.17, surface cracks occurred on the 1st day, and cracks occurred inside the wall at the interface as well. After the 2.5th day, interface cracks occurred at both ends of the wall structure and propagated into the wall along the interface direction. After interface cracks occurred, tensile stress near the crack decreased and changed to compressive stress. Element stress at the newly concrete gradually changed to tensile stress and also microcracks occurred at this time. Interface crack opening gradually increased on the 5th day and main crack occurred at the same day and same position with case 2. Main crack also propagated in the upward and downward direction at the same time until through crack occurred on the 7.5th day. When crack propagated downward and reached the interface, it can induced the interface crack inside the structure and element stress near this crack decreased close to zero.

Crack width, as shown in Fig.18, suddenly increased on the 7.5th day as well but it is larger than in case 2. From Fig.19, stress distributions are almost the same with case 2 before through crack occurring, but after crack occurring, the element stress in this case is faster released than case 2.

5. CONCLUSIONS

(1) New method to analyze thermal stress and crack behavior was developed and the method to obtain time dependent stress-strain relationship at early age concrete before and after cracking was

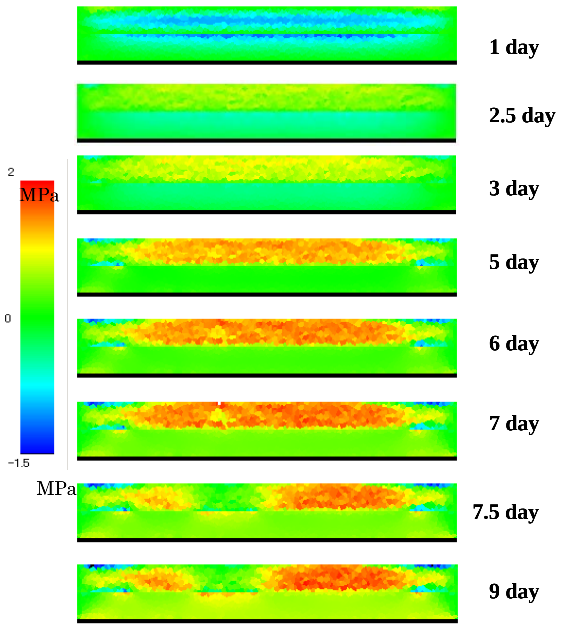


Fig.16 Horizontal stress distribution

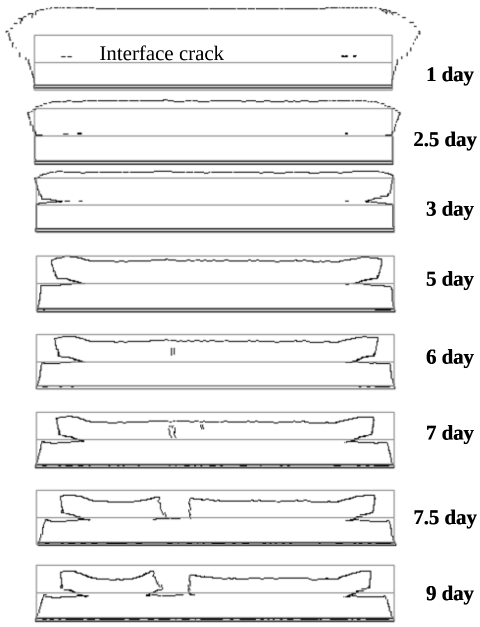


Fig.17 Deformation and Crack pattern

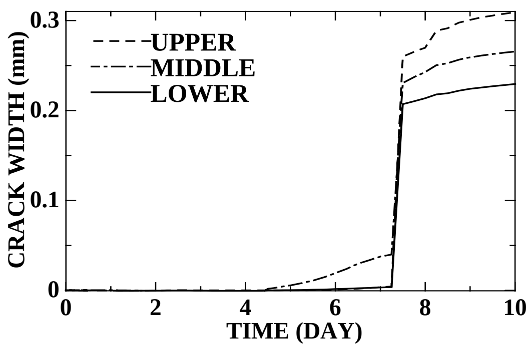


Fig.18 Crack width

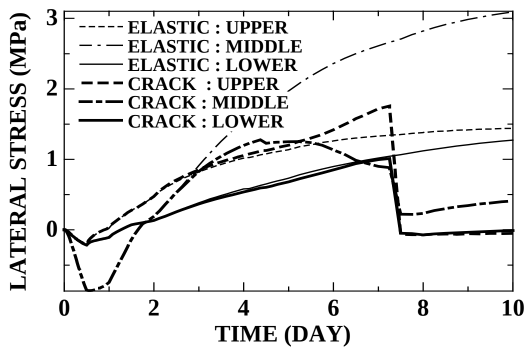


Fig.19 Horizontal stress

proposed based on the solidification concept.

(2) The combined method between RBSM analysis and truss model for analyzing mass transfer provides the reasonably good results corresponding to FEM method. However, small mesh for calculating mass transfer using RBSM method is necessary to obtain the same results with FEM due to the difference of shape function.

(3) The cracking behaviors of wall structure before and after crack occurring from the computational result reasonably correspond with real structure. Therefore, RBSM method combined with the truss element model can be used to analyze thermal stress analysis in mass concrete as well as thermal crack propagation.

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