- Technical Paper -

SHEAR CRACK WIDTH AND MAXIMUM LOAD OF RC COLUMN SUBJECTED TO CYCLIC LOADING

Marian DRAGOI*1 and Tatsuya TSUBAKI*2

ABSTRACT

In determining the degree of damage of a structure or a structural member caused by an earthquake it is important to know the maximum load to which the structure or structural member was subjected during the earthquake. Cracks in concrete structures can be easily observed and analyzed after earthquake. Starting from crack pattern and crack width, empirical relationships have been determined in order to estimate the maximum load that causes the cracks. The relationships have been established by analyzing data of six reinforced concrete columns having axial load and shear reinforcement ratio as parameters.

Keywords: shear crack width, shear reinforcement strain, maximum load, cyclic loading

1. INTRODUCTION

It is well known that if the diagonal tensile stress exceeds the tensile strength of concrete then diagonal cracks appear as a continuation of flexural cracks. Then, the strain in shear reinforcement near crack increases suddenly, and the diagonal tensile stresses are taken by shear reinforcement. By analyzing the distribution of the strain in shear reinforcement in case of reversed cyclic loading, an empirical formula is found for the estimation of load based on experimental results. Thus, knowing the strain in shear reinforcement, the maximum load experienced by a structure or structural member during an earthquake can be determined. Then, the maximum load is used in the evaluation of the damage [1].

Previous studies have been made by CEB [2] and Ueda [3] in order to predict shear crack width from the strain for RC beams. An empirical formula to predict strain in shear reinforcement knowing the shear crack width is also necessary for the case of RC columns.

After an earthquake, data such as crack width and crack pattern can be obtained. Starting from these data for a damaged structure, empirical formulas are determined in order to estimate the maximum load experienced by the structure. The empirical relationships are obtained by analyzing six RC columns [4] where axial load and shear reinforcement ratio are variable parameters. The correlation between the unloaded state (after the earthquake) and the loaded state (during the earthquake) is done by using a relationship between the crack widths corresponding to these states.

2. EXPERIMENT DATA AND ANALYSIS

2.1 Description of the experiment

Six RC columns of rectangular cross section having the size of 1100×300×300 mm and the footing size of 1300×400×600 mm were tested [4]. For the longitudinal direction, D13, and the transverse direction, D6, reinforcing bars have been used. The columns have been divided into groups according to shear reinforcement ratio, each group having variable axial load. The data concerning the RC columns are summarized in Table 1. The material properties for reinforcement are given in Table 2.

Cantilever type loading was used in the experiment and the load was applied at 900 mm from the bottom of the column. The specimens were subjected to reversed cyclic loading by increasing the load each cycle by 40 kN until 120 kN and then by 20 kN up to yield load, then controlling the deflection of the column by increasing the rotation angle with 1/200 rad each cycle up to 7/200 rad which represents the last loading stage.

*1 Graduate School of Engineering, Yokohama National University, JCI Member

*2 Professor, Dept. of Civil Engineering, Yokohama National University, Ph.D., JCI Member

Table 1 Details of RC columns

Specimen	Axial load (MPa)	Longitudinal reinforcement	Ratio (%)	Shear reinforcement (type-spacing)	Ratio (%)	Strength of concrete (MPa)		E (GPa)	Poisson ratio
						Compressive	Tensile		
I	0	D13×24	3.38	D6-100mm	0.21	32.2	2.47	27.3	0.19
II	2					32.3	2.71		
III	4					30.0	2.83		
IV	0			D6-50mm	0.42	30.9	2.57	28.1	0.18
V	2					30.8	2.74		
VI	4					30.2	2.79		

Table 2 Properties of reinforcement

Type of reinforcement	E (GPa)	Yield stress f_y (MPa)	Yield strain ε_y (μ)
D13	188	391	2080
D6	192	336	1778

2.2 Shear crack width

For loading and unloading stages of each loading cycle the crack width was measured by using a digital microscope.

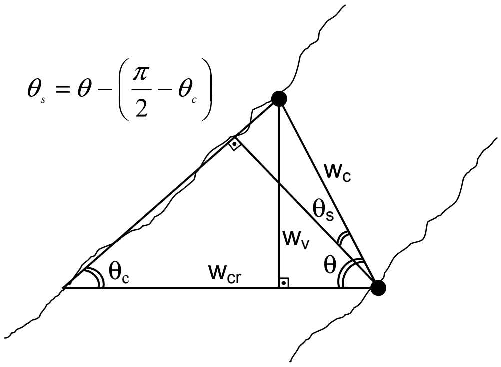

Fig. 1 Crack measurements

In case of shear cracks w_c, the distance between corresponding points and w_v, the vertical distance between these points have been measured. In Fig. 1 the description of the shear crack measurement is shown. The shear crack width along shear reinforcement w_{cr} is directly proportional to the strain in the shear reinforcement and is given by Eq. 1.

$$w_{cr} = w_c\left(\frac{\sin\theta}{\tan\theta_c} + \cos\theta\right) \quad (1)$$

where θ is a function of w_c and w_v, and θ_c is the crack angle obtained from crack pattern. The shear slip angle θ_s is shown in Fig. 1.

The crack patterns in the column part of the six specimens are shown in Fig. 2. These crack patterns correspond to the loading stages where the maximum load was reached. For these particular stages the stabilization of crack configuration was observed.

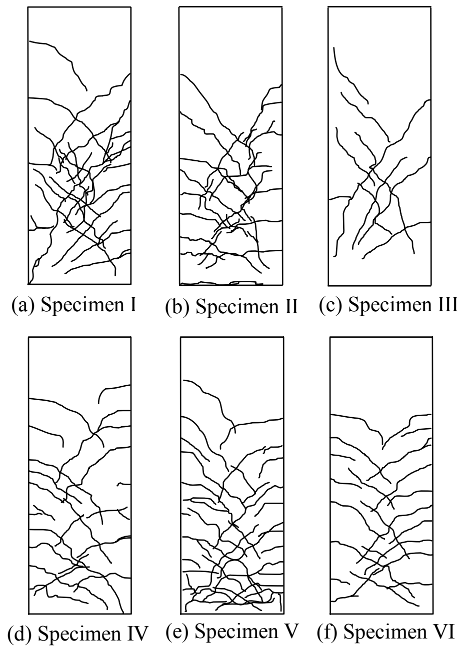

Fig. 2 Crack patterns

Data such as crack width and crack pattern correspond to the unloaded state where the structure or structural member is no longer subjected to seismic force. By using the experimental data, a correlation between the loaded and unloaded states is found. From regression analysis, a bilinear relationship is obtained to get the sum of crack width for loaded state S from that of unloaded state S_0 as shown in Fig. 3.

The sum of the crack widths along shear reinforcement is used in the analysis considering that the average strain in shear reinforcement is given by all shear cracks crossing the shear reinforcement. The relationships between the maximum crack width and the sum of crack widths crossing the shear reinforcement for loaded and unloaded states are given in Table. 3.

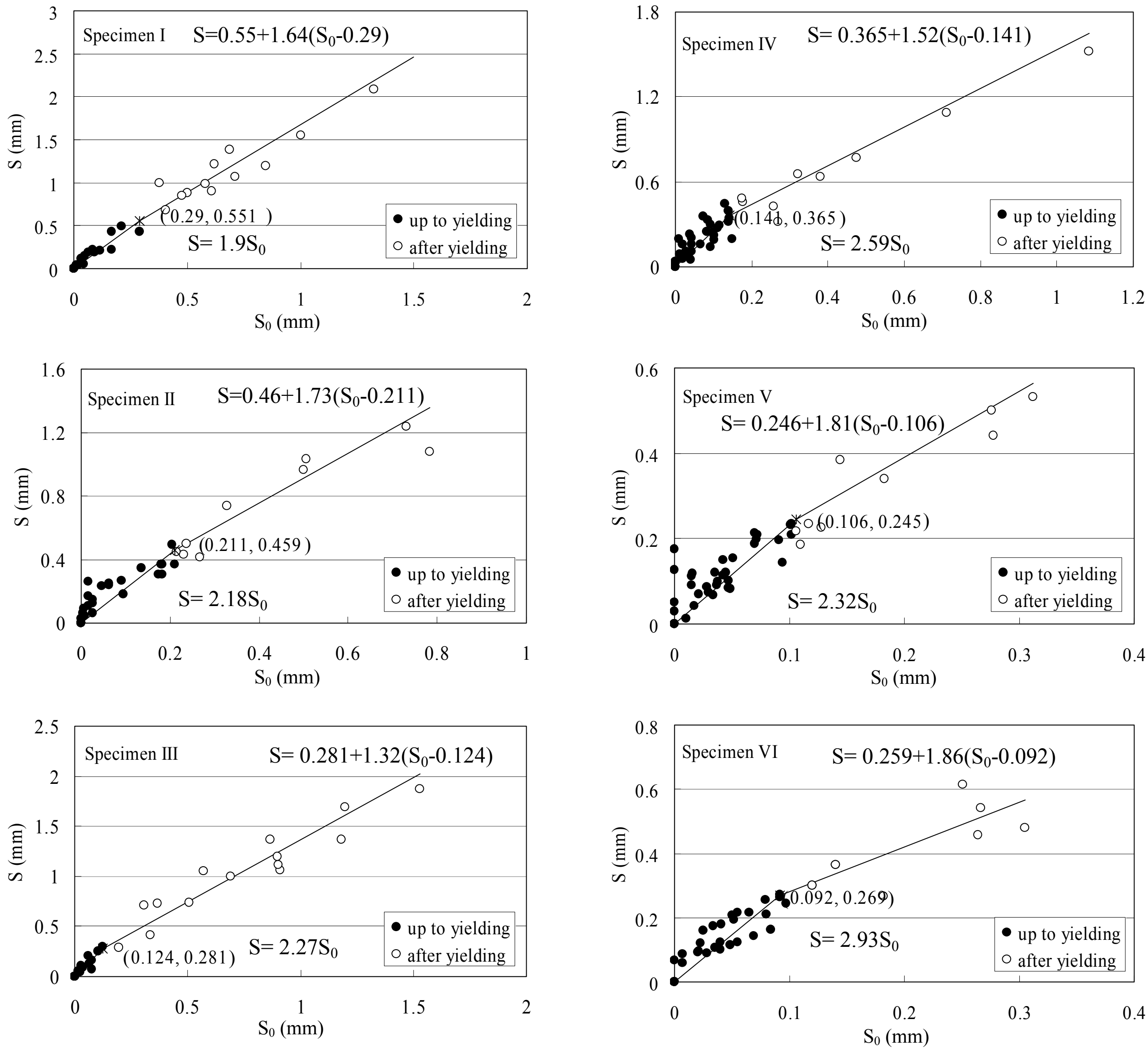

Fig. 3 Relationships between loaded and unloaded states for shear crack width
[( ): coordinates at the link point]

The bilinear relationship indicates the change of tendency by yielding of shear reinforcement.

Table 3. Crack width ratio

Specimen	I	II	III	IV	V	VI
$\frac{\max(w_{cr0})}{\Sigma w_{cr0}}$	0.62	0.65	0.47	0.48	0.56	0.52
$\frac{\max(w_{cr})}{\Sigma w_{cr}}$	0.57	0.55	0.47	0.54	0.53	0.50

2.3 Shear reinforcement strain

Strain in the shear reinforcement has been recorded during the experiment by using strain gauges which were placed on three shear reinforcing bars in the zone where shear cracks were expected. The length of the protection covering for the strain gauges was around 3 cm.

The arrangement of the reinforcement in the RC columns and the position of the strain gauges are presented in Fig. 4.

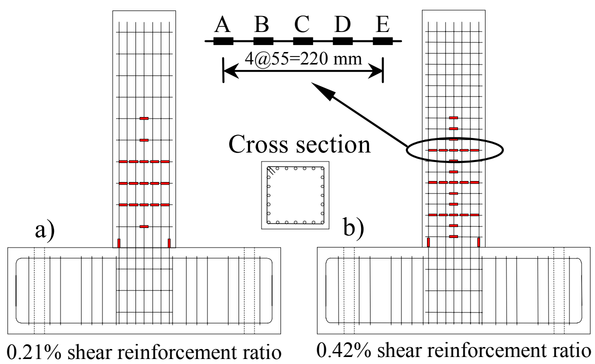

Fig. 4 Position of strain gauges

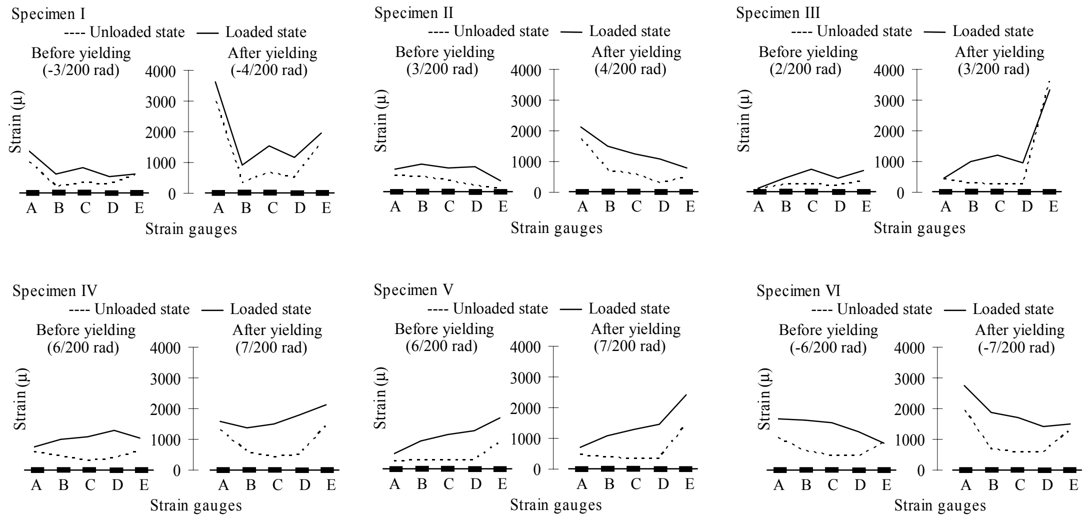

Fig. 5 Distribution of strain during loading

Considering the yield point of the shear reinforcement the distribution of the strain in the shear reinforcing bar having the same position is shown in Fig. 5 for all six columns. Measuring the strain for the loaded and unloaded states of each loading cycle and the shear crack width, regression analysis is performed to find the relationships between the strain in the shear reinforcement and the width of the shear crack crossing that shear reinforcing bar. In the analysis the average strain in the shear reinforcement with strain gauges and the sum of crack widths crossing the shear reinforcement are considered. The average strain has been obtained using Eq. 2.

$$\varepsilon_{av} = \frac{\varepsilon_A + \varepsilon_B + \varepsilon_C + \varepsilon_D + \varepsilon_E}{5} \tag{2}$$

where ε_A, ε_B, ε_C, ε_D, ε_E are the strain measured by the strain gauges.

The average strain in shear reinforcement is considered proportional to the sum of shear crack widths where the proportionality is given by coefficient α as in Eqs. 3 and 4 for unloaded and loaded states.

$$\varepsilon_{0av} = \alpha_0 S_0 \tag{3}$$

$$\varepsilon_{av} = \alpha S \tag{4}$$

It is observed that α is a function of the sum of crack widths as seen in Fig. 6. All the specimens show a similar variation of α for shear crack width. Such a variation of α can be explained by the fact that the length of degraded bond between steel and concrete is small at the beginning of loading, and small crack width gives high local strain. By the increase of load and reversed cyclic loading this length increases, and the local strain becomes smaller. α decreases rapidly up to a certain value of shear crack width, and then it starts to slightly increase or decrease depending on the bond degradation.

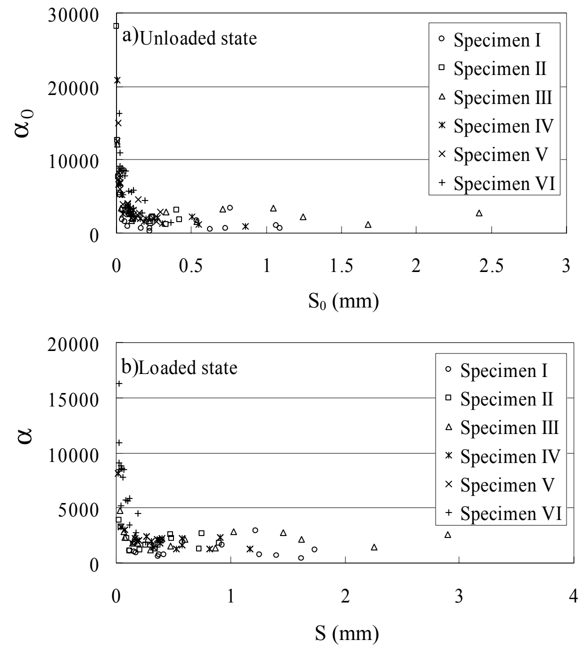

Fig. 6 Variation of α_0 and α coefficients

The variation of α is expressed by a bilinear function of shear crack width and the empirical formulas given by Eqs. 5 and 6 are found by regression analysis.

$$\alpha_I = p_{0I} + p_{1I} S_0 \tag{5}$$

$$\alpha_{II} = \alpha_{\lim in} + p_{1II}(S_0 - S_c) \tag{6}$$

By replacing S_0 with S the formula holds also

Table 4 Coefficients for unloaded state and loaded state

Coefficients		Unloaded state				Loaded state			
		p_{0I} (μ/mm)	p_{1I} (μ×mm^2)	p_{1II} (μ×mm^2)	S_c (mm)	p_{0I} (μ/mm)	p_{1I} (μ×mm^2)	p_{1II} (μ×mm^2)	S_c (mm)
a	a_0	-7629	-1311	4515	0.338	-1269	-467	585	0.553
	a_1	44728	8176	-13642	-0.500	12409	5285	-1238	-0.833
b	b_0	16785	1902	-1803	-0.215	1784	-3875	6068	-0.285
	b_1	-40095	-3785	2390	0.414	-5223	19642	-17761	0.650
c	c_0	-3317	-352	890	0.038	-806	-1393	-1004	0.058
	c_1	8052	957	-3190	-0.068	2064	6438	2190	-0.126

for loaded state. The coefficients of these two equations are obtained using the values from Table 4, for unloaded state and for loaded state, considering the following function:

$$f(N) = aN^2 + bN + c \tag{7}$$

where coefficients a, b, c are expressed as a linear function of shear reinforcement ratio ρ_S as in Eqs. 8-10.

$$a(\rho_s) = a_0 + a_1\rho_s \tag{8}$$

$$b(\rho_s) = b_0 + b_1\rho_s \tag{9}$$

$$c(\rho_s) = c_0 + c_1\rho_s \tag{10}$$

In Fig. 7 the comparison shows that for small values of crack width the predicted values are in good agreement with the experimental data.

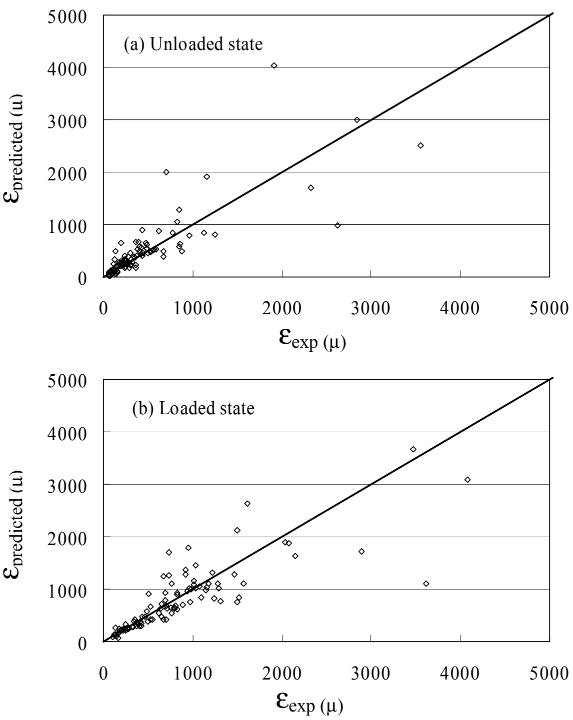

Fig. 7 Comparison between experimental and predicted strains

In the strain estimation formula the coefficients a, b, c are given by linear interpolation knowing that the shear reinforcement ratio is in between 0.21% and 0.42%. From the quadratic interpolation the coefficients for the empirical formula can be determined if axial load is in between 0 and 4 MPa.

2.4 Load

The maximum load in the experiment is a function of axial load as shown in Table 5.

Table 5 Maximum Load

Specimen	I	II	III	IV	V	VI
Max. Load	166	182	194	164	186	207

The strain-load curve is obtained from experimental data for all six specimens as shown in Fig. 8. The strain in the shear reinforcement having the same position is considered.

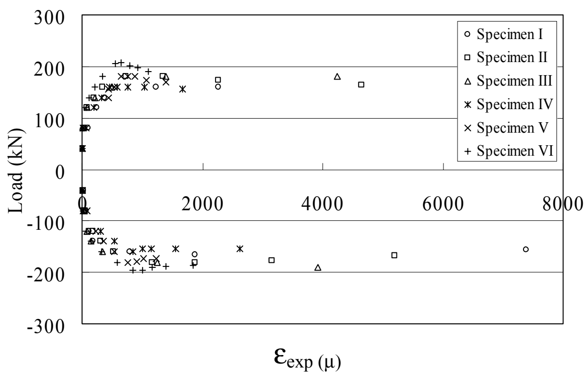

Fig. 8 Strain-load curve

The shape of the strain-load curve is defined by two equations. Up to the peak load the power function is used in the analysis to obtain the load from strain. The second part is considered to have a linear behavior.

The equation to express the load up to the peak has the following form:

$$V = a'\varepsilon^{b'} \tag{11}$$

where V is the predicted load, and ε is strain in shear reinforcement. Coefficients a' and b' are a function of axial load and shear reinforcement ratio and they represent the behavior of the RC column. The values of these coefficients to find the load up to the peak are given in Table 6. The contribution of concrete to shear strength V_c calculated by the JSCE code [5] is given for each specimen.

Table 6 Coefficients used up to peak

Specimen	N (MPa)	Vc (kN)	a' (kN/μ)	b'
I	0	111	47.769	0.168
II	2	120	46.824	0.191
III	4	124	69.787	0.125
IV	0	110	19.387	0.382
V	2	118	44.285	0.207
VI	4	124	47.316	0.224

After the peak, the load starts to decrease and the equation used to express this is given as follows:

$$V = c'\varepsilon + d' \quad (12)$$

Once the peak is reached the load decreases from d' value according to coefficient c', as shown in Table 7. For high shear reinforcement ratio, coefficient c' is small at N = 4 MPa, that means a rapid decrease of load with small increase in strain. After the peak the axial load makes the specimen reach the failure point faster.

Table 7 Coefficients used after peak

Specimen	N (MPa)	Vc (kN)	c' (kN/μ)	d' (kN)
I	0	111	-0.0053	182.90
II	2	120	-0.0119	212.30
III	4	124	-0.0104	233.04
IV	0	110	-0.0066	166.03
V	2	118	-0.0142	190.39
VI	4	124	-0.0480	249.50

Linear interpolation must be done to obtain the coefficients for other values of shear reinforcement ratio and axial load. First, linear interpolation is done for shear reinforcement ratio and the determined coefficient is used in linear interpolation for axial load. Comparing the experimental and analysis results, Fig. 9 shows that a good estimation of load is obtained using the empirical formula.

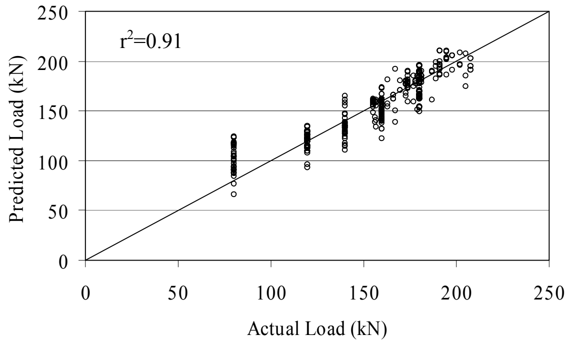

Fig. 9 Comparison between predicted load and actual load

3. CONCLUSIONS

(1) The formula for the ratio between the shear crack width for loaded state and that for unloaded state is a function of yield point of shear reinforcement.
(2) For the estimation of the strain of shear reinforcement the formula for the variation of strain - crack width ratio is presented to be related to the bond degradation mechanism.
(3) Considering the parameters used in the experiment the applicability of the proposed formulas to estimate the strain in shear reinforcement and maximum load has a limited range, 0 to 4 MPa for axial load and 0.21% to 0.42% for shear reinforcement ratio.
(4) The empirical formulas are not to be applied for large deformation where the crack width and the residual strain in shear reinforcement are large.

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