- Technical Paper -

MODELING OF STRESS-CRACK WIDTH RELATIONSHIP OF HIGH PERFORMANCE FIBER REINFORCED MORTAR UNDER UNIAXIAL TENSILE LOADING

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ABSTRACT

In this study, influence of fiber orientation on tensile behavior of High Performance Fiber Reinforced Mortar (HPFRM) was examined. A stress-crack width model has been proposed which can consider the influence of fiber orientation. The parameters in this study are volume of fiber and fiber orientation by the direction of flow at casting. Based on experimental results, the model parameters were empirically formulated. By probabilistic approach tensile softening curve was calculated. Proposed model was compared with experimental results and good agreement has been observed.

Keywords: tensile behavior, HPFRM, fiber orientation coefficient, stress-crack width relationship

1. INTRODUCTION

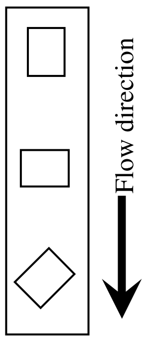
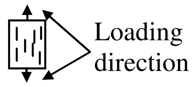
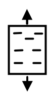
Recently, various types of High Performance Fiber Reinforced Mortar (HPFRM) with self-compacting ability has been developed. HPFRM is expected to improve tensile characteristics (ex. tensile strength, softening) by the incorporation of fiber in the matrix [1]. This material has some developed property, for example, high strength, high fluidly, high ductility. Those properties are enable to endow positive development in matrix, for example, rationalization of cross section, power saving of working and making durable construction. According to previous study fiber orientation of material with self – compacting ability is influenced by flow direction [1] and mechanical characteristics of HPFRM by considering fiber orientation have investigated [2]. From the study it was confirmed that mechanical characteristics of HPFRM has significantly influenced by fiber orientation. It was also confirmed that neither the volume fraction of fiber nor the number of fiber, orientation coefficient is the appropriate index to represent the mechanical characteristics of HPFRM. With this background in this study, uniaxial tension softening model of HPFRM has been proposed which can consider the influence of fiber orientation.

2. INFLUENCE OF FIBER ORIENTATION ON TENSILE SOFTENING BEHAVIOR [2]

2.1 Outline of Experiment

In this study, to make HPFRM ordinary Portland cement, fine aggregate, additive, steel fiber and water were mixed and cast. The target compressive strength was 100 MPa at 28 days after casting and W/B ratio was 21%. Three different volume fractions (0.5%, 1.0% and 1.5%) of fiber were used to make HPFRM and the length/diameter ratio of the steel fiber was 13/0.13. The flow direction of mortar was controlled by pouring directly into the concrete mold (400mm ×1800mm ×100mm). After pouring, three small steel frames (200mm×180mm×100mm) were installed into the concrete mold so the specimens with three values of angle against the flow direction (0°, 45° and 90°) were obtained. After demolding and curing, three test pieces (35mm×35mm×150mm) for each angle were prepared for the tensile test from the 200mm×180mm×100mm size specimens. The parameters in this study were mix proportion of fiber volume and fiber orientation according to the direction of the flow of casting (0, 45 and 90 degree). The main parameters studied in the experimental program are summarized in Table 1.

Table 1 Parameters of the experiment

Case No.	Image of flow direction	Image of fiber orientation
Case A		
Case B		
Case C		

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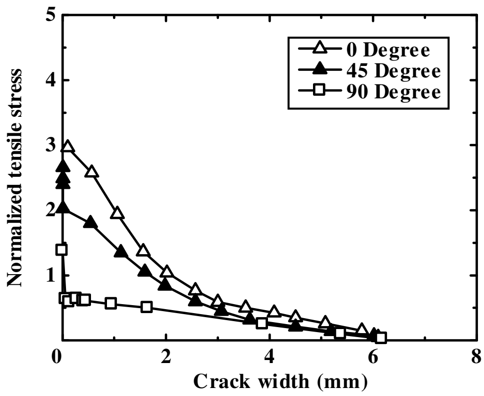


Fig.1 Relationship between normalized tensile stress and crack width (Fiber volume: 1.5%).

2.2 Experimental Results

The observed tensile behavior by a digital camera and strain gages are shown in Fig. 1. Because of the variation in the compressive strength of the each batch of mortar, measured tensile stresses were normalized by the tensile strength of the plain mortar. For all volume fraction of fiber it was observed that, specimen in which fibers are parallel to the direction of the tensile force shows higher tensile strength and improves post-peak behavior [2]. For all volume fractions of fibers, 0 degree orientation of fiber shows more ductile tension softening. Stress reduction for 0 degree orientation was gradual but stress reduction for 45 degree and 90 degree orientation was rather significant.

In case of crack pattern, specimens with 0 degree fiber orientation show multiple crack patterns. After first crack also shows some hardening and then finally failed by single crack about in the middle section of the specimens. Specimens with 45 and 90 degree fiber orientation also shows multiple cracking but comparatively less than 0 degree cases and finally failed by single crack. For others mechanical test results, (Tensile strength, fracture energy, maximum crack width) and microscopic test results are referred to previous publication by the authors [2].

3 TENSION SOFTENING MODEL OF HPFRM

3.1 Flow of Calculation

To calculate the softening behavior for any given crack width, W Eq. 1 has been used in this study as shown below. The flow to calculate stress for a given crack width W, Fiber volume V_f, and Casting direction, $\alpha_{casting}$, is shown in Fig. 2.

$$\sigma_W = \left\{ \sum_{i=1}^{N(W)} P_i(W) \times \eta_i \right\} \Big/ A \qquad (1)$$

where,

σ_W : Stress at any cross section at a crack width of W

$N(W)$: Number of fiber at any crack width W

$P_i(W)$: Pullout force of single fiber

η_i : Orientation coefficient of fiber i

A : Cross sectional area of the specimen

If V_f and $\alpha_{casting}$ is known then mean and standard deviation of number of fiber and orientation angle can be calculated as shown in Eqs.2, 3, 4 and 5 which were founded from experimental observation. By using the mean and standard deviation the distribution of number of fiber and orientation angle can be obtained by Eq. 6. From the distribution of number of fiber and orientation angle, number of fiber and their orientation coefficient in each small area can be calculated by Eqs. 7, 8. When all the parameters are known, stress in a small area can be calculated as shown in calculation flow by Eq. 9. This procedure will continue j times (j= 1 to M, M is number of small areas). Total response of stress for any given crack width will be the summation of individual response of all the small areas in a cross section as shown in Eq. 10. If total cross sectional area of a specimen is A and total stress is σ then stress-crack width curve for area A will be the summation of the stress-crack width curve of the small areas A_j (A_1, A_2, $A_3 \ldots A_M$) as shown in Fig. 3.

In this study, mean and standard deviation of number of fiber and orientation angle was proposed with variation of maximum, average and minimum response. So nine different softening responses can be found from nine different combinations of mean and standard value of number of fiber and orientation angle as the combinations are shown in Table 2.

3.2 Distribution of Number of Fiber and Orientation Angle

The model parameters, number of fiber and orientation coefficient are functions of volume fraction of fiber and casting direction. To quantify the number of fiber and orientation angle of fiber, probabilistic approach was considered in this study. To find the appropriate probability distribution function of number of fiber and orientation angle, several probability distribution functions (Normal, Poisson's, Weibull, Logarithmic distribution functions) were tried. It was concluded that there is no unique distribution function that can represent all the case of number of fiber and orientation angle of fiber [2]. Among various distribution functions normal distribution function showed better representation of the model parameters.

Table 2 Combination of mean and STD of number of fiber and orientation angle

Combination Of Mean and STD		Number of fiber (Mean and STD*) Maximum (A)	Average (B)	Minimum (C)
Orientation angle (Mean and STD)	Maximum(D)	D ×A	D ×B	D ×C
	Average (E)	E ×A	E ×B	E ×C
	Minimum(F)	F ×A	F ×B	F ×C

*STD: Standard deviation

Flow of the calculation

For a Given,
Crack width, W

Calculate mean and standard deviation of Number of fiber (N_f) and Orientation angle (φ)

$$\mu(N_f) = m_1 \times V_f \quad (2)$$
$$\sigma(N_f) = m_2 \times V_f \quad (3)$$
$$\mu(\varphi) = m_3 \times \alpha_{casting} + C_1 \quad (4)$$
$$\sigma(\varphi) = m_4 \times \alpha_{casting} + C_2 \quad (5)$$

where,
μ : Mean value of N_f and φ
σ : Standard deviation of N_f and φ
$\alpha_{casting}$: Casting direction

Calculate distribution of number of fiber (N_f) and orientation angle (φ)

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \quad (6)$$

where,
$P(x)$: Probability distribution
σ : Standard deviation of N_f and φ
μ : Mean value of N_f and φ

Calculate Number of fiber and Orientation coefficient
Number of fiber,

$$N_f(j) = Aj \times n_f \times P(x) \quad (7)$$

Orientation coefficient,

$$\eta_{i(j)} = \frac{1}{N}\sum_{i}^{N_f(j)} \cos\varphi \quad (8)$$

where,
N : Total number of fiber
Aj : Small area
n_f : Number of fiber / unit area
$N_f(j)$: Number of fiber in Small area Aj

Pullout force of single fiber for any given crack width w

Calculate up to j times, j = 1 to M (M: Number of small area)

Calculate Stress of a small area (Aj),

$$\sigma(j) = \left\{\sum_{k=1}^{N_f(j)} P(W) \times \eta_{i(j)}\right\} \Big/ A \quad (9)$$

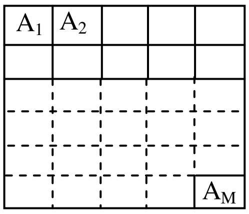
where,
$\eta_{i(j)}$: Orientation coefficient of fiber in small area A_j

Calculate Total Stress for any given crack width W,

$$\sigma_W = \sum_{j=1}^{M} \sigma(j) \quad (10)$$

where,
$\sigma(j)$: Stress at small area A_j (j = 1 to M)

Fig. 2 Flow of calculation of stress-crack width relationship



Small area, Aj
j = 1, 2, 3 …… M

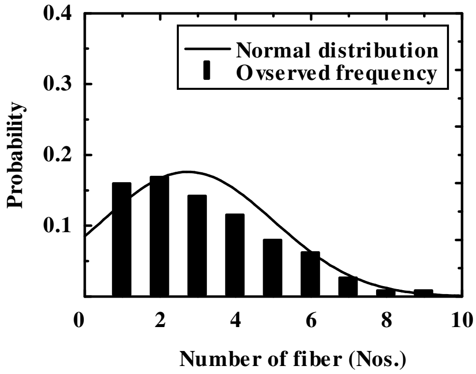
Stress (σ_1) in small area A_1,

$$\sigma_1 = \{P(W) \times N_f(1) \times \eta_{i(1)}\} / A_1$$

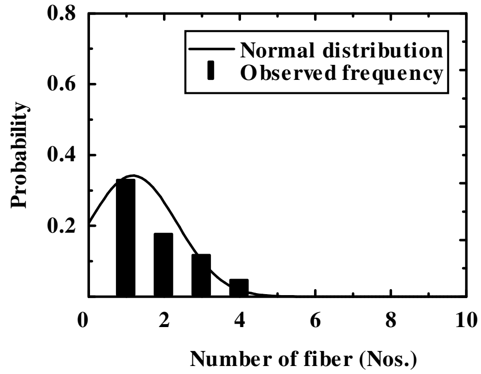
Total stress (σ),

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3 + \dots\dots\dots + \sigma_M$$

Fig. 3 Schematic of small areas in a cross sectional area of a specimen



(a) V_f: 1.0%, Casting direction: 0°



(b) V_f: 1.5%, Casting direction: 90°

Fig.4 Comparison between normal distribution of fibers and actual distribution of fibers

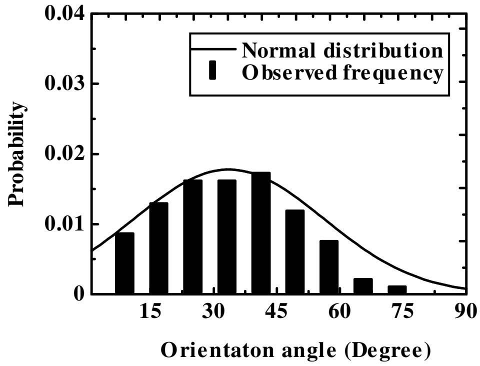


Fig.5 Comparison between normal distribution of orientation angle and actual distribution of orientation angle

Figure 4 shows the comparison of distribution of number of fiber by normal distribution function and actual distribution of fiber. Fig. 5 shows the distributions of orientation angle by normal distribution function and actual distribution of orientation angle. From the comparison it was clarified that, if the volume fraction of fiber in matrix and casting direction of any HPFRM member is known then the distribution of fibers and their orientation angle will follow normal distribution. So to use the normal distribution function, mean and standard deviation of number of fiber and orientation angle are proposed in this model.

Theoretically, the number of fiber and orientation angle of fiber should be same at any cross section of the matrix. However, in reality it is impossible that the number of fiber and their orientation angle will be same at any cross section of the matrix as they are not independent to volume of fiber and casting direction. To propose the model, this fact was taking into account by probabilistic approach. In addition to this, variation of number of fiber and orientation angle in a cross section is also consider by taking maximum, minimum and average of mean and standard deviation of number of fiber and orientation angle.

3.3 Formulation of the Model

By microscopic investigation the number of fiber and orientation angle of fiber was measured in this study. Six specimens were investigated per case. Total 54 specimens were investigated for nine cases [2]. From the microscopic investigation results, empirically mean and standard deviation of number of fiber is proposed to get the normal distribution of number of fiber in a cross section. Similarly, for orientation angle the mean and standard deviation of orientation angle also empirically formulate based on microscopic investigation, although the standard deviation was found almost constant for different mix proportions used in this study. So the standard deviation of orientation angle is proposed as constant. The coefficient of m, m_1, m_2 and m_3 of Eqs. 2, 3, 4 and 5 are shown in Table 3 and Table 4.

In this study, "Maximum" is empirically formulated from the maximum value of number of fiber and their orientation angle from the investigated six specimens per case. "Average" and "Minimum" was also formulated similarly. In case of number of fiber, the normal distribution curve transform into a curve that describes the number of fiber per unit area in small areas. Likewise, for orientation angle the normal

Table 3 Coefficient of mean and standard deviation of number of fiber

Number of fiber, Mean, $\mu(N_f) = m_1 \times V_f$ Standard deviation, $\sigma(N_f) = m_2 \times V_f$			
	Maximum	Average	Minimum
m_1	-0.1530	-0.0139	-0.0089
m_2	-0.0095	-0.0097	-0.0085

Table 4 Coefficient of mean and standard deviation of orientation angle

Orientation angle, Mean, $\mu(\varphi) = m_3 \times \alpha_{casting} + C_1$ Standard deviation, $\sigma(\varphi) = m_4 \times \alpha_{casting} + C_2$			
	Maximum	Average	Minimum
m_2	0.1919	0.14	0.109
m_3	-0.0097	-0.0097	-0.0097
C_1	0.074	0.06	26.3
C_2	26.95	26.95	26.95

distribution curve transform into a curve that describes the orientation coefficient of fiber in the small area. Further, this curve transform into a curve that describes the orientation coefficient of fiber to the number of fiber those are present in the small areas. In this approach precisely the number of fiber and their orientation coefficient in a small area was achieved. Fig. 6-9 shows how the normal distribution curve has been transformed into other form. Basic equations for transformation are shown in calculation flow. The third parameter of the model is single fiber pullout force. Sum of individual responses of all the fibers in cracked surface will be the total crack bridging force. To predict the stress-crack width relationship, single fiber pullout behavior was taken from experiment [1] shown in Fig. 10. The bi-linear lines are the arithmetic average value of single fiber pullout force for different embedded length.

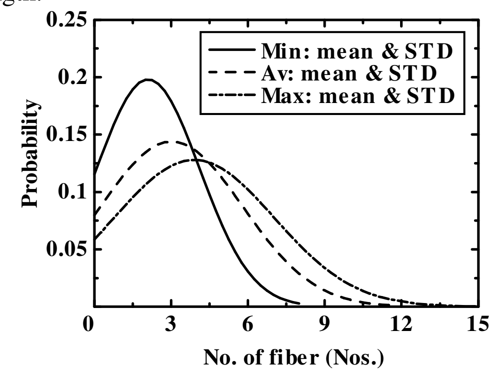


Fig. 6 Normal distribution of fiber by different mean value (V_f : 1.5% , Casting direction: 0°).

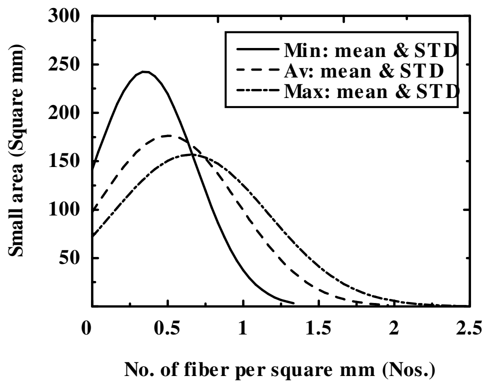


Fig. 7 Relationship between a small area and number of fiber per mm^2 (V_f: 1.5%, Casting direction: 0°).

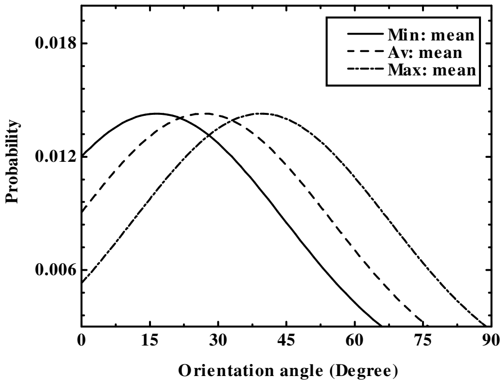


Fig. 8 Normal distribution of orientation angle of fiber (V_f: 1.0%, Casting direction: 0°).

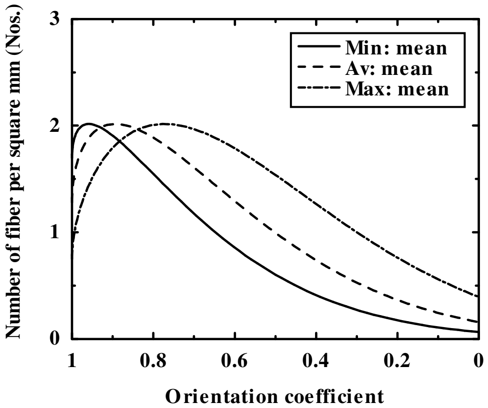


Fig. 9 Relationship between orientation coefficient and number of fiber per square mm (V_f: 1.0%, Casting direction: 0°).

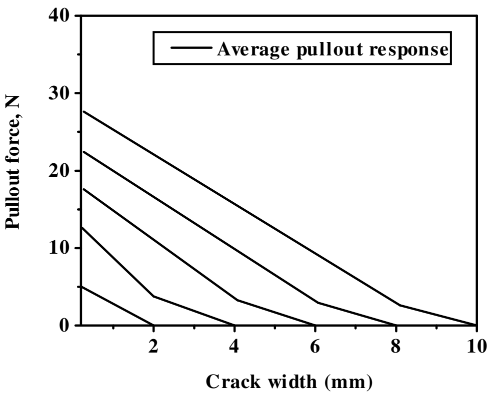
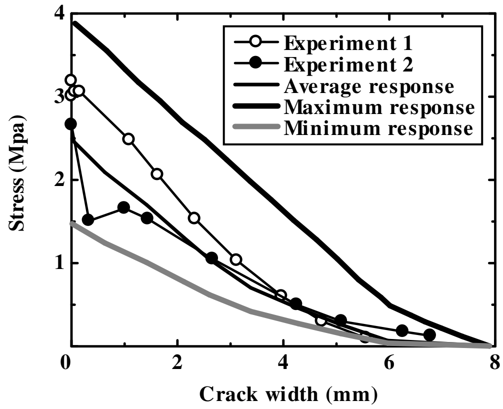


Fig.10 Pullout behavior of single fiber for different crack width

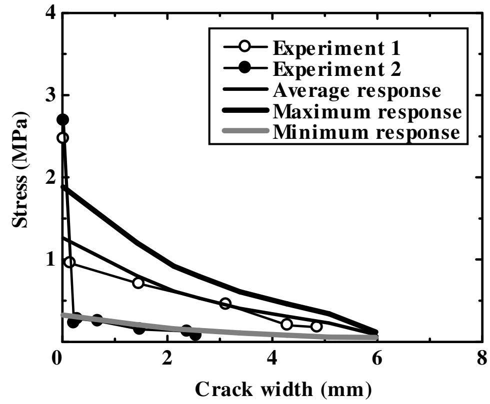
3.4 Comparison Between Predicted and Experimental Results

Experimental and predicted tensile softening behavior of HPFRM by the proposed model is compared to verify the model for different volume fraction of fiber and different casting direction shown in Figs. 11-13.For each cross section three softening curve was predicted. One of them is average softening curve which is calculated by the average distribution of number of fiber and orientation angle of fiber. The other two were maximum and minimum softening curve of a cross section and they were calculated by the maximum and minimum distribution of number of fiber and orientation angle of fiber (D × A and F × C in Table 2).

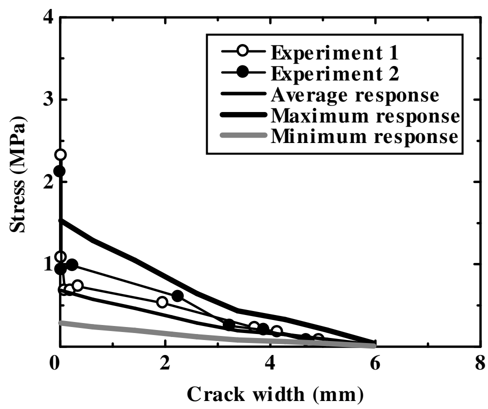
The predicted softening curve suggested that the proposed model can predict the experimental result with a higher degree of accuracy and the variation of experimental results can be predicted very well by the region of maximum and minimum softening curve which endow a region of variation of softening behavior in a cross section for any volume fraction of fiber and any castings direction.



(a) V_f : 0.5%, Casting direction: 0°

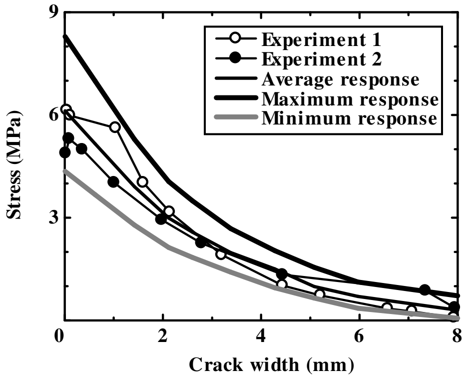


(b) V_f : 0.5%, Casting direction: 45°

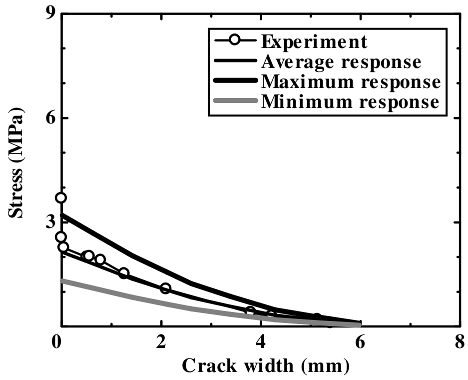


(c) V_f : 0.5%, Casting direction: 90°

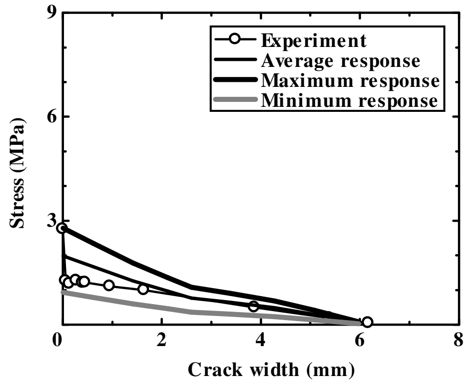
Fig. 11 Relationship between stress-crack width for 0.5 % volume of fiber



(a) V_f : 1.0%, Casting direction: 0°

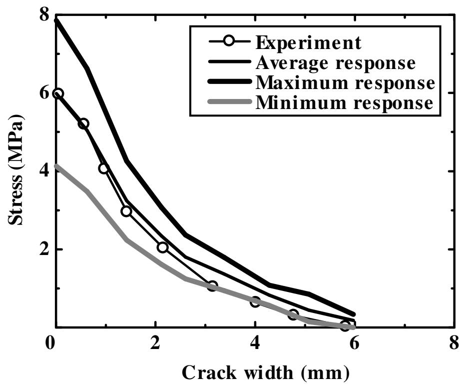


(b) V_f : 1.0%, Casting direction: 45°

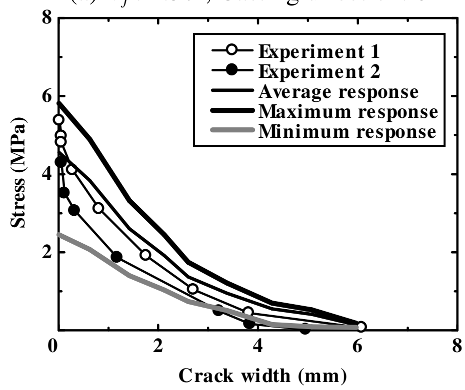


(c) V_f : 1.0%, Casting direction: 90°

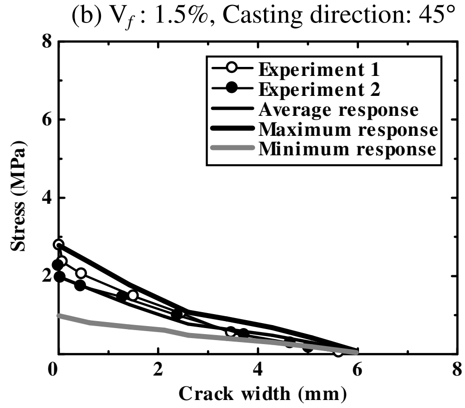
Fig. 12 Relationship between stress-crack width for 1.0 % volume of fiber



(a) V_f : 1.5%, Casting direction: 0°



(b) V_f : 1.5%, Casting direction: 45°



(c) V_f : 1.5%, Casting direction: 90°

Fig. 13 Relationship between stress-crack width for 1.5 % volume of fiber.

4 CONCLUSIONS

The following conclusions are drawn based on the findings of this study:

(1) The actual fiber orientation has strong influence on the tensile characteristics of HPFRM. So for that, in the prediction of structural performance of HPFRM members, the fiber orientation has to be considered.

(2) Normal distribution function shows better representation of model parameters.

(3) The proposed model can predict the stress-crack width relationship with higher degree of accuracy.

(4) A maximum and minimum softening curve for any cross section endow a region of variation of softening behavior which has significant impact on design purpose.

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