Analytical Study on the Thixotropy of Fresh Concrete Using Discrete Element Method

Guodong CAO[*1], Zhuguo LI[*2], Yuanqiang TAN[*3]

ABSTRACT

Fresh concrete is a typical thixotropic material with a time-dependent rheological characteristic. In this study, a general Discrete Element Method (DEM) model for fresh concrete was firstly established and the material parameters were calibrated by the rheological test. Then, a thixotropy DEM model of fresh concrete was proposed by introducing a time-dependent particle contact model into the general DEM model. The changes of fresh concrete's lateral pressure on the wall and yield stress with rest time were successfully addressed with this thixotropy DEM model.

Keywords: thixotropy, fresh concrete, discrete element method, lateral pressure, yield stress

1. INTRODUCTION

In fresh concrete, the irreversible hydration and reversible flocculation take place simultaneously, these processes result in the slump loss. However, the flocculent particles can be dispersed by an agitation or shearing. It is usually thought that the thixotropic behavior of fresh concrete results from the flocculation and dispersion of cement particles. Thixotropy includes two main aspects: (1) structural build-up when concrete is at rest and (2) structural break-down when concrete is under a shear or flow. Fresh concrete can exhibit different rheological behaviors when at rest than when flowing. This difference is due to thixotropy, which can have important consequences for such applications as formwork pressure, multi-lift casting, slip form paving, pumping, and segregation resistance.

In order to improve the workability of the fresh concrete, it is necessary to clarify the thixotropic characteristic of fresh concrete. Some researchers have studied the thixotropy of fresh concrete with theoretical analysis and experimental investigation [1] [2]. However, the size of particles in the theoretical analysis is from micro level to centimeter level. Also, because of the hydration effect, the accurately experimental investigation is in fact difficult. As a complement, the numerical simulation of the fresh concrete behavior could provide a powerful tool to understand the macroscopic material behavior [3], [4].

The Discrete Element Method (DEM) has been widely used to study the behaviors of various particulate materials [5-7]. Several numerical investigations, using the DEM, have also been reported on the flow behavior of fresh concrete [3], [8-12]. Tan et al. [3] proposed a two dimensional DEM model to study the effects of the pumping velocity and the angle of bend on the abrasion of pump pipes. A three dimensional DEM was used to simulate the flow behaviors of self-compacting concrete under various consistency and rheological tests: slump flow, L-flow and V-funnel [8]. The numerical approach was also adopted to simulate the flow behaviors of the fresh concrete with varied consistencies during transportation, placement and compaction. The correlation between the mix design and rheological properties of fresh concrete was also investigated by DEM [9], [10]. A three dimensional and parallel DEM code has been developed to simulate the rheological behaviors of fresh concrete, of which the numerical results were well in accordance with the corresponding experimental results [11]. Shyshko and Mechtcherine [12] developed a material model of DEM for fresh concrete to study the interaction between neighboring spherical entities that represent coarse aggregates with a virtual covering of matrix mortar. These studies have shown that DEM is possible to simulate the flow behavior of fresh concrete. DEM provides a better chance to understand thixotropy of fresh concrete.

In this paper, based on the three-dimensional particle flow code (PFC3D) [13], in which a parallel contact model and a viscous damping model are integrated, a general DEM model was established for fresh concrete , the parameters of the contact model were calibrated by the rheological tests. Then, we modified the general DEM model to get a thixotropy DEM model by introducing a time-dependent parameter into the contact model of the general DEM model. Using this thixotropy DEM model, the variation of static yield stress and lateral pressure of fresh concrete at rest with the elapsed time was examined analytically.

2. DISCRETE ELEMENT METHOD

*1 Graduate School of Science and Engineering, Yamaguchi University, JCI Student Member
*2 Associate Prof., Graduate School of Science and Engineering, Yamaguchi University, JCI Member
*3 Prof., School of Mechanical engineering, Xiangtan University, Non-member

The alternate calculations are performed in the DEM between the applications of the Newton's second law to the particles and a force-displacement law at the contacts. The force-displacement law is used to update the contact forces resulting from the relative motion at each contact, while the Newton's second law is used to determine the motion of each particle, caused by the contact and body forces acting upon it. The contact between two neighboring particles occurs only at a point, The relationship between the particle motion and the forces or moment is expressed as Eq.(1) and Eq.(2) [13].

$$F=m(x''-g) \tag{1}$$

$$M=I\omega' \tag{2}$$

where,

F	: contact force
m	: mass of the particle
x''	: translational acceleration
g	: acceleration of the gravity
M	: resultant moment acting on the particle
I	: moment of inertia
ω'	: angular acceleration.

3. RHEOLOGICAL MODEL

3.1 The general DEM model

Fresh concrete consists of two phases: coarse aggregate and matrix mortar. To reach a compromise between computation accuracy and time in the numerical simulation, we assumed that the matrix mortar exists at the form of particles with the dimension of 10~20mm. The size distribution of coarse aggregate is 10~40mm. The contact forces of the particles can be resolved the elastic and viscous components [12]. In this study, the parallel contact model (see Fig.1(a)) and viscous damping model (see Fig.1(b)) were employed to represent the elastic and viscous action, respectively. The parallel contact model contains five parameters: normal and shear stiffness k^n and k^s, normal and shear strength σ_c and τ_c, and bond-radius multiplier λ. The bond radius R is equal to $\lambda min(R^a, R^b)$, in which R^A and R^B being the radius of the bonded particle a and b, respectively.

Once the parallel contact bond is formed, the contact force F_i and moment M_i are initialized to zero. Subsequent relative motion between the two bonded particles results in an increment in F_i and M_i, in normal and shear directions, as shown in Eq.(3)~(6).

$$\Delta F_i^n = -k^n A \Delta U_i^n \tag{3}$$

$$\Delta F_i^s = -k^s A \Delta U_i^s \tag{4}$$

$$\Delta M_i^n = -k^s J \Delta \theta_i^n \tag{5}$$

$$\Delta M_i^s = -k^n I \Delta \theta_i^s \tag{6}$$

where,

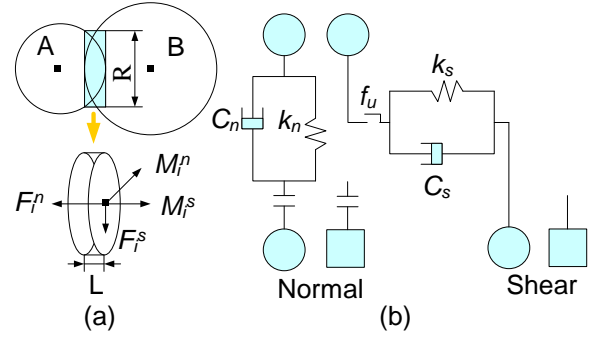|	
ΔF_i^n, ΔF_i^s	: normal and shear forces
ΔM_i^n, ΔM_i^s	: normal and shear moments

Fig.1 (a) The parallel contact model, (b) The viscous damping model (circle and square denotes particle and wall, respectively)

ΔU_i^n, ΔU_i^s	: normal and shear relative displacements
$\Delta \theta_i^n$, $\Delta \theta_i^s$	: normal and shear relative rotations between two bond particles

The A, J, I of the parallel contact bond cross-section can be expressed as follows:

$$A = \pi R^2 \tag{7}$$

$$J = 1/2 \pi R^4 \tag{8}$$

$$I = 1/4 \pi R^4 \tag{9}$$

where,

A	: area of parallel contact bond cross-section
J	: polar moment of inertia
I	: moment of inertia

The maximum tensile and shear stresses acting on the bond periphery (via beam theory) are calculated to be:

$$\sigma_{max} = \frac{F_i^n}{A} + \frac{|M_i^s|}{I}R \tag{10}$$

$$\tau_{max} = \frac{|F_i^s|}{A} + \frac{|M_i^n|}{I}R \tag{11}$$

If $\sigma_{max} \geq \sigma_c$ or $\tau_{max} \geq \tau_c$, then the bond breaks, the contact forces disappear. σ_c and τ_c are normal and shear strength, respectively.

The viscous damping is defined by the normal and shear damping coefficients (see Fig.1(b)): C_n, C_s(in this study C_n=0.7, C_s=0.3). k_n, k_s, and f_u are the normal, shear stiffness and friction coefficient of particles, respectively.

3.2 Calibration and validation

The calibration of the parameters of the contact models is usually performed with simple experiments in the lab. The material parameters for the simulation

Table 1. Mix proportions of concrete

W/C	Mass in cubic meter (kg/m³)						
	C	BFS	FA	G	S	W	A
0.68	235	62	51	1287	755	160	6.96

[Notes] C: cement, BFS: blast furnace slag powder, FA: fly ash, G: coarse aggregate, S: sand, W: water, A: Water-reducing agent

Table 2. The input parameter values of particle contacts

Contact	C-C	C-M	M-M
Normal stiffness k^n	—	200000	6000
Shear stiffness k^s	—	40000	1200
Normal strength σ_c	—	5500	2200
Shear strength τ_c	—	1000	500
Multiplier λ	—	0.5	0.3
Local damping	0.3	—	—

[Notes] C-C, C-M, M-M: the contacts between coarse aggregate particles, coarse aggregate and mortar particles, mortar particles, respectively.

Fig.2 (a) The L-flow apparatus, (b) The ICAR rheometer
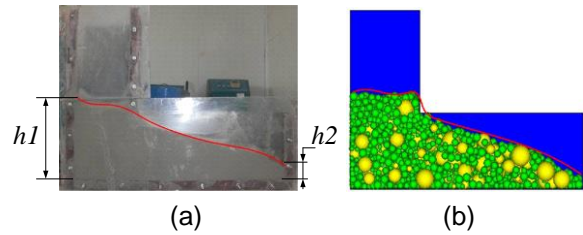
Fig.3 The final shape of the flow in the L-flow apparatus

of the calibration experiments are iteratively adjusted until the results of the experiments and the simulations match. The mix proportions of the fresh concrete used in the calibration experiments are shown in Table 1. The slump value of used concrete is 180mm.

The apparatuses used in the calibration experiments are presented in Fig.2, which are a L-flow apparatus and a vane impeller rheometer, respectively. The L-flow apparatus currently is used to evaluate the consistency of fresh concrete. After the fresh concrete sample stopped to flow in the horizontal section of the apparatus, as shown in Fig.3, the heights of the concrete in the vertical section h_1, and at the end of the horizontal section h_2 were measured. The ratio $PA=h_2/h_1$ is then obtained.

In the flow simulation of the L-flow, the number of the used coarse aggregate particles and the mortar particles were 146 and 1949, respectively. The volume ratio of the coarse aggregate particles to the mortar particles was 0.6, which was the same to the volume ratio of coarse aggregate and matrix mortar in the concrete. After all the particles were formed, 5000 cycles were executed to make the adjacent particles to contact. A series of the flow simulation were performed through adjusting the input parameters. Fig.3(b) presents a numerical flow simulation result,
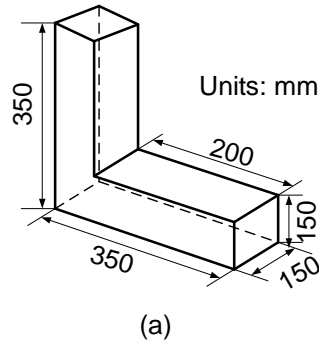
the final shape of which was almost the same to that of the experimental result (see Fig.3(a)). The ratio PA was 0.21 for the experiment, and was 0.18 for the simulation. Based on this simulation, five sets of input parameter values were selected.

The PA value obtained from the L-flow test is mainly depended on the yield stress of fresh concrete, In order to improve the accuracy of the general DEM model, another calibration experiment was conducted subsequently, using a rheometer.

The rheometer with van impeller, shown in Fig.1(b), is able to supply the Bingham constants: yield stress and plastic viscosity [14]. We firstly used this rheometer to measure the Bingham constants of the fresh concrete. The average yield stress and the average plastic viscosity of 5 times measurement were 439.7Pa, and 4.6Pa s, respectively.

Then, we conducted a flow simulation of the same concrete in the rheometer, with 909 coarse aggregate particles and 12133 mortar particles. The five sets of input parameter values, obtained from the L-flow test simulation were used in the numerical calculations of Bingham constants. At last, we determined a set of input parameter values that could make the calculating results of the Bingham constants closer to the measuring results. The calculating results of yield stress and plastic viscosity were 454.3Pa and

Table 3. The input parameter values of the particles

Coarse aggregate			Matrix Mortar		
Normal stiffness k_n (N/m)	Shear stiffness k_s (N/m)	Frictional coefficient f_u (dimensionless)	Normal stiffness k_n (N/m)	Shear stiffness k_s (N/m)	Frictional coefficient f_u (dimensionless)
10000	1000	0.15	1000	100	0.08

4.9Pa s, respectively. Table.2 and Table.3 show the obtained input parameters for the contacts and the particles through the calibrations mentioned above.

3.3 The thixotropy DEM model

Thixotropy phenomenon is one of characters of fresh concrete, which is generally considered to result from the flocculation of cement particles. The particle flocculation leads to an increase in the contact force between the adjacent particles. However, in the general DEM, the particle contact force of fresh concrete at rest doesn't vary with the elapsed time. Thus, the general DEM can't simulate the thixotropic behaviors of fresh concrete. The variation of the particle contact force with the evolution of particle flocculation is not clear quantitatively, and its clarification is a difficult problem. In our numerical simulation, we supposed that the contact force is an increment function of the elapsed time t in a standstill state, as follows:

$$F_i^t = F_i^0 + \Delta F_i \qquad (12)$$
$$\Delta F_i = f(t, F_i^0) \qquad (13)$$

where,

F_i^t : No.i contact force in t moment
F_i^0 : No.i contact force in initial moment
ΔF_i : an increment in the contact force

In the general DEM model, the contact force is described with the parallel model and viscous damping model. This external force and moment acting on the two adjacent particles are related to the maximum normal and shear stresses at the interface of the two particles. If either of these maximum stresses exceeds its corresponding limit, the parallel contact will be broken (see section 3.1). Therefore, in our thixotropy DEM model, two parameters of the parallel model: normal strength and shear strength were taken into account, and a simple linear relation was used, as shown in Eq.(14) and Eq.(15).

$$\sigma_c^t = \sigma_c^0 + k_\sigma t \qquad (14)$$
$$\tau_c^t = \tau_c^t + k_\tau t \qquad (15)$$

where,

k_σ, k_τ : proportional constant
σ_c^0, σ_c^t : normal strength in initial and t moment
τ_c^0, τ_c^t : shear strength in initial and t moment

4. VARIATION OF LATERAL PRESSURE

In the casting process of fresh concrete, the formwork must support the lateral pressure that results from fresh concrete. Because of the thixotropic character, the lateral pressure changes with the elapsed time.

In the experiment, the slump value of fresh concrete is the same to previous value (180mm). We
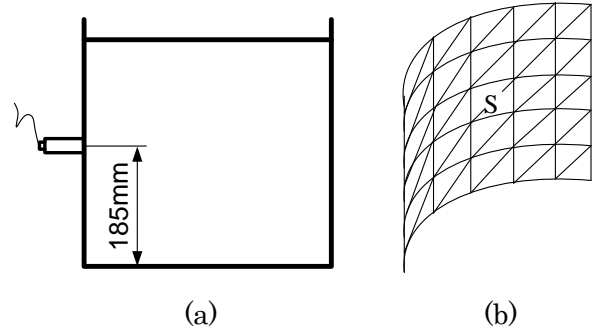

Fig.4 The position of lateral pressure measurement schematically (a) in the lab, (b) in numerical simulation

measured the lateral pressure of the fresh concrete in a standstill state on the van rheometer' wall from 40min later after mixing. The pressure sensor was installed in the middle of the van rheometer (see Fig.4(a)). Before the measurement, the fresh concrete in the rheometer was sheared adequately to destroy the particle flocculation structures. 63min later, the concrete sample was mixed again for 3 min. The measuring results are shown Fig.5. With the increase of rest time, the lateral pressure decreased, but right after re-mixing, sharply increased then began to decrease again. Due to the formation of the particle flocculate structure at rest, fresh concrete can support part of its own weight [9], the lateral pressure decreased with the rest time from 40min to 64min, as shown in Fig.5. After the re-mixing, the lateral pressure rose up to a high level that was a little less than the initial value. It is shown that the thixotropy of fresh concrete is a reversible process within a period. The difference between the measured lateral pressure at 66min and the initial value may be caused by the hydration.

In PFC, the rheometer container' wall was divided into a series of triangle (see Fig.4(b)). Using the input parameters determined by the calibrations and the particle contact force model shown in Eq.(14)
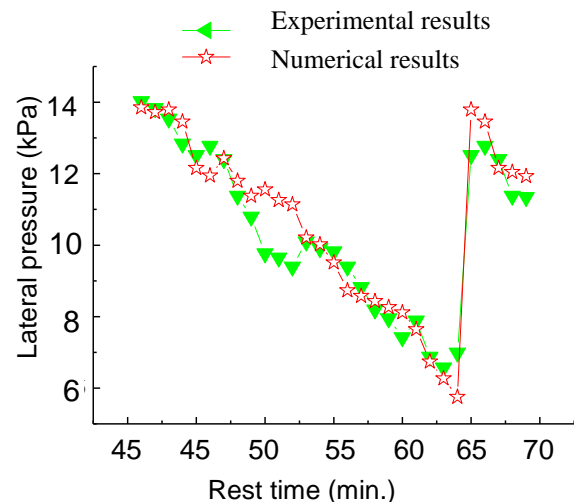

Fig.5 Numerical and experimental Results of lateral pressure changing with rest time

and Eq.(15), the stress state of the fresh concrete was examined, and the normal stress of middle position (the "S" point in Fig.4(b)) was record and expressed as the lateral pressure. The lateral pressure's calculating results are also shown in Fig.5. The calculating results are consistent with the experimental results. Such a good correspondence of numerical and experimental results shows that the thixotropy DEM model proposed in this study can be used to predict the lateral pressure change over time.

5. VARIATION OF STATIC YIELD STRESS WITH REST TIME

A typical rate-controlled stress growth test plot is shown in Fig.6, consisting of three portions [14]. In the initial linear region, the material behaves elastically. Next, as bonds between solid particles are broken, the plot transitions from the linear region to the curved viscoelastic region. Finally, when a sufficient number of bonds are broken to achieve flow, the yield stress is reached, indicated as the maximum value on the curve. Two different yield stresses are indicated in Fig.6. The first yield stress, which occurs at the transition from elastic to viscoelastic behavior, is referred to as the static yield stress $\tau_{0(s)}$ and increasing with the resting time. The peak shear stress in referred to as the dynamic yield stress, which does not depend
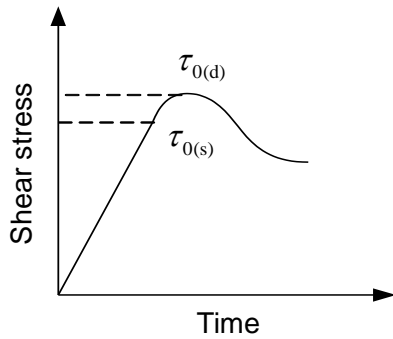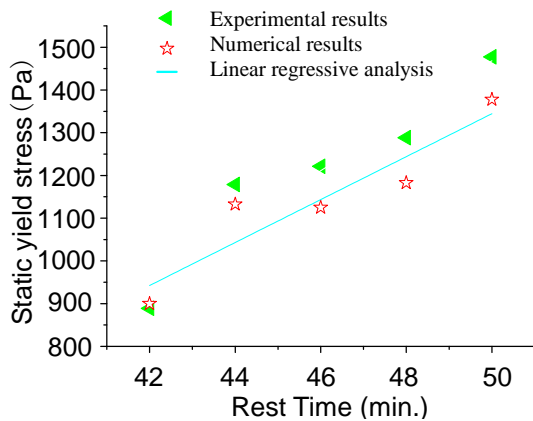
on the material flow history [15]. The dynamic yield stress $\tau_{0(d)}$ is generally used as the indication of yield stress because it is associated with full structural breakdown and the beginning of plastic flow.

Static yield stress test is used to measure thixotropy of fresh concrete [17]. This method involves shearing a material initially at rest by increasing the shear strain of the sample very slowly and measuring the shear stress to create a shear stress-shear strain relation. By slowly increasing the shear stress of the material until the structure breaks, the material structure is characterized by the peak stress, which is the static yield stress.

Eric P. K. and David W. F. present a measuring method of $\tau_{0(s)}$ using their developed van impeller rheometer [14]. After the given rest time, the van impeller is slowly turned at 0.025rps to measure the peak torque to break the material structure. The maximum value T_{max}, is recorded, and further the static yield stress $\tau_{0(s)}$ is calculated according to Eq.16 [14], [16].

$$\tau_0 = \frac{T_{max}}{\frac{\pi}{2}d^2\left(h+\frac{d}{3}\right)} \tag{16}$$

where,

d	: van impeller diameter (=125mm)
h	: van impeller height (=125mm)

We used this method to measure the variation of the static yield stress of the concrete shown in Table 1, during it was in the standstill state. The experimental results are respectively shown in Fig.7.

The numerical analyses of the static yield stress were performed by inputting the parameters determined through the calibrations and using the thixotropy DEM model. The numerical results are also indicates in Fig.7.

As shown in Fig.7, the numerical results of $\tau_{0(s)}$ versus the elapsed time in the standstill state, are well consistent with their experimental values. With an increase in the elapsed time, the $\tau_{0(s)}$ increased. Especially, the increase of $\tau_{0(s)}$ was greater after 48 minutes. From this result, it is thought that the thixotropy DEM model would be applied to simulate the thixotropy phenomenon of fresh concrete.

We did a linear regressive analysis toward the numerical results of the static yield stress $\tau_{0(s)}$, the relationship between the $\tau_{0(s)}$ and the elapsed time was obtained, as shown in Eq.(17). This relationship is the same to that proposed by Roussel [2], as shown in Eq.(18).

Fig.6 A typical rate-controlled stress growth

Fig.7 Measuring results of static yield stress with rest time

$$\tau_0(t) = 842 + 50t \qquad R^2 = 0.827 \tag{17}$$

$$\tau_0(t) = \tau_0 + A_{thix}t \tag{18}$$

where,

A_{thix}	: flocculation rate.

6. SUMMARY

In this study, fresh concrete was considered to be a two-phase system: coarse aggregate and matrix mortar, the matrix mortar was represented by one kind of imaginary particles for making the general DEM model to be applied to the numerical analysis of fresh concrete. Then two rheological tests were carried out to calibrate the material parameters of the general DEM model.

Furthermore, taking the increment of the contact force between the particles of fresh concrete at rest with the rest time into account, the thixotropy DEM model for fresh concrete was developed, and the variations of the static yield stress and the lateral pressure with the rest time were investigated. With the rest time, the static yield stress increased, but the lateral pressure decreased. The linear relationship between the static yield stress and rest time was obtained.

ACKNOWLEDGEMENT

REFERENCES

[1] Li, Z. G., Ohkubo, T., Tanigawa, Y., "Theoretical analysis of time-dependence and thixotropy of fluidity for high fluidity concrete," *Journal of materials in civil engineering*, Vol. 16, No.3, 2004, pp. 247-256.

[2] Roussel, N., "A thixotropy model for fresh fluid concretes: theory, validation and applications," *Cement and Concrete Research*, Vol. 36, No.10, 2006, pp.1797-1806.

[3] Tan, Y. Q., Zhang, H., Yang, D. M., et al, "Numerical simulation of concrete pumping process and investigation of wear mechanism of the piping wall," *Tribology International*, Vol. 46, No.1, 2012, pp. 137-144.

[4] Zhang, H., Tan, Y. Q., Yang, D. M., et al, "Numerical investigation of the location of maximum erosive wear damage in elbow: Effect of slurry velocity, bend orientation and angle of elbow ," *Powder Technology*, Vol. 217, 2012, pp. 467-476.

[5] Zhang, H., Tan, Y. Q., Li, M., "A Numerical Simulation of Motion of Particles under the Wafer in CMP," *Proceedings of International Conference on the Computer Science and Software Engineering,* Vol. 3, France, 2008, pp. 31-34.

[6] Zhang, H., Trias, Miquel, F. X., Tan, Y. Q., et al, "Parallelization of a DEM/CFD code for the numerical simulation of particle-laden turbulent flows," *Proceedings of International Conference on Parellel Computational Fluid Dynamics.* Vol.23, Barcelona, 2012, pp. 1-5.

[7] Zhang, H., Tan, Y. Q., Shu, S., et al, "Numerical investigation on the role of discrete element method in combined LBM-IBM-DEM modelling," *Computers & Fluids*, Vol. 94, No. 2, 2014, pp. 37-48.

[8] Noor, M., "Three-Dimensional Discrete Element Simulation of Flowable Concrete," *Ph. D. Dissertation*, Department of Civil Engineering, Graduate School of the University of Tokyo, 2000.

[9] Shyshko, S., Mechtcherine, V., "Continuous numerical modelling of concrete from fresh to hardened state," *IBAUSIL*, internationale Baustofftagung Weimar, Germany, 2006, pp.20-23.

[10] Mechtcherine, V., Shyshko, S., De, Schutter, G., et al, "Simulating the behaviour of fresh concrete using distinct element method," *Proceedings of the 5th International RILEM Symposium on Self-Compacting Concrete*, Vol. 1, Ghent, 2007, pp. 467-472.

[11] Zhang, W. C., "Parallel Discrete Element Simulation and Its Application to the Study of Solid-Liquid Flow Behavior," *Ph. D. Dissertation,* National Taiwan University, 2009.

[12] Shyshko, S., Mechtcherine, V., "Developing a Discrete Element Model for simulating fresh concrete: Experimental investigation and modelling of interactions between discrete aggregate particles with fine mortar between them," *Construction and Building Materials*, Vol. 47, 2013, pp. 601-615.

[13] Inc, I. C. G., "PFC2D user's manual (Version 3.1)," ICG, Minneapolis, 2004.

[14] Koehler, E. P., Fowler, D. W., "Development of a portable rheometer for fresh portland cement concrete," *ICAR Report*, 2004, pp.105-426.

[15] Roussel, N., "Steady and transient flow behavior of fresh concrete pastes," *Cement and concrete research*, Vol. 35, No. 9, 2005, pp. 1656-1664.

[16] Papenhuijzen, J., "The role of particle interactions in the rheology of dispersed systems," *Rheologica Acta*, Vol. 11, No. 1, 1972, pp. 73-88.

[17] Billberg, P., "Stress-Strain Measurement Of Fresh SCC at Rest Using a Concrete Viscometer," RHEO Iceland, *Proceedings of RILEM International Symposium on Rheology of Cement Suspension such as Fresh Concrete*, Reykjavik, Iceland, 2009.8, pp. 265-272.