- Technical Paper -

EVALUATION OF SHEAR CRACK WIDTH IN PARTIALLY PRESTRESSED CONCRETE MEMBERS

Eakarat WITCHUKREANGKRAI *1, Hiroshi MUTSUYOSHI *2, Mayuko TAKAGI *3 and Sudhira DE SILVA *3

ABSTRACT

This paper presents a review of available prediction formulae in the literature to estimate shear crack width in concrete members. Influencing factors affecting shear cracking behavior are extensively discussed, and shear crack widths and crack spacing calculated from each design equation are compared with those obtained from available experimental results. Finally, a rational and simplified equation for the calculation of shear crack width for partially prestressed concrete members is proposed.

Keywords: shear crack width, stirrup strain, partially prestressed concrete.

1. INTRODUCTION

Crack control is an important issue for serviceability and durability requirements in the design of structural concrete members. For decades, numerous research works have been conducted to study the cracking behavior and crack control in reinforced concrete (RC) and partially prestressed concrete (PPC or PRC) members. Most of them have been focused on members subject to tension or bending, while very few studies have been conducted on members subject to shear or torsion [1].

Mechanism of diagonal or shear cracks in concrete members subject to combined load, i.e., tension, shear and flexure, has not yet been clearly investigated because of the complexity of stress and strain distributions in the regions of such cracks. The formation and extent of shear cracks cannot be reliably predicted, causing the analytical prediction of shear crack width very complicated.

Although some prediction formulae for estimating shear crack width have been proposed by some investigators, most of them are based on semi-empirical method. Moreover, such prediction formulations are originally developed for RC members, hence, parameters concerning the effect of prestress are not taken into account. Their applicability for predicting shear crack width and crack spacing in PRC members needs to be carefully investigated.

2. APPROACH FOR CALCULATION OF SHEAR CRACK WIDTH

A review of available prediction formulae in the literature shows that, although each formula contains a different set of influencing parameters, they can be mainly classified into two categories according to the main variable considered in the approach: principle tensile strain or stirrup strain.

2.1 Approach based on Principal Tensile Strain

The concept of this approach is based on the smeared crack model where the actual complex crack pattern is idealized as a series of parallel cracks occurring at angle θ to the longitudinal reinforcement [2]. This means that diagonal cracks are uniformly distributed with constant crack angel and crack spacing, similar to the crack pattern of membrane element subject to normal and shear stresses. By neglecting the effect of tension stiffening, the average width of diagonal cracks can be assumed equal to the product of the principal tensile strain, ε_1, and the average spacing of diagonal cracks, $s_{m\theta}$

$$w_m = \varepsilon_1 \cdot s_{m\theta} \tag{1}$$

The average spacing of diagonal cracks can be related to the crack spacing in the horizontal

---

*1 Civil Engineering Technology Division, Shimizu Corporation, Ph.D., JCI Member
*2 Graduate School of Science and Engineering, Saitama University, Professor, JCI Member
*3 Graduate School of Science and Engineering, Saitama University, Graduate Student, JCI Member

and vertical directions which depends upon the crack control characteristics of both the longitudinal and transverse reinforcement, as shown in Fig. 1. The CEB-FIP Model Code 1990 [3] provides an expression for average spacing of diagonal crack as:

$$s_{m\theta} = \frac{1}{\dfrac{\sin\theta}{s_{mx}} + \dfrac{\cos\theta}{s_{my}}} \tag{2}$$

Since the crack spacing is influenced by several factors, the estimation of its exact value is rather uncertain and intractable. Hence, the use of formulae based on semi-empirical statistic analysis of experimental results is considered acceptable in design practice [2].

Table 1 summarizes available formulae for predicting the average diagonal crack spacing. Parameters affecting the crack spacing proposed by Collins and Mitchell [2] are illustrated in Fig. 2. For simplification, Collins and Mitchell [2] further assume that the diagonal crack spacing, $s_{m\theta}$, can be taken as 300 mm. Study by Yoon et al. [4] has showed that it is acceptable to assume that the longitudinal crack spacing is equal to the effective depth d and the transverse crack spacing is equal to the stirrup spacing s_y. Colotti and Spadea [5] adopted a similar concept that, for a member subjected to combined action of bending, shear and/or torsion, the diagonal crack width can be taken as the product of the principal tensile strain and the diagonal crack spacing. The equation to predict diagonal crack spacing was taken from that proposed by Iori and Dei Poli [6].

In these approaches, it can be observed that although the value of principal tensile strain can be analytically predicted, the diagonal crack spacing is estimated according to the empirical formulae for crack spacing in members subject to axial tension.

2.2 Approach based on stirrup strain

As discussed earlier, in the smeared-crack concept, diagonal cracks are modeled as uniformly distributed cracks and the stress and strain are averaged over the cracked member containing steel reinforcements in both horizontal and vertical directions. In the web of a slender beam, however, only shear reinforcements normally exist. Thus, after the formation of shear cracks, stirrups will mainly subject to tension and become effective to restrain the crack opening and crack propagation. This implies that the diagonal crack is primarily controlled by the web reinforcement rather than the longitudinal steel.

Test results by many investigators [7-9]

Table 1 Summary of formulae for average diagonal crack spacing (principal tensile strain)

Researchers	Average diagonal crack spacing, $s_{m\theta}$	Note
Collins and Mitchell (1991)	$s_{m\theta} = \dfrac{1}{\dfrac{\sin\theta}{s_{mx}} + \dfrac{\cos\theta}{s_{my}}}$ $s_{mx} = 2\left(c_{nx} + \dfrac{s_x}{10}\right) + 0.25k_1\dfrac{d_{bx}}{\rho_{ex}}$ $s_{my} = 2\left(c_{ny} + \dfrac{s_y}{10}\right) + 0.25k_1\dfrac{d_{by}}{\rho_{ey}}$ For simplification: $s_{m\theta} = 300$ mm	d_b is bar diameter; c_n is distance to reinforcement; s is bar spacing; ρ_{ex} is the ratio of steel reinforcement to the effective concrete area ($\rho_{ex} = A_{sx}/A_{ce}$); ρ_{ey} is effective stirrup ratio ($\rho_{ey} = A_w/b_w s_y$); k_1 is a coefficient for bonding property of the bars (k_1 = 0.4 for deformed bars, k_1 = 0.8 for plain bars)
Yoon et al. (1996)	$s_{m\theta} = \dfrac{1}{\dfrac{\sin\theta}{s_{mx}} + \dfrac{\cos\theta}{s_{my}}}$	$s_{mx} = d$; d is effective depth $s_{my} = s_y$: s_y is stirrup spacing
Iori and Dei Poli (1985)	$s_{m\theta} = s_z$ for $\dfrac{s_z}{s_y} < 0.55$ $s_{m\theta} = s_y$ for $\dfrac{s_z}{s_y} > 1.80$ $s_{m\theta} = \dfrac{s_z + s_y}{2\sqrt{2}}$ for $0.55 \le \dfrac{s_z}{s_y} < 1.80$	s_z and s_y are the center-to-center bar spacing in the z and y directions, respectively. However, in case that the longitudinal bars are concentrated at the top or bottom level of the cross-section, s_z is calculated using the equation adopted in the Eurocode 2.

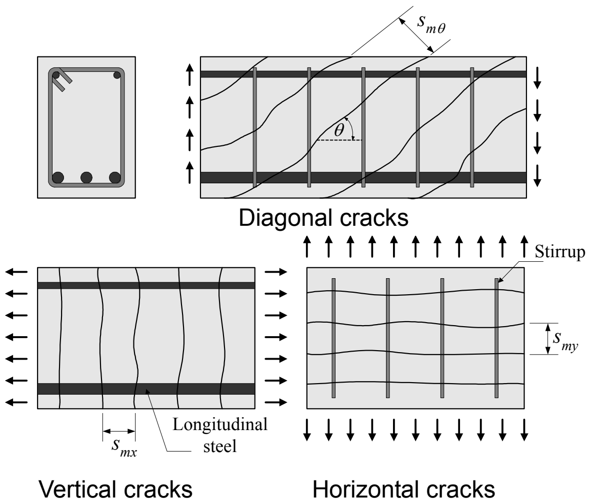


Fig. 1 Modeling of cracks (smeared cracks)

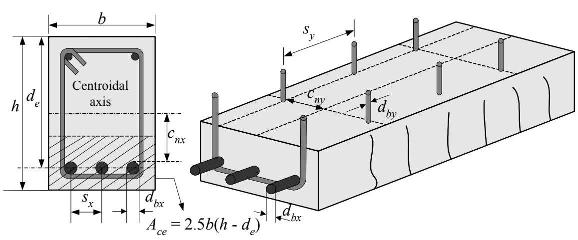


Fig. 2 Parameter influencing crack spacing

have shown that strain in stirrup is the most important factors affecting the shear crack width in RC beams. Although, the shear crack widths can be related to the strain in stirrup at the level of the crack, there is a high variation of shear crack widths at a particular stirrup strain. For this reason, no general conclusion can be made on the exact relationship between shear crack width and stirrup strain. Some researchers [10, 11] suggested a simple linear relationship, while a second-order polynomial curve was adopted by some other investigators [7, 8].

Table 2 shows a summary of available formulae for predicting shear crack width based on the concept of using stirrup strain. It should be noted that the formulae are empirically obtained from the fitting of experimental data of shear crack width and stirrup strain, hence, there is no explicit expression of shear crack spacing.

3. EVALUATION OF SHEAR CRACK WIDTH MODEL

Experimental works conducted aiming at investigating the shear crack width and crack spacing of PRC beams are very limited. Even for RC beams, very few studies have been carried out. In this study, to evaluate the accuracy of existing approaches for estimating shear crack width in PRC members, the results of test conducted at Saitama University [9] are used for comparison. Brief details of experiment are explained in the following section.

Test specimens consisted of two RC and six PRC beams, having a rectangular cross-section of 200x300 mm. The typical layout and cross-section details of specimens are shown in Fig. 3. All specimens were statically loaded up to failure with a shear span-to-effective depth ratio (*a/d*) of 3.0. Experimental variables included the stirrup spacing (s_y=75 and 125mm), the presence of prestressing bar, the amount of prestressing force ($\sigma_{c,ps}$=2.5 and 5.0 MPa), and the distribution of compressive stress due to prestress. A smaller amount of stirrups was provided in the left span to ensure the occurrence of critical shear cracks, thus enabling the measurement of such cracks. At each loading stage, crack widths were determined using digital microscope with an accuracy of 1/1000 mm, and the crack pattern was recorded so that the crack development can be investigated. Note that the use of digital microscope beneficially enables the measurement of shear crack width at any location. Discussion of the test results will be made on the shear crack angles and spacing, the distribution of crack width along shear span, and

Table 2 Summary of formulae for average shear crack width (stirrup strain)

Researchers	Average shear crack width, w_m	Note
CEB-FIP Model Code (1978)	$w_m = S_{rm} \cdot \varepsilon_{wm}$ $S_{rm} = 2\left(c_y + \frac{s_y}{10}\right) + k_1 k_2 \frac{d_{by}}{\rho_{ey}} \le \frac{d-x}{\sin\alpha_s}$ $\varepsilon_{wm} = \varepsilon_w \left[1 - \left(\frac{V_{cd}}{V_s}\right)^2\right] \ge 0.4\varepsilon_w$	c_y is side concrete cover; s_y is stirrup spacing; d_{by} is diameter of stirrup; k_1 = 0.4 for deformed bars, k_1 = 0.8 for plain bars; k_2 = 0.25; ρ_{ey} is effective stirrup ratio, $\rho_{ey} = A_w/[(c_y+8d_{by})(15d_{by})]$, d is effective depth, x is depth of neutral axis, α_s is angle between stirrup and member axis
Hassan et al. (1991)	$w_m = \frac{\phi K_1 K_2 S}{1000 K_{fc} K_p}$ $K_p = \left(\rho_{ey}/0.004\right)^{1.3}$, $K_{fc} = \left(\frac{f'_c}{19.6}\right)^{2/3}$	For plain bar: $K_1 = 2.4$, $S = 4\times10^3\varepsilon_w + 20\times10^6(\varepsilon_w)^2$ For deformed bar: $K_1 = 2.0$, $S = 8\times10^3\varepsilon_w + 2\times10^6(\varepsilon_w)^2$ K_2 = 1.2 for vertical stirrup and 1.0 for inclined stirrup
Shinomiya et al. (2002)	$w_m = l_{av}\varepsilon_w$ $l_{av} = 2\left(\frac{c_s + c_b}{2} + \frac{s}{10}\right) + 0.1\frac{d_{by}}{\rho_{ey}}$	$\rho_{ey} = A_w/[(2c_b+d_{by})b]$; b is beam width; $c_b = (s_y-d_{by})/2$; c_s is side concrete cover; s_y is stirrup spacing; s is distance between stirrup legs, s = b-2c_s-d_{by}
Fukuyama et al. (2000)	$w_m = \frac{0.9D}{\sqrt{2}} \cdot \varepsilon_w$	D = beam depth
Piyamahant (2002)	$w_m = \frac{2d_{by}S}{(f'_c/20)^{2/3}}$; $S = \varepsilon_w(6 + 3500\varepsilon_w)$	f_c' is compressive strength of concrete (N/mm^2)

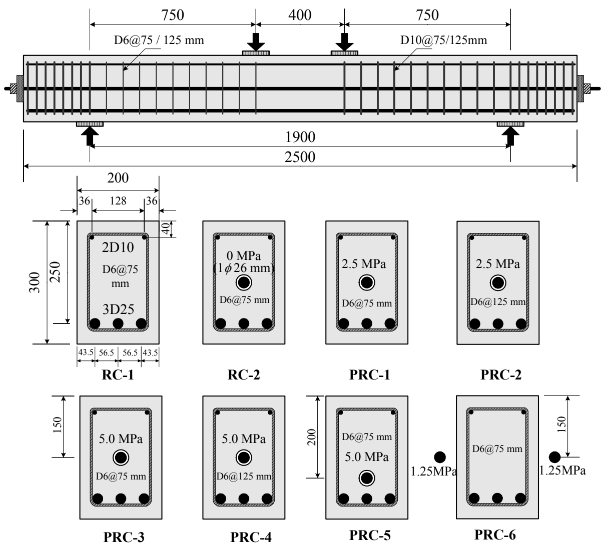


Fig. 3 Typical layout and cross-sectional details of test specimens

the relationship between shear crack width and stirrup strain.

3.1 Shear crack angles and crack spacing

Fig. 4 shows crack patterns at the load when the strains in stirrups crossed by critical shear cracks reached the yield point. In all specimens, shear cracks are extensions of flexural cracks occurred in the shear span (flexural shear cracks). It can be seen that the angles of shear cracks are not constant and varied along the shear span: cracks become steeper as they get closer to the support. A summary of shear crack angles is shown in Table 3. It can be seen that the average of crack angel can be taken as approximately 35° for all specimens.

To check the applicability of smeared crack model for estimating the crack spacing of flexure-shear cracks, crack spacing calculated by existing formulae is compared with that obtained from the test results as shown in Table 4. Note that the vertical crack spacing, s_{mx}, was measured at the centroid of concrete section while the horizontal crack spacing, s_{my}, was measured at the location of stirrup No. 3, for beam RC-1. From the results of comparison, it is evident that the prediction of shear crack spacing by Collins and Mitchell is greatly overestimated, while the simplified models proposed by Yoon et al. and Iori and Dei Poli show a better agreement.

3.2 Variation of shear crack width along shear span

The variation of the shear crack widths along shear span is carefully investigated by plotting the measurements of crack widths at each stirrup with increasing load (Fig. 5). Moreover, the average strains of each stirrup are also plotted for comparison. It can be seen that the variation of crack widths is almost similar to the distribution of stirrup strains, implying that shear crack widths have a close relationship with stirrup strains. By increasing the level of prestress ($\sigma_{c,ps}$), shear crack widths and stirrup strains tend to decrease and show a rather uniform variation along the shear span. This clearly indicates the impact of $\sigma_{c,ps}$ on the shear cracking behavior in PRC members.

3.3 Shear crack width and stirrup strain relationship

Fig. 6a shows the relationship between shear crack width and stirrup strain for beams PRC-1 ($\sigma_{c,ps}$=2.5 MPa, s=75mm) and PRC-2 ($\sigma_{c,ps}$=2.5 MPa, s=125mm). It can be seen that, at a particular stirrup strain, shear crack widths show significant variation. According to the regression analysis, the best fitting curve can be obtained from a linear curve, which can be used to represent the relationship between average values of shear crack widths and stirrup strains. From Fig.

Table 3 Summary of crack angles

Beam	θ_1	θ_2	θ_3	θ_4	θ_{av}
RC-1	43.6	29.3	-	-	36.5
RC-2	43.1	28.2	-	-	35.7
PRC-1	44.0	37.3	23.0	-	34.8
PRC-2	47.6	35.3	27.0	-	36.6
PRC-3	48.5	37.2	19.7	-	35.1
PRC-4	44.6	36.0	22.8	-	34.5
PRC-5	42.8	38.7	28.6	27.3	34.4
PRC-6	41.0	38.5	24.4	-	34.6

Table 4 Comparisons of crack spacing

	Exp.	Collins & Mitchell	Yoon et al.	Iori & Dei Poli
S_{mx}	150.1	227.4	250	-
S_{my}	104.2	295.1	75	-
$S_{m\theta}$	85.6	187.3	76.3	70.2

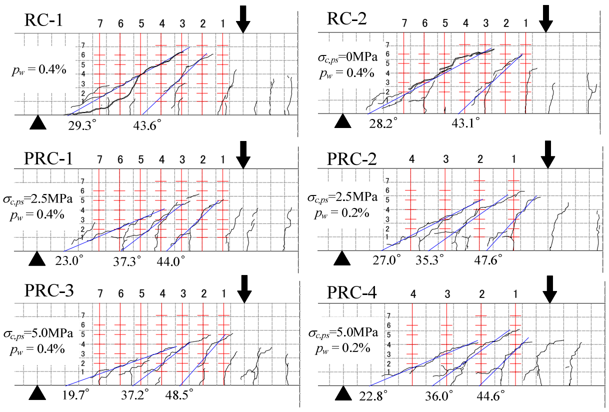


Fig. 4 Crack patterns and crack angles

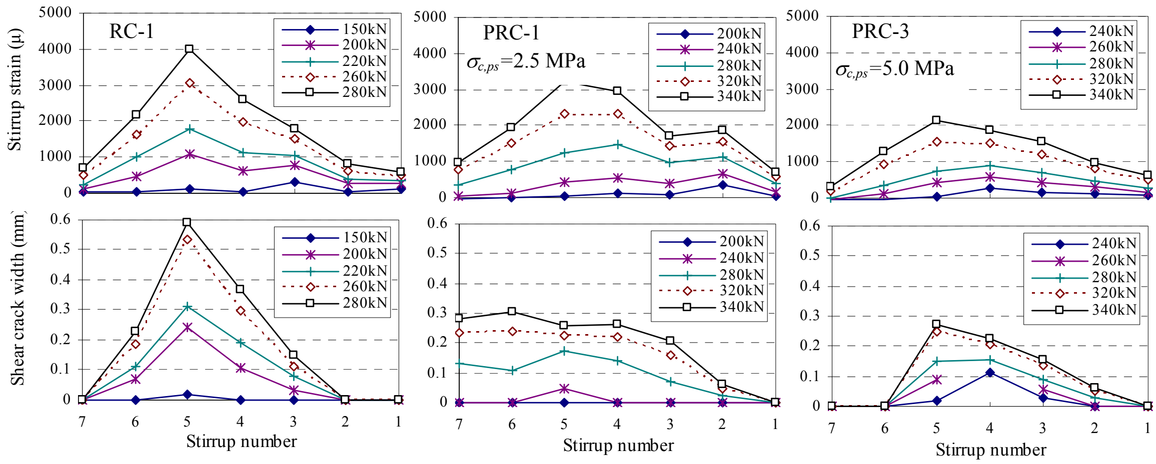
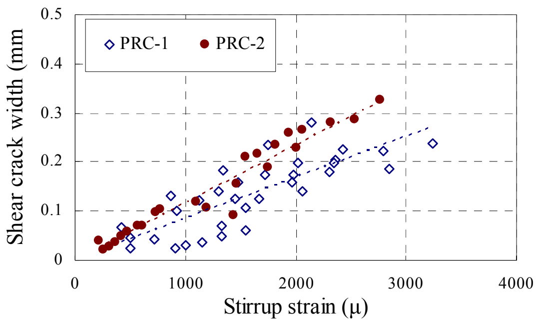
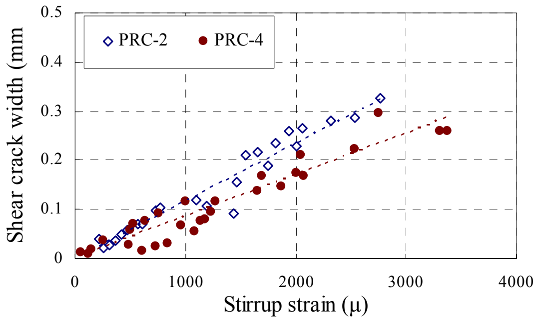


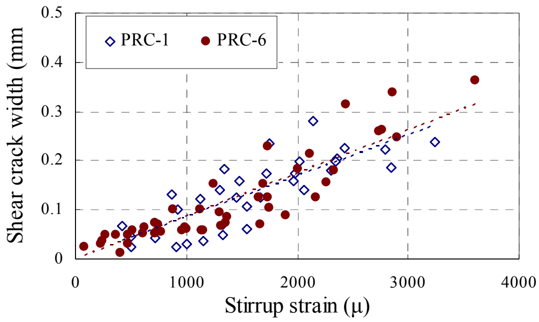
Fig. 5 Variation of shear crack width and stirrup strain along shear span



(a) Effect of stirrup spacing (stirrup ratio)



(b) Effect of prestress level ($\sigma_{c,ps}$)



(c) Effect of prestressing bar as tension steel

Fig.6 Shear crack width vs. stirrup strain

6a, beam PRC-2 yields a larger shear crack width at the same stirrup strain, implying that the stirrup spacing has an important effect on w-ε_w relationship in PRC members. In beams with small stirrup ratio (PRC-2 and PRC-4), increasing the prestress level can also effectively reduce the shear crack widths, as can be seen from Fig. 6b.

The contribution of prestressing tendon to control shear cracks can be considered consisting of the axial compressive force and the presence of prestressing steel as tension reinforcement. The impact of provision of prestressing bar at the centroid of beam section can be discussed by comparing the w-ε_w relationship of beams PRC-1 and PRC-6, as illustrated in Fig. 6c. Note that in beam PRC-6 the prestressing bars are provided outside the concrete section, thus only axial compressive forces are acting at both ends of the beam. Fig. 6c shows that the w-ε_w relationships of both specimens are almost the same, implying that the provision of prestressing bar at mid-depth of concrete section is ineffective in aiding control of shear crack in beams. The reason may be due to the inferior bond property of prestressing bar compared to that of deformed stirrups.

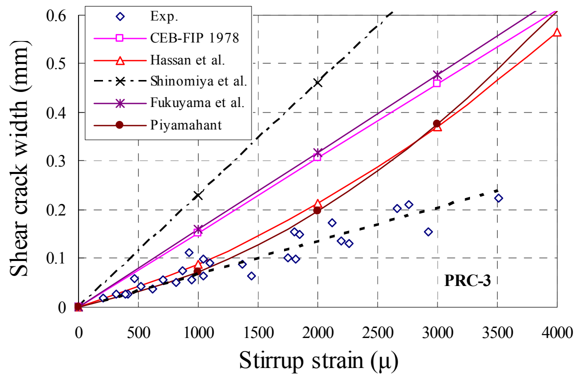


Fig. 7 Comparison of various prediction formulae

To verify the applicability of existing formulae for predicting shear crack width in PRC members, shear crack widths calculated by various prediction equations are compared with the test results, as illustrated in Fig. 7 for beam PRC-3. It is apparent that crack-width predictions from all equations appear to be overestimated, although the predicted crack widths are average crack widths, not the maximum ones. This may be attributed to the fact that parameters related to the prestressing force ($\sigma_{c,ps}$), which can affect the w-ε_w relationship, are not taken into account in any existing prediction formulae.

4. PROPOSED EQUATION FOR SHEAR CRACK WIDTH

The proposed equation for calculating shear crack width in PRC members is developed based on the assumption of a linear relationship between shear crack width and stirrup strain. As discussed in the previous section, influencing factors affecting w-ε_w relationship are stirrup spacing (or stirrup ratio, p_w) and compressive stress at centroid of concrete section due to prestress, $\sigma_{c,ps}$. The factors k_w and k_p, which are functions of stirrup ratio and $\sigma_{c,ps}$, respectively, were chosen to obtain a better fitting curve for the test results used in this study. To incorporate the effect of crack angles, shear crack spacing parameter is also included, by determining the longitudinal crack spacing, s_{my}, and the transverse crack spacing, s_{mx}, from the provisions suggested in the CEB-FIP Model Code 1978. The average shear crack width can then be expressed as:

$$w = 0.75 k_w k_p s_{m\theta} \varepsilon_w \qquad (3)$$

where $k_w = \left(\dfrac{0.004}{p_w}\right)^{2/3}$ $\quad k_p = \left(\dfrac{1}{1+\sigma_{c,ps}/f'_c}\right)$

Table 5 Accuracy of proposed equation

Beam	CEB-FIP 1978		Hassan et al.		Proposed Eq.	
	Avg.	Std.	Avg.	Std.	Avg.	Std.
RC-1	0.50	0.13	0.68	0.13	0.94	0.07
RC-2	0.59	0.14	0.81	0.13	1.13	0.07
PRC-1	0.55	0.11	0.80	0.09	1.07	0.06
PRC-2	0.71	0.13	0.96	0.11	1.03	0.09
PRC-3	0.46	0.14	0.67	0.12	0.95	0.07
PRC-4	0.51	0.15	0.66	0.15	0.79	0.10
PRC-5	0.62	0.12	0.92	0.10	1.24	0.06
PRC-6	0.55	0.13	0.83	0.11	1.08	0.07

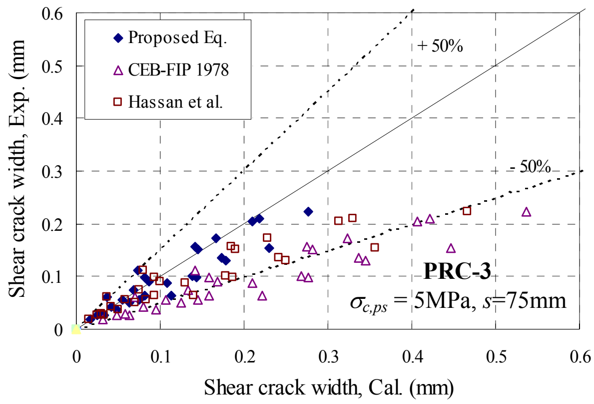


Fig. 8 Accuracy of proposed equation (PRC-3)

A comparison of predicted and measured shear crack widths is summarized in Table 5, and the accuracy of proposed equation for calculating shear crack width in beam PRC-3 is illustrated in Fig. 8. Note that the comparison was carried out up to the yield strain of stirrups. It can be seen from Table 5 and Fig. 8 that the predictions from the proposed equation show a better agreement compared to those obtained from other equations. However, it should be noted that the proposed equation is based on a rather limited number of test data and range of variability of the parameters studied. A comparison with other test results, currently available in literature, is needed to verify the effectiveness and the accuracy of the proposed method.

5. CONCLUSIONS

Prediction formulae available in the literature for calculating shear crack width and crack spacing can be mainly classified into two categories according to the main variable considered in the approach: principle tensile strain or stirrup strain. The approaches using principal tensile strain (smeared crack concept) seem to be more rational than those based on stirrup strain because the crack control characteristic of reinforcement in both horizontal and vertical directions can be taken into account. However, the test results have shown that stirrup strain is the most important factor affecting shear crack width in both RC and PRC members. Other influencing factors that should be taken into account were found to be the stirrup ratio and the prestress level. By taking the above parameters as variables, a simplified equation for predicting shear crack width in PRC members was proposed. The shear crack widths computed from the proposed equation showed a better correlation with the test results compared to other prediction formulae.

REFERENCES

[1] Adebar, P.: Diagonal Cracking and Diagonal Crack Control in Structural Concrete, ACI SP-204: Design and Construction Practices to Mitigate Cracking, pp. 85-106, 2001

[2] Collins, M.P. and Mitchell, D.: Prestressed Concrete Structures, Prentice-Hall Inc, Englewood Cliffs, N.J., 766 pp., 1991

[3] CEB: CEB-FIP Model Code 1990, CEB Bull d'Information, No. 203, 204 and 205, July 1991

[4] Yoon, Y.S., Cook, W.D., and Mitchell, D.: Minimum Shear Reinforcement in Normal, Medium, and High-Strength Concrete Beams, ACI Structural Journal, Vol. 93, No. 5, pp. 576-584, Sept.-Oct. 1996

[5] Colotti, V. and Spadea, G.: An analytical model for crack control in reinforced concrete elements under combined forces, Cement & Concrete Composites, Vol. 27, pp. 503-514, 2005

[6] Iori, I. and Dei Poli, S.: Sulla torsione di elementi strutturali in cemento armato. L'Industria Italiana del Cemento, Vol. 2, pp. 121-129, 1985 (in Italian)

[7] Hassan, H. M., Farghaly, S. A., and Ueda, T.: Displacements at Shear Crack in Beams with Shear Reinforcement under Static and Fatigue Loadings, Proceedings of JSCE, No. 433, Vol. 15, pp. 215-222, 1991

[8] Piyamahant, S.: Shear Behavior of Reinforced Concrete Beams with a Small Amount of Web Reinforcement, M.Eng. Dissertation, Kochi University of Technology, Japan, 2002

[9] De Silva, S.: Investigation on Shear Cracking Behavior of Partially Prestressed Concrete Beams, M.Eng. Dissertation, Saitama University, Japan, 2005

[10] Fukuyama, H., Suwada, H., Iso, M., Matsuzaki, Y., Nakano, K., and Kasahara, M.: Evaluation of Damage Limit State of RC Elements due to Shear Crack Width, Proceedings of AIJ, Vol. C-2, pp. 13-14, 2000 (in Japanese)

[11] Shinomiya, K., and Watanabe, F.: Study on Prediction of Shear Crack Width for Reinforced Concrete Beams, Proceedings of Architecture Institute of Japan, Vol. C-2, pp. 295-296, 2002 (in Japanese)