- Technical Paper -

3D NONLINEAR FINITE ELEMENT ANALYSIS OF CONCRETE UNDER DOUBLE SHEAR TEST

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ABSTRACT

This paper presents a study on shear strength of lightweight and normal aggregate concrete, mortar and paste specimens under double shear test. A linear relationship between compressive and shear strength has been found for all materials. 3D nonlinear finite element analysis closely predicted experimental shear strengths. Some aspect of failure mechanism was discussed by investigating deformation, stress and strain distribution of the numerical model.

Keywords: Shear strength, double shear, smeared crack model, nonlinear finite element analysis

1. INTRODUCTION

Concrete in structural members rarely failed due to shear stress. This is because concrete material is weak in tension and comparatively strong in shear. This leads many researchers think that unlike strength in tension and compression, concrete has no characteristic value for shear [1]. In principle, shear can be considered as a biaxial tension-compression state of stress. Fig.1 illustrates this idea. The closed curve obtained from experimental study is a typical biaxial strength envelop for concrete subjected to proportional biaxial loading. This concept of multiaxial behavior considered concrete as elastic-fracture material so before limit stress is reached concrete remains intact.

However the situation is not as simple as that. It is also well known from experiments that in the microlevel when stress is high, micro-cracks have occurred and the behavior of concrete is nonlinear. In certain stress regime these micro-cracks will distribute in a very narrow band, so in the macro-scale after concrete has fractured, shear failure may occur in a weak plane in material along the shear band.

Several researchers believe in the existence of shear fracture in concrete. The mechanism underlying their claims is shown in Fig.2. Shown in Fig.2(a) is Bazant [2] definition of a shear crack. A precursor to the development of macro shear is an array of tensile (mode I) mircocracks propagated in a very narrow region. At failure, the inclined cracks are supposed to join, in order to form a single shear plane as indicated. A second approach to explain



Fig.1 Biaxial strength of concrete



Fig.2 Definition of shear failure

shear failure is an idea of Fictitious Crack Model proposed by Hillerborg [3] as shown in Fig.2(b). In this approach a process zone, which is a region in front of stress free macrocrack where microcracks distributed, is subjected to in plane shear. The third situation occurs when the shear zone is not sufficiently confined that means a mixed mode state of stress developed in concrete. A curve crack will propagate possibly like a sketch in Fig.2(c).

Shear in existing cracks has led to vivid discussions in the past decade. Some shear testing methods have been proposed and applied to concrete such as single shear test, double shear test, indirect single or double shear tests and punch-through shear test etc. However researchers raised doubt about possibility to realize pure shear (mode II) damage and concluded that all of the proposed testing methods yielded a mixed mode failure of concrete [4]. While

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the debate of shear failure mode continues, there is a practical interest in shear strength of concrete.

This paper focuses on shear strength of different types of concrete, mortar and paste materials. Nonlinear finite element (FE) analysis has been implemented to verify experimental results and simulate failure process of test specimens.

2. SHEAR STRENGTH DEFINITIONS

Shear strength of a material is often used to covers several concepts such as 1) strength against pure shear, 2) shear stress required for failure without normal stress, 3) shear diagram on solid interface depending on normal stress, 4) Mohr's stress envelop [6]. According to Everling [7] shear strength τ can be defined as the breaking shear force T applied to an imposed surface A supporting no normal force, that is:

$$\tau = T/A$$
 (1)

One common way to define concrete shear strength is based on failure curves of concrete in σ - τ space. Fig.3 schematically shows such a curve. This curve is an envelope of a series of Mohr's circles obtained from shear test of concrete specimens under uniaxial and multiaxial stress state. The value of shear, where the envelope crosses vertical axis, is defined as the shear strength, which can generally be expressed by the following formula[1]:

$$\boldsymbol{t}_c = k \sqrt{f_c f_t} \tag{2}$$

where,

 τ_c is shear strength (N/mm²) f_c and f_t are compressive and tensile strength respectively (N/mm²) k is a coefficient obtained from experiment

A particular case of k=0.5 gives Mohr's formula of shear strength which denoted by τ_0 in Fig.3. It is to note that definition using failure envelope doesn't give consistent shear strength. Different testing methods yield different values of shear strength.

From classical school of linear elastic fracture, in pure shear mode of failure, crack should initiate and propagate under uniform shear stress. Crack growth should also be confined to a plane. A limit shear stress in such circumstance is shear strength.

In this study Formula (1) is used to calculate shear strengths. The shear force was obtained using double shear test recommended by Uomoto [5].

3. TEST PROGRAMS

3.1 Detail of Tested Specimens

The experimental work consisted of specimens



Fig.3 Failure envelop in σ - τ space

Series	W/C	Q tt.	Description						
n a m e	(%)	(unit)							
P - 1	86	12	Cement paste						
P - 2	58	12							
P - 3	43	12							
N S - 1	86	12	Normal aggregate						
N S - 2	58	12	mortar						
N S - 3	43	12							
L S - 1	86	12	Lightweight aggregate						
L S - 2	58	12	mortar						
L S - 3	43	12							
N S N G - 1	86	12	Normal weight						
N S N G - 2	58	12	concrete.						
N S N G - 3	43	12							
N S L G - 1	86	12	LWA concrete of						
N S L G - 2	58	12	normal fine and LW						
N S L G - 3	43	12	coarse aggregates						
L S N G - 1	86	12	LWA concrete of LW						
L S N G - 2	58	12	fine and normal coarse						
L S N G - 3	43	12	aggregates						
L S L G - 1	86	12	LWA concrete of LW						
L S L G - 2	58	12	fine and coarse						
L S L G - 3	43	12	aggregates						

Table 1 Test specimens

made by four types of concretes, two types of mortar and cement paste materials. **Table 1** shows different types of material together with W/C ratio and quantity of specimens for each series of tests. There are 21 series of tests. Each series has 12 specimens and the total number of specimens is 252. Shear test specimen is a block of dimensions $10 \times 10 \times 40$ cm. In addition, for each series of tests, there were three standard cylinder specimens for compressive strength and Young's modulus tests. All specimens were cast and cured in water under standard room temperature for 28 days. Materials used for mortar and concrete mixture are presented in Table 2. The selection of mix proportions for paste, mortar and concrete specimens is presented in Table 3.

3.2 Test Arrangements and Procedure

Photo 1 shows loading arrangement of the doubleshear test. A specimen was symmetrically positioned within a shear device. The shear device consists of two parts, a base and a top part. The outer surface of shearing edges of the top part is coincided with the inner surface of shearing edges of the base. The intersection of this surface and the specimen is the tested section, that is the section used to calculate shear strength. Along two tested sections there were strain gauges glued on the surface of the specimen. These gauges were used to monitor and adjust eccentric loading. Monotonic, force-controlled loading was applied with speed of increment of shear stress of 0.1N/mm² per second.

3.3 Selection of Test Data

Data of those only specimens with damaged surface closed to plane, that means satisfied condition that crack should confined to a plane, were selected for further analysis. A number of specimens with curved damaged surface or inclined surface, which is not coincided with test section, were excluded. Detail discussion of damage patterns of tested specimens and procedure for selection of the test results can be found in our previous published paper [4].

4. TEST RESULTS AND DISCUSION

4.1 Shear strength vs. compressive strength

Fig.4 shows plots of shear strength versus compressive strength for all selected specimens. It can be seen from the distribution of data points that shear strength increases with increment of compressive strength. Linear regression analysis showed that concrete and mortar specimens have



Photo 1 Test arrangement



Fig.4 Compressive-Shear strength relationship

N a m e	Symbol	Туре	N o te s				
Cement	С	Normal Portland cement	Density $3.16(g/cm^3)$				
F in e aggregate	S	N atural sand Lightweight (A sanolite)	Dry density 2.60(g/cm ³) Dry density 1.90(g/cm ³), absorption 20.2 %				
Coarse aggregate	G	N atural crushed rock Lightweight (A sanolite)	D ry density 2.63(g/cm ³) Dry density 1.58(g/cm ³), absorption 28.9 %				
A d m ix tu re	LS	Plasticizer	D ensity 2.70 (g/c m ³)				
A d m ix tu re	S P	High performance water reducing agent	S P 8 N				
	A F	Antifoam agent	M icro-air 404				
	M C	Vicious agent	90EMP-15000				

Table 2 Materials for concrete mixture

Series name	W/C	s/a	W eight per unit volume (kg/m ³)							
	(%)	(%)	W	С	LS	S _N	S _L	G _N	G L	S P
P - 1	86			672	566					1.34
P - 2	58	-	578	1003	283	-	-	-	-	5.02
P - 3	43			1335	0		l			10.7
N S - 1	86			305	257					0.61
N S - 2	58	-	256	456	129	1418	-	-	-	2.28
N S - 3	4 3			606	0					4.85
L S - 1	86	-		305	257					0.61
L S - 2	58	-	256	456	129		955	-	-	2.28
L S - 3	4 3			606	0					4.85
N S N G - 1	86			192	162					2.67
N S N G - 2	58	48	165	287	8 1	892	-	977	-	3.81
N S N G - 3	4 3			381	0					4.95
N S L G - 1	86			192	162					1.14
N S L G - 2	58	48	165	287	8 1	892	-	-	587	2.29
N S L G - 3	4 3			381	0					3.43
L S N G - 1	86			192	162					0.76
L S N G - 2	58	48	165	287	8 1	-	652	977	-	1.52
L S N G - 3	4 3			381	0					2.29
L S L G - 1	86			192	162					0.76
L S L G - 2	58	48	165	287	8 1	-	652	-	587	1.91
L S L G - 3	4 3			381	0		1			3.05

nearly identical relationship between shear and compressive strength which can be presented by the solid line, whereas paste specimens show slightly difference as shown by dotted line.

4.2 Relationship between compressive, tensile and shear strengths

Relationship between compressive, tensile and shear strength of concrete is shown in Fig.5. In this figure the coefficient k in Formula (2) was plotted against compressive strength of tested specimens. It can be seen that distribution of coefficient k is roughly in a range from 0.4 to 0.9. Average value of k for all specimens in the experiment is 0.62.

Shown in the same figure solid lines presenting two cases of coefficient k proposed by Morsh and Mohr. Most of the data points are bounded by these two values of k. Similar result was reported for different shear test methods in a previous experimental study [8].

5. FINITE ELEMENT SIMULATION

Failure process of concrete under increasing shear load would be well understood if we could measure stress distribution of the tested specimen. However such a measurement is extremely difficult. FE analysis is another approach to simulate this process. In this study three-dimensional nonlinear FE analysis has been implemented for tested specimens. Concrete behaviour and crack development are described using the rotating crack model [9]. In this model the orientation of crack is aligned to the principal stress direction, only normal stress occurred on the crack surfaces, and the crack experienced pure mode I. Therefore, no additional hypotheses are necessary to characterize the shear behaviour, which is generally difficult to model. However, like any other smeared crack model where cracks are idealized as being distributed or smeared over the whole element, the rotating crack cannot describe exact size and location of cracks. There is also no such definition like crack width in this model. Only a region or elements where cracks occurred can be detected.

5.1 Analytical Model

Fig.6 shows three-dimensional model of the double shear test. Concrete specimen is modeled using 3D solid 8 nodes isoparametric element having dimension of $5 \times 5 \times 5$ mm. Upper part of loading device and supports are modeled using rigid element.

Actual boundary conditions of the model are quite complicated. Friction between upper and lower





shearing edges (see Photo 1) depends on the level of vertical loading. However the contact area is comparatively small therefore this friction effect is ignored in the analysis. Shearing edges have common nodes with tested block. Lateral expansion of specimen is allowed by modeling support edges as rollers in horizontal direction or x direction in Fig.6.

A forced displacement is applied in the middle point of the upper part and the push-over analysis is implemented until the force P at the same point decreased 95% compared to its maximum value. The analysis made use of FINAL program developed by the Obayashi Corporation.

5.2 Constitutive model and failure criteria

Concrete in the experiment has undergone compression with almost no horizontal confinement. For such a low level of confinement, a modified Ahmad triaxial stress-strain relationship proposed by Naganuma [10] is promising because it showed a good agreement with experimental results. This stress-strain model is selected for constitutive model of concrete. The Ottosen four parameters concrete failure criterion is used in this study. According to Naganuma [10] this criterion also well corresponds with experiment data of low confined concrete under triaxial stress state. It is already known that the cracked concrete can still carry some tensile stress in the direction normal to the crack. The phenomenon is characterized by concrete intrinsic property called fracture energy, which is an area under tensile stressstrain curve. In this study, a model proposed by Izumo [11] was used for concrete stress-strain relationship in tension, which can be written as follow:

$$\frac{\boldsymbol{s}_{t}}{f_{t}} = \left(\frac{\boldsymbol{e}_{cr}}{\boldsymbol{e}_{t}}\right)^{c}$$
(3)

Where, f_t : Tensile strength under biaxial loading; \boldsymbol{e}_t : Average tensile strain in the direction perpendicular to crack; \boldsymbol{e}_{ck} : Average tensile strain when crack occurred; c: parameter representing condition of bond between rebars and concrete (c=1.0 for the case of plain concrete was used in this study).

5.3 Failure process

To investigate the failure process of tested specimens a representative case of analysis is shown hereafter. NSNG-3 is selected for this purpose. This is a normal concrete series of specimens with the following properties obtained from experiment:

Compressive strength:65.35 N/mm²Splitting tensile strength:4.06 N/mm²Young's modulus :30.91 kN/mm²Poisson ratio:0.18

Fig.7 shows analytical relationship of load P at the middle node of the upper part of the loading device (see Fig.6) and displacement of the same node. Load-displacement relationship is linear up to 60% of its maximum load value then the first crack occurred in the middle of bottom face of the specimen. The first crack was initiated by tensile stress concentrated in the middle of specimen. Fig.8a shows distribution of principal stress vectors when the first crack has occurred. Compressive load from upper shearing edge transferred to lower shearing edge through a compression core. A large compressive stress exists near shearing edges. However roughly between supports there was a flow of pure tensile stress causing vertical bending crack as can be seen in Fig.8b (compressive and tensile stress vectors are not in the same scale). The double shear test is obviously influenced by bending stress. The length of the bending crack however did not increase but confined within 2cm.

The next crack occurred along the sheared section in the side faces(Y=0mm and Y=100mm) of specimen and closed to the supports. This crack initiated also by the tensile stress concentrated along the sheared section and grew from the bottom to the top of the specimen. The crack however developed under mix mode of failure. Fig.9b shows principal strain distribution at the peak load. Those elements with large horizontal strain (tensile) are places where crack occurred due to mostly tensile stress.However close to shearing edges, especially upper edge, are



Fig.7 Analytical load-displacement relationship of NSNG-3





Photo 2. Damaged specimen

places where tensile strain is not large. In fact in these region crack occurred under shear stress. Fig.9a shows distribution of shear stresses in the same plane of side face of specimen. Shear stress did not uniformly distribute along the shear section but concentrated close to shearing edges causing crack at the final moment of failure. Maximum shear stress was $7.91N/mm^2$ which is 1.8 time of the split tensile strength of the specimen. Actual damaged of tested speciment can be seen in Photo 2.

5.4 Average shear strength

Analytical average shear strengths obtained by dividing maximum load by twice the area of shear section are plotted together with linear regression line of experiment data in Fig.10. The solid line, the same line for mortar and concrete in Fig.4, is plotted here again for convenience. From the figure it is clear that distribution of shear strengths obtained from FE analysis distributed quite closely to experimental line. However analytical results showed lower strengths compared to experiment one. This is probably due to difference between analytical and actual specimen's boundary conditions. If the friction effect at the supports can be taken into account then constraint of lateral expansion could be higher which could give raise to the strength of specimen. Also actual experiment could be less influenced by bending stress because of the higher actual lateral boundary confinement.

6. CONCLUSIONS

In this paper three dimensional nonlinear finite element analysis is carried out for concrete and mortar failing in double shear experiment. Following can be concluded:

- (1) Concrete and mortar has the same linear relationship between shear and compressive strengths which is slightly different from that of paste material. For low compressive strength paste has higher shear strength compared to concrete but for higher compressive strength opposite tendency has been found.
- (2) FE simulation showed that material in double shear test failed due to tensile and shear stresses. The test was influenced by bending stress causing crack in the middle of specimen.
- (3) Shear strengths obtained from analysis are closely approximate actual experimental strengths. Analytical strengths are slightly lower than that of experiment giving conservative evaluation of shear strength.

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