- Technical Paper -

EVALUATION OF SHEAR CRACK SPACING PREDICTION MODELS IN REINFORCED CONCRETE BEAMS

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ABSTRACT

An overview of existing shear crack spacing prediction models in reinforced concrete members is presented in this paper. The influential parameters on spacing of shear crack in each model are examined based on the shear cracking behavior clarified by the authors' previous study [1]. A comprehensive evaluation of these prediction models is carried out by comparing shear crack spacing calculated from the prediction models with those obtained from seven experimental investigations, which shows the necessity of more rational prediction model.

Keywords: reinforced concrete, shear crack spacing, shear crack width, stirrup strain.

1. INTRODUCTION

Cracks in concrete structures are unavoidable due to the low tensile strength of concrete. This issue considers one of the major problems that reduce the life time of concrete structures. A huge number of investigations in last decades were concerned with the cracking behavior and crack control in reinforced concrete (RC) members due to its harmful effects on structural performance such as serviceability and durability requirements. This interest has been significantly increased by a trend toward adapting the design codes into performance-based design. Under the performance-based design, crack width is related to various required performances such as appearance, water/air-tightness of concrete structures.

The guidelines prescribed in existing design codes are meant mostly for tensile and flexural crack width. These guidelines were experimentally obtained and cannot be applied directly to prediction of shear crack width, because shear cracking is caused by a different mechanism [1].

Extensive efforts have been carried out by many researchers in last decades [1-14] to clarify shear cracking behavior and its influential parameters in RC members. Zakaria et al. [1] have explained well the shear cracking behavior by carrying out a detailed experiment to show the effects of the various influential parameters on the shear crack spacing and the relationship between shear crack width and stirrup strain at the intersection with shear cracks. It was concluded that shear reinforcement characteristics, such as side concrete cover to stirrup, stirrup spacing and/or stirrup configuration, and longitudinal reinforcement ratio have a significant effect in controlling the shear crack spacings and openings. Shear cracks width increases approximately in proportion to both the strain of shear reinforcement and with the spacing between shear cracks, implying that the stirrup strain and the shear crack spacing are main factors on shear crack opening prediction. To predict the shear crack width effectively, a reliable prediction model of the shear crack spacing is needed.

Various prediction models have been proposed to calculate shear crack spacing [7-14] in RC members. In spite of these efforts, the factors affecting the shear crack spacing have not been well considered in the existing prediction models.

The objective of this paper is to evaluate the existing models for calculating shear crack spacings in RC beams in order to examine their reliability based on the comparison with the experimental data obtained from seven experimental sets on shear cracking behavior.

2. EXISTING SHEAR CRACK SPACING PREDICTION MODELS

It was found in the previous works [9, 10] that the shear crack spacing ($s_{m\theta}$) can be related to the crack control characteristics of both the longitudinal and transverse reinforcement, which can be represented by vertical and horizontal crack spacing (s_{mx} and s_{my}), as shown in Fig. 1. The vertical and horizontal crack spacings are the spacings that would occur under the tension in the direction perpendicular to the longitudinal and transverse reinforcement. Fig. 2 illustrates the influential parameters related to cross-sectional properties which affect shear crack spacing in RC beams.

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Table 1 Overview of existing models predicting average shear crack spacing

Models	Proposed Equations	Definition of Parameters
CEP-FIP Model (1978)	$s_{rm} = 2(c_s + \frac{s_y}{10}) + k_1 k_2 \frac{d_{by}}{\rho_y}$	s_{rm} is the average crack spacing; c_s is the side concrete cover to the shear reinforcement; s_y is the shear reinforcement spacing; d_{by} is the diameter of shear reinforcement; ρ_y is the ratio of the amount of transverse reinforcement to the effective concrete area within $7.5d_{by}$; k_1 is a coefficient that represents the bond properties of the bars (k_1 = 0.4 for deformed bars, k_1 = 0.8 for plain bars); k_2 = 0.25 for pure tension.
CEP-FIP Model (1990)	$l_{s,\max} = \frac{1}{\frac{\sin\theta}{l_{sx,\max}} + \frac{\cos\theta}{l_{sy,\max}}}$ $l_{sx,\max} = \frac{\varphi_{sx}}{3.6\rho_{sx}}$ $\quad l_{sy,\max} = \frac{\varphi_{sy}}{3.6\rho_{sy}}$ $s_{m\theta} = \frac{2}{3} l_{s,\max}$	$l_{s,max}$ is the maximum shear crack spacing; $l_{sx,max}$ is the maximum vertical crack spacing; $l_{sy,max}$ is the maximum horizontal crack spacing; $s_{m\theta}$ is the average shear crack spacing; θ is shear crack angle; φ_{sx} is the diameter of longitudinal reinforcing bar; φ_{sy} is the diameter of transverse reinforcing bar; ρ_{sx} is the ratio of the amount of longitudinal reinforcing steel to the effective concrete area ($\rho_{sx} = A_s/A_{cx,ef}$), as shown in Fig. 2; ρ_{sy} is the ratio of the amount of transverse reinforcing steel to the effective concrete area ($\rho_{sy} = 0.5A_w/2.5s_y(c_s+0.5\varphi_{sy})$).
Collins and Mitchell Model (1991)	$s_{m\theta} = \frac{1}{\frac{\sin\theta}{s_{mx}} + \frac{\cos\theta}{s_{my}}}$ $s_{mx} = 2(c_x + \frac{s_x}{10}) + 0.25k_1 \frac{d_{bx}}{\rho_x}$ $s_{my} = 2(c_y + \frac{s_y}{10}) + 0.25k_1 \frac{d_{by}}{\rho_y}$	c_x is the distance to the longitudinal reinforcement; c_y is the distance to the shear reinforcement; s_x is the longitudinal reinforcement spacing; s_y is the shear reinforcement spacing; d_{bx} is the diameter of longitudinal reinforcement; d_{by} is the diameter of shear reinforcement; ρ_x is the ratio of the amount of longitudinal reinforcementel to the effective concrete area ($\rho_x = (A_s+A_{ps})/A_{cx,ef}$), as shown in Fig. 2; ρ_y is the ratio of the amount of transverse reinforcement to the effective concrete area ($\rho_y = A_w/b_w s_y$), where b_w is the web width; k_1 is a weighted factor that represents the bond properties of the bars (k_1 = 0.4 for deformed bars, k_1 = 0.8 for plain bars).
Yoon et al. Model (1996)	$s_{m\theta} = \frac{1}{\frac{\sin\theta}{s_{mx}} + \frac{\cos\theta}{s_{my}}}$	$s_{m\theta}$ is the average shear crack spacing; θ is shear crack angle, and $s_{mx} = d_e$; d_e is the effective depth $s_{my} = s_y$; s_y is the shear reinforcement spacing
Shinomiya et al Model (2002)	$l_{av} = 2(\frac{c_s+c_a}{2} + \frac{s}{10}) + 0.1\frac{d_{by}}{\rho_y}$	l_{av} is the average crack spacing; c_s is the side concrete cover to the shear reinforcement; $c_a = (s_y-d_{by})/2$; s_y is the shear reinforcement spacing; d_{by} is the diameter of shear reinforcement; s is the distance between stirrup legs ($s = b_w-2c_s-d_{by}$); b_w is the beam width; ρ_y is the ratio of the amount of transverse reinforcement to the effective concrete area ($\rho_y = A_w/[(2c_a+d_{by})b_w]$).
Colotti and Spadea Model (2005)	$s_{m\theta} = s_x$ for $\frac{s_x}{s_y} < 0.55$ $s_{m\theta} = s_y$ for $\frac{s_x}{s_y} > 1.80$ $s_{m\theta} = \frac{s_x + s_y}{2\sqrt{2}}$ for $0.55 \le \frac{s_x}{s_y} < 1.80$	$s_{m\theta}$ is the average shear crack spacing; s_x and s_y are the center-to-center bar spacing for the longitudinal and shear reinforcement, respectively. If the longitudinal bars are concentrated at the top or bottom level of the cross-section (the case of a beam with longitudinal reinforcement in bending), s_x is replaced by the value of the spacing, s_{rm}, calculated according to the Eurocode 2 (1991).
Witchukreangkrai et al. Model (2006)	$s_{m\theta} = \frac{1}{\frac{\sin\theta}{s_{mx}} + \frac{\cos\theta}{s_{my}}}$ $s_{mx} = 2(c_b + \frac{s_x}{10}) + k_1 k_2 \frac{d_{bx}}{\rho_x}$ $s_{my} = 2(c_s + \frac{s_y}{10}) + k_1 k_2 \frac{d_{by}}{\rho_y}$	c_b is the bottom concrete cover to the longitudinal reinforcement; c_s is the side concrete cover to the shear reinforcement; s_x is the longitudinal reinforcement spacing; s_y is the shear reinforcement spacing; d_{bx} is the diameter of longitudinal reinforcement; d_{by} is the diameter of shear reinforcement; ρ_x is the ratio of the amount of longitudinal reinforcement to the effective concrete area within $7.5d_{bx}$; ρ_y is the ratio of the amount of transverse reinforcement to the effective concrete area within $7.5d_{by}$; k_1 is a coefficient that represents the bond properties of the bars (k_1 = 0.4 for deformed bars, k_1 = 0.8 for plain bars); k_2 = 0.25.
De Silva et al. Model (2008)	$s_{m\theta} = \frac{1}{\frac{\sin\theta}{s_{mx}} + \frac{\cos\theta}{s_{my}}}$ $s_{mx} = 0.36\times[2(c_x + \frac{s_x}{10}) + 0.25k_1 \frac{d_{bx}}{\rho_x}]$ $s_{my} = 0.36\times[2(c_y + \frac{s_y}{10}) + 0.25k_1 \frac{d_{by}}{\rho_y}]$	c_x is the distance to the longitudinal reinforcing bar; c_y is the distance to the shear reinforcement; s_x is the longitudinal reinforcement spacing; s_y is the shear reinforcement spacing; d_{bx} is the diameter of longitudinal reinforcement; d_{by} is the diameter of shear reinforcement; ρ_x is the ratio of the amount of longitudinal reinforcement, as shown in Fig. 2; ρ_y is the ratio of the amount of transverse reinforcement to the effective concrete area ($\rho_y = A_w/b_w s_y$); k_1 = 0.4 for deformed bars, k_1 = 0.8 for plain bars.

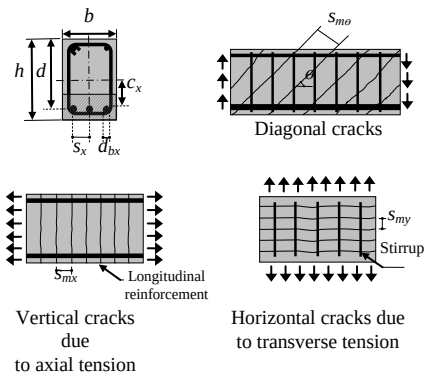


Fig. 1 Characteristics of shear crack spacing [10].

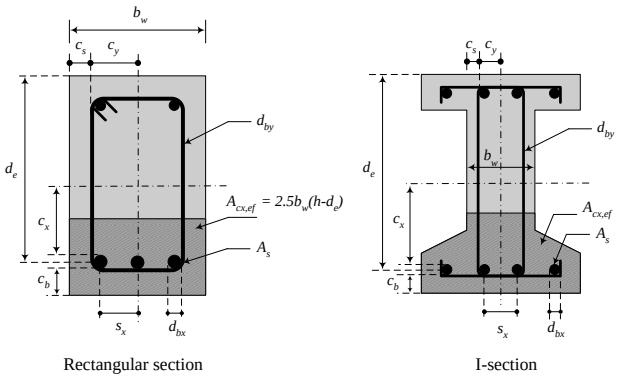


Fig. 2 Parameters affecting shear crack spacing.

Table 1 presents the summary of the existing shear crack spacing prediction models [7-14]. In these models, various parameters on shear crack spacing in RC members are proposed. However, none of the models considers fully all the influential factors affecting the shear crack spacing, which are stated in the authors' study [1], as shown below.

CEB-FIP model Code 1978 [8] accounts only for the characteristics of the shear reinforcement by considering side concrete cover to shear reinforcement, spacing, diameter and ratio of shear reinforcement as the influential parameters. The model provides an upper limit, beyond which the effect of spacing does not change, for the shear reinforcement spacing with 15 times the reinforcement diameter. Conversely, the model does not consider the crack control characteristics of the longitudinal reinforcement, such as the distance to the reinforcement (c_x, see Fig. 2) as stated in previous works [1, 7, 10]. Only reinforcement diameter and reinforcement ratio of both the longitudinal and shear reinforcement are considered in CEB-FIP model Code 1990 [9]. However, other parameters as illustrated by Zakaria et al. [1], such as the cover to the reinforcement and reinforcement spacing, are not taken into account. The parameters in Collins and Mitchell model [10] are not well selected. The distance to the shear reinforcement, c_y is adopted in the model instead of the side concrete cover to shear reinforcement, c_s which was found to have significant effect in the studies by Zakaria et al. [1] and De Silva et al. [7]. Also, there is no upper limit for the reinforcement spacing. Yoon et al. model [11] considers the effective depth, d_e and the spacing between shear reinforcement as the main parameters. Other influential factors on shear crack spacing are not included in the model because of its simple model nature. As the same concept of CEP-FIP model [8], Shinomiya et al. model [12] does not account for the effect of longitudinal reinforcement characteristics. Colotti and Spadea model [13] provides a simple expression that only considers spacing of longitudinal and shear reinforcement. Witchukreangkrai et al. [14] accounts for the crack control characteristics of both longitudinal and shear reinforcement. However, the model does not consider the distance to the longitudinal reinforcement, c_x. The upper limit for the reinforcement spacing is suggested

Table 2 Details of investigated specimens

Reference	Specimen	Overall height, h (mm)	Web width, b_w (mm)	Effective depth, d_e (mm)	Longitudinal reinforce-ment ratio, ρ_t %	Shear reinforce-ment ratio, ρ_w %	Side concrete cover to stirrup, c_s (mm)	Stirrup spacing, s_y (mm)
Hassan and Ueda (1978)	VDS10 right	500	200	440.00	2.87	0.31	25.0	230.0
	VDS10 left	500	200	440.00	2.87	0.84	25.0	85.0
	VDS13 right	500	200	440.00	2.87	0.31	25.0	410.0
	VDS13 left	500	200	440.00	2.87	0.82	25.0	155.0
	VPS10 right	500	200	440.00	2.87	0.33	25.0	190.0
	VPS10 left	500	200	440.00	2.87	0.91	25.0	70.0
	VPS13 right	500	200	440.00	2.87	0.31	25.0	430.0
	VPS13 left	500	200	440.00	2.87	0.83	25.0	160.0
Hisada (1999)	No. 1	400	160	345.50	3.07	0.50	20.0	79.2
	No. 2	400	160	345.50	3.07	1.00	20.0	39.6
	No. 3	400	160	345.50	3.07	0.50	20.0	79.2
	No. 4	400	160	345.50	3.07	1.00	20.0	39.6
Fukuyama et al. (2000)	No. 1	270	200	225.50	1.69	0.21	22.5	150.0
	No. 5	270	200	241.50	1.65	0.43	21.0	75.0
	No. 8	270	200	225.50	1.69	0.21	22.5	150.0
	No. 11	270	200	241.50	1.65	0.43	21.0	75.0
Witchukreangk-rai et al. (2004)	S-1	300	300	250.00	3.43	0.21	29.5	100.0
	S-2	300	300	250.00	3.43	0.11	29.5	200.0
De Silva (2005)	RC-1	300	200	250.00	3.04	0.42	25.0	75.0
	RC-2	300	200	224.12	4.58	0.42	25.0	75.0
De Silva et al. (2008)	IRC-1	500	150	450.00	3.00	0.34	25.0	125.0
	IRC-2	500	150	450.00	3.00	0.34	69.0	125.0
	IRC-3	500	150	408.49	4.17	0.34	25.0	125.0
Zakaria et al. (2009)	A_2 left	350	200	280.00	2.83	0.72	25.0	100.0
	A_2 right	350	200	280.00	2.83	0.72	25.0	100.0
	A_3 left	500	200	432.00	2.84	0.72	25.0	100.0
	A_3 right	500	200	432.00	2.84	0.72	25.0	100.0
	A_4 left	750	200	669.00	2.84	0.72	25.0	100.0
	B_1 left*	500	200	432.00	2.84	0.72	40.0	100.0
	B_1 right	500	200	432.00	2.84	0.72	40.0	100.0
	B_2 left	500	200	432.00	2.84	0.36	60.0	200.0
	B_2 right	500	200	432.00	2.84	0.72	60.0	100.0
	B_3 left	500	200	432.00	2.84	0.36	80.0	200.0
	B_3 right	500	200	432.00	2.00	2.84	0.72	80.0
	C_1	500	200	450.00	2.00	1.62	0.72	25.0
	C_2	500	200	427.00	2.00	2.30	0.72	25.0
	C_3	500	200	417.00	2.00	3.64	0.72	25.0

as in CEB-FIP model [8]. The model of De Silva et al. [7] applies the same concept as that of Collins and Mitchell model [10] but introducing a reduction factor for both vertical and horizontal crack spacing.

3. DESCRIPTION OF INVESTIGATED SPECIMENS AND EXPERIMENTAL DATA

The experimental data used in this study were collected from seven experimental investigations for RC specimens [1-7]. The test results of 37 beam specimens obtained from the available literature were used to compare among the different models predicting shear crack spacing in RC beams to evaluate their accuracy and validity. Those experimental results include the average measured shear crack angle and shear crack spacing in each shear span of each specimen. Details of the investigated specimens are given in Table 2.

Both the shear crack angle (θ_{exp}) with the member axis and shear crack spacing ($s_{m\theta\text{-}exp}$) in the direction vertical to the shear crack were measured at the height of the centroid of beam section.

4. EVALUATION OF THE EXISTING SHEAR CRACK SPACING PREDICTION MODELS

The accuracy of the existing eight models for predicting shear crack spacing is examined by comparing with the available experimental data described in section 3 as shown in Fig. 3.

The prediction results by CEB-FIP model [8] in Fig. 3 (a) show the lowest dispersion among all the investigated models with the coefficient of variation of 17.6%. The model overestimates the experimental results for shear crack spacing with the average of experimental-to-predicted ratios of 0.67. The reason for this overestimation is that CEB-FIP model [8] does not account for the effects of crack control characteristics of the longitudinal reinforcement and shear crack angle as well (see section 2).

Fig. 3 (b) presents the comparison results for CEP-FIP model [9] which show a relatively high dispersion with the coefficient of variation of 29.1%, and the underestimation for most of the cases with the average experimental-to-predicted ratios of 1.89. The reason for the inaccurate predictions of CEB-FIP model [9] is due to the fact that only the reinforcing bar diameter and reinforcement ratio are considered in the model.

Collins and Mitchell model [10] exhibits a high dispersion with the coefficient of variation of 21.3%, as shown in Fig. 3 (c). Many cases of experimental results are overestimated with the average shear crack spacing ratio of 0.75.

It is interesting to notice that Collins and Mitchell model [10] includes the distance from the vertical center line of the beam cross section to shear reinforcement (c_y in Fig. 2) as a parameter. The greater the distance is, the bigger shear crack spacing is. Among the right shear span of specimens B_1, B_2 and B_3 of the study by Zakaria et al. [1] whose web width is 200 mm, the side concrete cover is only the variable and 40, 60 and 80 mm respectively, meaning that the distance, c_y is 120, 80 and 40 mm respectively. The measured average shear crack spacing is 154.3, 171.8 and 195.6 mm respectively. This implies that side concrete cover is more influential than the distance, c_y. The main reason for the overestimation by Collins and Mitchell model may be the adopted definition of the effective concrete area around the shear reinforcement ($A_{cy,ef} = b_w s_y$) without the upper limit, as given in Table 1.

It can be inferred from Fig. 3 (d) that the prediction results of Yoon et al. model [11] give a relatively high dispersion with the coefficient of variation of 30.2% and that it has a relatively high underestimation of the experimental results with the average experimental-to-predicted ratios of 1.26. The main reason for the high scatter is that the model considers only the shear reinforcement spacing and the effective depth (s_y and d_e, respectively in Fig. 2) as the influential parameters for shear crack spacing.

From the comparison results presented in Fig. 3 (e), it can be concluded that Shinomiya et al. model [12] is the most conservative model whose average experimental-to-predicted shear crack spacing ratios is 0.55. Most of the experimental results are overestimated with the coefficient of variation of 31.8%. The reason for the overestimation by Shinomiya et al. model is due to the limited parameters, which are the side concrete cover to shear reinforcement, the diameter and spacing of shear reinforcement, the distance between stirrup legs, and the ratio of shear reinforcement, in the model as shown in Table 1.

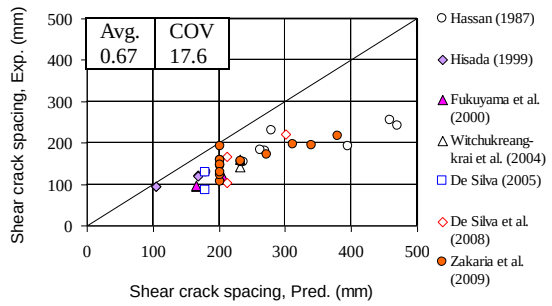
Colotti and Spadea model [13] shows prediction results similar to those of CEB-FIP model [9] in terms of underestimation and high dispersion. The coefficient of variation is 26.5% and the average of experimental-to-predicted shear crack spacing ratios is 1.60 as shown in Fig. 3 (f). The simplicity of the model, which only considers the longitudinal and shear reinforcement spacing as shown in Table 1, is considered the main reason for the high dispersion.

Fig. 3 (g) illustrates that Witchukreangkrai et al model [14] exhibits a rather high scatter with the coefficient of variation of 19.8% and small underestimation with the average of experimental-to-predicted ratios of 1.29**.** The reason of the underestimation is the fact that the model does not take into account for the effect of the maximum distance to the longitudinal reinforcement, c_x as explained in section 2.

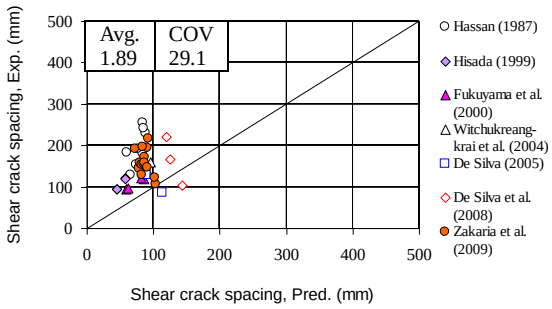
Witchukreangkrai et al. model [14] includes the bottom concrete cover to the longitudinal reinforcement (c_b in Fig. 2) as a parameter whose increase widens shear crack spacing. Among the left shear span of specimens A_2, A_3 and A_4 of the study conducted by Zakaria et al. [1], the beam effective depth is only the variable and 280, 432 and 669 mm respectively, giving the distance, c_x of 68.8, 139.5 and 246.6 mm respectively and the bottom concrete cover of 31.2, 25.5 and 29.4 mm respectively. The measured average shear crack spacing is 105.7, 143.3 and 191.2 mm

respectively. This implies that distance to the longitudinal reinforcement, c_x is more influential than the bottom concrete cover.

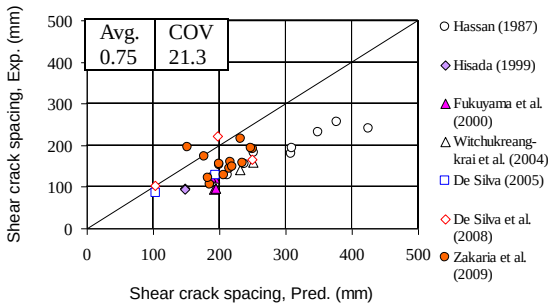
The prediction results by De Silva et al. model [7] in Fig. 3 (h) present a high dispersion with the coefficient of variation of 21.2%. The model underestimates the experimental results in many cases with the highest average of experimental-to-predicted



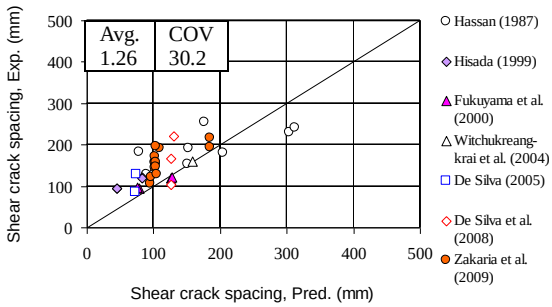
(a) Prediction results of CEB-FIP model (1978)



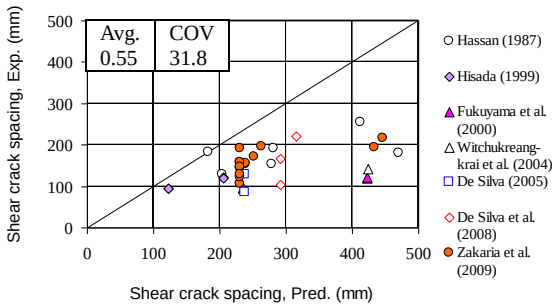
(b) Prediction results of CEB-FIP model (1990)



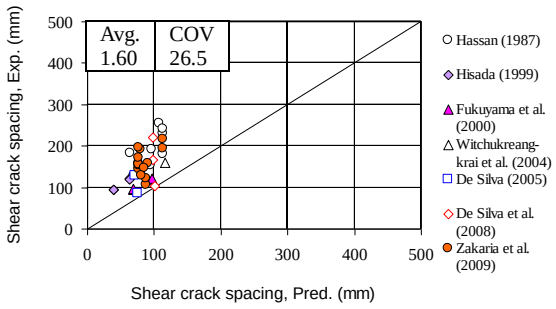
(c) Prediction results of Collins and Mitchell model (1991)



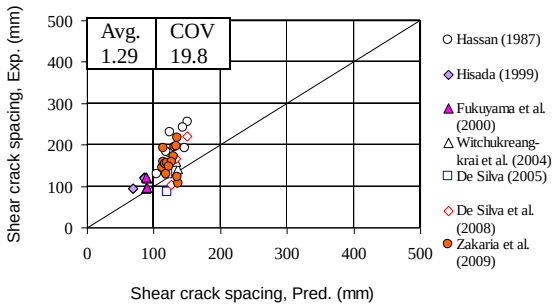
(d) Prediction results of Yoon et al. model (1996)



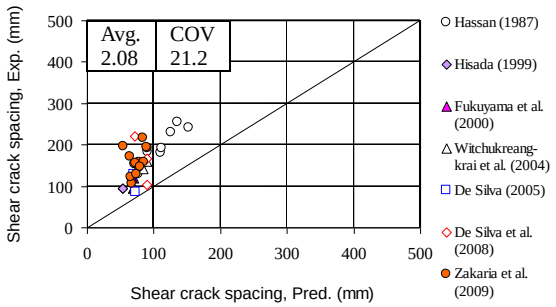
(e) Prediction results of Shinomiya et al. model (2002)



(f) Prediction results of Colotti and Spadea model (2005)



(g) Prediction results of Witchukreangkrai et al. model (2006)



(h) Prediction results of De Silva et al. model (2008)

Fig. 3 Accuracy of existing predication models for shear crack spacing

average shear crack spacing ratios of 2.08. De Silva et al. model is a model with simple modification of Collins and Mitchell model [10] by adding the reduction factor of 0.36, which was determined from a regression analysis of results of the experiment by De Silva et al [7], for crack spacings in longitudinal and transverse directions (s_{mx} and s_{my} in Fig. 1) as shown in Table 1. Introducing the reduction factor of 0.36 solves the overestimation with Collins and Mitchell model but causes the underestimation for most of the cases.

The above comparisons of prediction results by eight existing models clearly show that none of the models can provide good average of experimental-to-predicted ratio as well as small coefficient of variation. As a result, there is a necessity to propose a reliable model to estimate accurately average shear crack spacing in RC beams.

5. CONCLUSIONS

This paper carried out the comprehensive examination of the existing prediction models in the literature for shear crack spacing in reinforced concrete beams. The discussion on the influential parameters in these models is given based on the clarification of shear cracking behavior and its influential parameters. It was found that most of the existing prediction models consider the shear crack spacing to be a function of the vertical crack spacing, s_{mx} (or crack spacing in the direction normal to shear reinforcement) and the horizontal crack spacing, s_{my} (or crack spacing in the direction normal to longitudinal reinforcement) with consideration of shear crack angle. The experimental data used in this study were collected from seven experimental investigations on shear cracking behavior for RC specimens. The test results of 37 beam specimens obtained from the available literature were used to compare among the different models predicting shear crack spacing in RC beams to evaluate their accuracy and reliability. The comparisons of prediction results by eight existing models clearly show that none of the models can provide good average of experimental-to-predicted ratio as well as small coefficient of variation. To predict the shear crack width accurately, a reliable prediction model of the shear crack spacing is necessary.

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