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- Technical Paper -

DYNAMIC ANALYSIS FOR RC COLUMNS WITH CIRCULAR CROSS SECTION USING MULTI-DIRECTIONAL POLYGONAL 3D LATTICE MODEL

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ABSTRACT

The study performs nonlinear dynamic analysis for reinforced concrete (RC) columns using the 3D lattice model. For analysis, a model based on geometrical equivalence between circular and square cross sectional areas and a new multi-directional polygonal discretization model with more realistic target geometry are proposed. The applicability of the models is examined performing the dynamic analysis on a circular cross sectional RC column tested by E-Defense. The results show acceptable agreement in behavior after cracking using the proposed multi-directional polygonal model.

Keywords: reinforced concrete, columns, circular cross section, 3D lattice model, dynamic analysis

1. INTRODUCTION

The highly nonlinear behavior that reinforced concrete (RC) structures exhibit in the occurrence of an earthquake is at the top of priorities in the analytical development of numerical techniques to study seismic behavior. Many techniques are available to perform seismic analysis of RC structures; nonetheless, it remains highly difficult to grasp structural behavior in the after cracking range, especially in the case of shear failure. Columns are especially vulnerable and strong against lateral loads induced by seismic action result in many times of the shear capacity as a dominant mode of failure.

In seismic design of RC columns, the geometry of the section has a strong influence on the shear capacity of the member. However the majority of the codes simply assume that the shear capacity of a circular cross section equals the capacity of an equivalent rectangular section [1]. On the other hand, because of the complexity of analysis and increasing computational requirements by the analytical models caused by high number of degrees of freedom, it can be useful the application of relatively simple models. A 2D lattice model has been proposed [2] and further enhanced into 3D lattice model [3], which allows the reasonable prediction of shear behavior and reduced degrees of freedom by adopting an arch and truss analogy in structural discretization.

With the above in mind, this study is focused on the applicability of the 3D lattice model to perform dynamic analysis on circular cross section columns considering geometrical properties of circular cross section columns in analytical discretization with special consideration for the shear response characteristics of columns under seismic excitation.

2. ANALYTICAL MODEL

2.1 Outlines of 3DLattice Model

The development of the 3D lattice model is based on the 2D lattice model [3]. Fig.1 shows the schematic representation of RC column using the 2D lattice model, which allows the representation of elements in terms of concrete and reinforcement. Here it is assumed that the cross sectional depth of the lattice model corresponds to the effective depth of cross section d. In the 2D lattice model, the concrete region is divided in arch and truss part.

The value t is defined as the ratio of the width of the arch part to that of cross section b, where $0<t<1$. The widths of the arch part and truss part are given as $b \times t$ and $b \times (1-t)$. In the 3D lattice model, presented in Fig.2 for a RC column, the truss action in a 3D space is represented using an orthogonal coordinate system defined by three planes x-y, y-z and z-x, respectively. Two crossing diagonal members are located in each truss plane to create a unit consisting of 12 diagonal members [3].

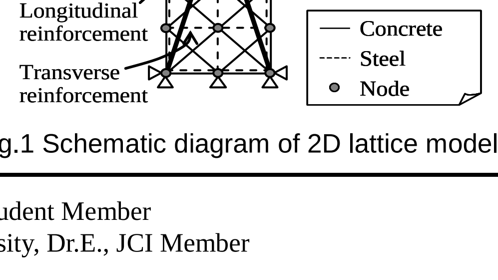

Fig.1 Schematic diagram of 2D lattice model [3]

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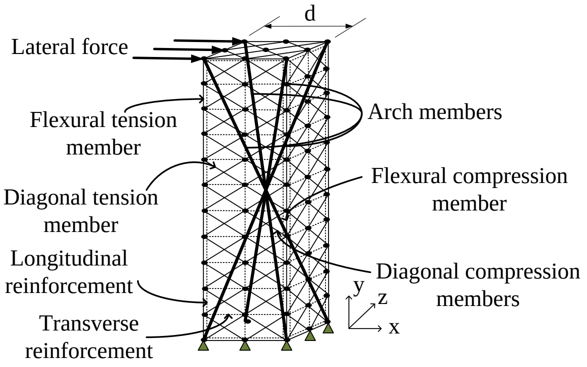


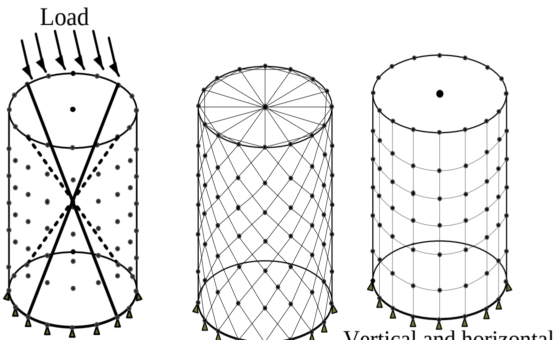
Fig.2 Diagram of 3D lattice model [3]

As previously stated, in the 3D lattice model the shear resisting mechanism is represented by truss and arch action, and it is assumed that the arch action is defined by four arch members that connect the loading point with the bottom of the column at the opposite corner. The truss action can be seen as an idealized compressive strut.

2.2 Multi-directional Polygonal 3D Lattice Model

Circular reinforced concrete columns are favored for bridge piers, because of relative simplicity of construction as well as omnidirectional strength characteristics under wind and seismic loads [4]. However, under these circumstances the columns' flexural and very especially shear resistance are very important. Furthermore the design procedures adopted for such columns are also important, as one of the cornerstones of seismic design of columns is to prevent brittle failure, that is, ensure ductile behavior of columns in the event of earthquakes. With that, addressing the complexities of seismic response of columns with relative degree of simplicity is essential.

Based on previous studies [3], the 3D lattice model has shown satisfactory capabilities of prediction of shear behavior of RC structural members with relative simplicity in analytical procedure. Based on the above an analysis concept has been developed using the 3D lattice model for circular cross section RC columns based on more realistic multi-directional polygonal discretization. The modeling will be explained later on.



Arch members

Truss members

Vertical and horizontal members

Fig.4 Schematic diagram of multi-directional polygonal 3D lattice model

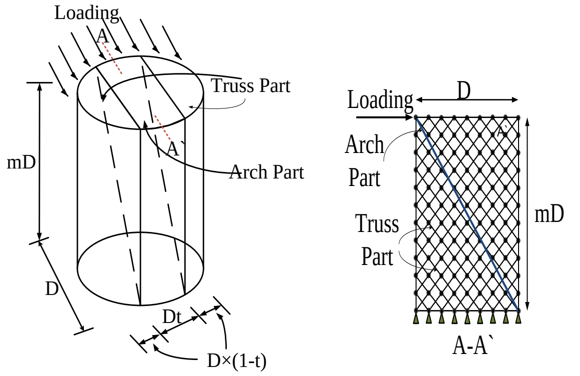


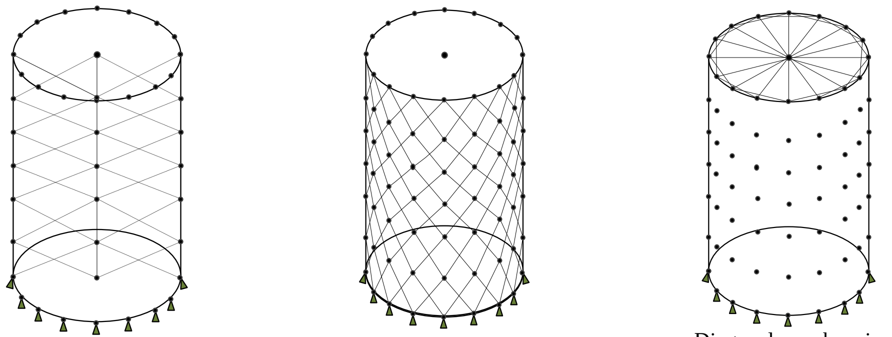
Fig.3 Arch and truss discretization in the 3D lattice model for circular column

(1) Modeling of Lattice Model Members

In the multi-directional polygonal 3D lattice model the shear resisting mechanism is based upon the arch and the truss assumption as generally presented in Fig.3. The discretization from solid concrete to the 3D lattice model is performed so that the actual cross-sectional diameter, *D* of the analytical model corresponds to that of the target, that means that the target defines mesh size and model height.

The arch action is assumed to be represented by four arch members connected from the loading point to the opposite bottom of the column which are representative of internal stress flow. This assumption is largely a simplification from the general 3D lattice model where it is assumed that the compressive portion in arch action is sufficiently represented in the pair of arch members. Fig.4 illustrates a schematic representation of arch members for a circular cross section column as well as truss members in 3D space. Here, the effect of vertical force is considered by distributing the tributary deck mass over the column by all nodes in the first layer at the top of the model. The detailed representation of the diagonal members is shown in Fig.5. The diagonal members which include a part of representation of truss action consist of three parts, which are inner diagonal members (IDM), surface diagonal members (SDM) and diagonal members in transverse direction (DMT) respectively. In the model the inner diagonal members are assumed to be modeled as the truss action in the RC column. It is assumed that the modeling of diagonal members is preceded by the distribution of sixteen peripheral nodes for every horizontal layer. The distance between two nodes is equal to half of cross sectional diameter, 0.5*D*, the in-plane nodal positions are set according to their polar coordinates defined by $x = r\cos\phi$ and $y = r\sin\phi$ in an orthogonal system, where r is the radius of cross-section and ϕ is the internal angle on the cross section defined by a triangle formed by joining two successive node to each other and the center node.

In the case of inner diagonal members, each node at the surface is directly connected to the lower or upper node in the central group on nodes. For the surface diagonal members the nodes are connected successively along the surface area and in the case of diagonal members in transverse direction, sixteen



Inner diagonal members (IDM)
Surface diagonal members (SDM)
Diagonal members in transverse direction (DMT)

Fig.5 Representation of the truss elements

peripheral nodes are connected to a center node for every layer of nodes, and every node is connected to the second following node forming an enclosed truss system.

In the lattice model, the height of the analytical model does not always correspond to the height of the target, but rather the closest dimension. This is due to the fact that distance between two nodes in vertical direction is fixed as half of the diameter.

Longitudinal reinforcement is represented as vertical reinforcement member along the sixteen nodes per layer defining the geometry of the model. Regarding transverse reinforcement, it is represented in the form of horizontal reinforcement members uniformly distributed at intervals of 0.5*D* throughout the model as the intervals of arrangement are not taken into account. The representation of vertical and horizontal members is detailed in Fig. 4.

(2) Cross-Sectional Area of Lattice Members

The determination of cross-sectional area of arch and diagonal members in the multi-directional 3D lattice model is performed considering that the diameter of the model is invariant for the definition of geometry properties of the analytical model, which are the height, mesh size and distribution of nodes. Therefore the notion of analytical diameter is introduced. An analytical diameter of column cross section is defined as the diameter obtained from analytical conditions to calculate the cross-sectional area of arch and truss members. The analytical diameter is obtained considering two cases: the first one considers the shear resisting capacity of concrete where the concept of effective shear area of concrete is used. In the analysis, the gross cross sectional area of solid concrete considered in order to assemble the 3D lattice model is reduced to an effective portion corresponding to 0.6 to 0.8 times the cross-sectional gross area [5].

A second approach used is to consider the effect of effective stiffness in dynamic response of RC columns. For that, the reduction of the flexural stiffness *EI* thought the inertia of the column is considered. In this study, reduction factors of moment of inertia ranging from 0.4 to 0.7 times gross moment of inertia [6] are considered. For the determination of the stiffness matrix in the arch members, it is assumed that a single arch member is representative of the stress flow for analysis purposes. The equivalence of global stiffness of structural systems in 2D and 3D is assumed [3].

The cross sectional area of arch members is given as follows:

$$A_{arch-3D} = \left(\frac{2+m^2}{1+m^2}\right)^{3/2} \cdot A_{arch-2D} \tag{1}$$

$$A_{arch-2D} = D^2{}_{ana}\, t \cdot \sin\theta \tag{2}$$

where,
$A_{arch\text{-}3D}$: Cross sectional area of arch member in 3D
$A_{arch\text{-}2D}$: Cross sectional area of arch member in 2D
D_{ana} : Analytical diameter of the column model
t : Ratio of width of arch to truss members
m : Set so that $m \times D_{ana}$ = Model height
θ : Inclination of the arch member

The determination of the cross sectional area of the diagonal members is given as follows:

$$A_{IDM} = A_{SDM} = \frac{D_{ana}(1-t)}{2} \cdot \frac{D_{ana}}{2} \sin 45^\circ \tag{3}$$

$$A_{DMT} = \frac{a}{2m} \cdot \frac{D_{ana}}{2} \sin 45^\circ \tag{4}$$

where,
A_{IMD} : Inner diagonal members
A_{SMD} : Surface diagonal members
A_{DMT} : Diagonal members in transverse direction
D_{ana} : Analytical diameter of the column model
t : Ratio of width of arch to truss members
m : Set so that $m \times D_{ana}$ = Model height

It is noticeable in Eq. (3), that the cross sectional area of inner diagonal members and surface diagonal members are calculated using the same equation. This simplification is due to the fact that in the pre-analysis process, it was verified that omnidirectional properties of a circular cross section meant that a single formula for the diagonal members in longitudinal direction to the transverse plane could be adopted for simplification of analytical procedure.

(3) Material Constitutive Models

a) Concrete

For the arch and diagonal members of concrete, the compression model accounts for lateral restraint effect by shear reinforcement. In this model in order to consider the effect of confinement by transverse reinforcement, the stress-strain relationship proposed by Mander et al. [7] is used. The compressive softening behavior of cracked concrete is given according to Vecchio et al. [8] who proposed that the ability of fully cracked concrete to resist compressive stress decreases as the transverse tensile strain ε_t increases. The flexural compression member is assumed to be cover concrete that is unconfined and becomes ineffective after the compressive strength is reached. The stress-strain relationship of uncracked concrete is here represented by a quadratic curve [8] and an exponential curve.

The flexural tension members of concrete are assumed to be located near reinforcement. In this region, the concrete behaves with the reinforcements and continues to contribute tensile force even after cracking due to the bond effect between the concrete and reinforcements. Therefore, after cracking, the tension stiffening model [9] is applied to the flexural tension members. On the other hand, for the diagonal tension members that consist of concrete far from reinforcement the tension softening curve, the so-called 1/4 model proposed by Rokugo et al.[9] is applied. The fracture energy of concrete G_F, is assumed to be 0.1 N/mm.

b) Reinforcing Bar

The envelope stress-strain relationship of reinforcement is modeled as a bi-linear. The tangential stiffness after yielding is set as $0.01E_s$, where E_s denotes the Young's modulus of reinforcement. After yielding, the stiffness of reinforcement decreases when the stress stage changes from tension to compression, while similar behavior is observed when the stress stage changes from compression to tension. In the analysis, the Bauschinger effect is considered by using a numerically improved model of reinforcement [10].

3. DYNAMIC ANALYSIS FOR CIRCULAR CROSS SECTION RC COLUMNS

3.1 Analytical Target [11]

The analytical target is a circular cross section column named C1-5 tested using a shake-table by E-Defense. The specimen, shown in Fig.6 is a cantilever circular column with diameter 2000 mm. The heights of the column and footing correspond to 7500 mm and 1800 mm respectively.

The longitudinal and transverse reinforcement have a nominal strength of 345 MPa (SD345) and the design concrete strength of 27 MPa. Sixty four deformed 35 mm diameter longitudinal bars are presented in two layers, while deformed 22 mm circular ties are set at 150 mm and 300 mm intervals in the outer and inner longitudinal bars.

Column C1-5 was excited using a near-field ground motion which was recorded at the JR Takatori Station during the 1995 Kobe earthquake. In the shake table test input ground motion, the maximum amplitude of input acceleration in longitudinal direction is 6.21 m/s^2 while in transverse direction is 7.88 m/s^2. In that way the maximum amplitude of acceleration, time history and boundary conditions for both directions are considered in response analysis in both directions. From the damage point of view, buckling of longitudinal reinforcement was observed, however in this study this effect is not taken into consideration for simplicity in analysis.

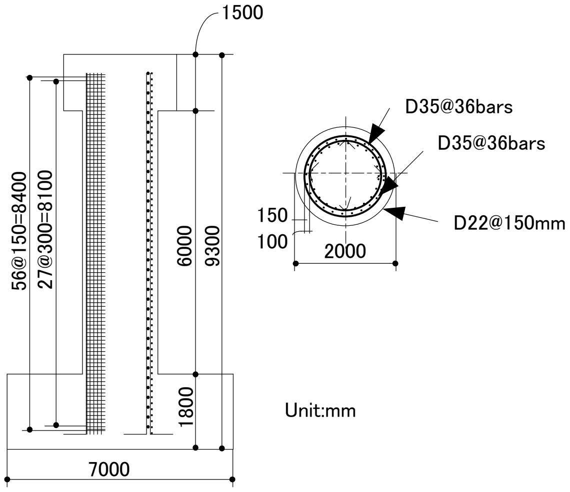


Fig.6 Column C1-5 details [11]

In the experimental program, C1-5 has been excited under different conditions, in this study the analysis will focus on C1-5(1) which corresponds to the first excitation. In the experiment a two-deck tributary mass of 307 ton is used to the column and considered in the analysis for axial load effect by distributing along the nodes in the top layer number one of lattice members in the analytical models.

3.2 Discretization of Analytical Models

In order to analyze column C1-5 two analytical situations have been set. First is the conversion of circular cross sectional shape into an equivalent rectangular cross sectional shape, based on the fact that most design methods for shear and flexure of RC members are mainly based on the rectangular cross-sectional shape for the analysis Upon this, the 3D lattice model developed in previous studies [3] is used in an analytical model further addressed as AM-1 shown in Fig.7.

The second analysis corresponds to the application of the multi-directional polygonal 3D lattice model, newly developed for circular cross section columns. Here, the shear capacity of reinforced concrete is considered by assuming that of the actual resisting area of concrete is reduced in seismic response. It is in that way that an analytical model addressed as the AM-2 is developed, where an effective area of concrete corresponding to 0.7 of the gross area of concrete is assumed to determine the cross-sectional area of the lattice members.

The reduction in flexural stiffness is considered to perform analysis. Assuming that there is stiffness degradation in seismic response, an analytical model AM-3 is developed; where in the case effective area moment of inertia corresponding to 0.7 of the gross moment area of inertia is assumed for the analytical diameter used to determine the cross-sectional area of

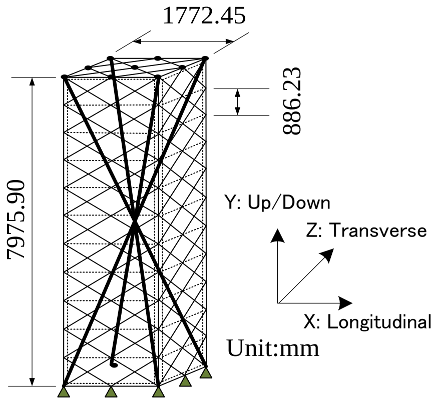


Fig.7 Analytical model AM-1

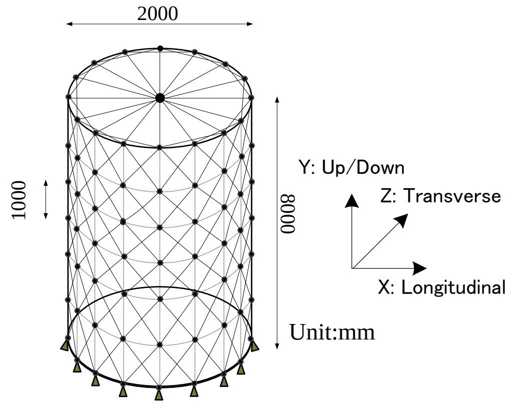


Fig.8 Analytical models AM-2 and AM-3

lattice model elements. It should be noted that the general geometry of AM-2 and AM-3 is the same as detailed in Fig.8. Additionally, for simplicity, the pull-out effect of longitudinal reinforcement is not considered in the analysis for all analytical cases.

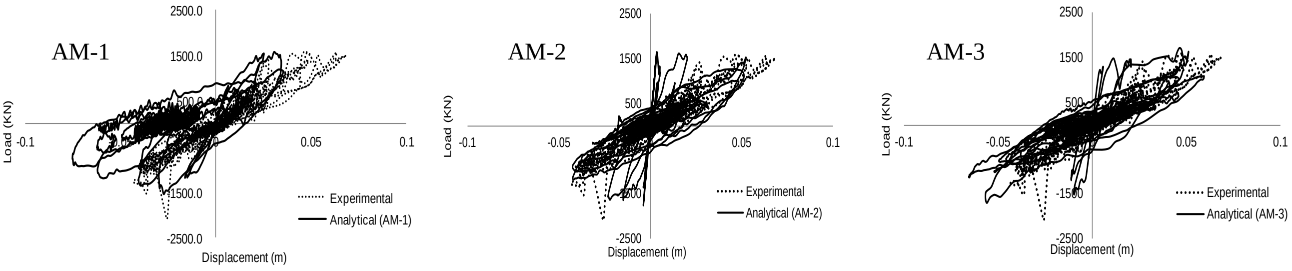
3.3 Analytical Results and Discussion

The analytical results using analytical models AM-1, AM-2 and AM-3 are shown in Fig.9 and 10 respectively, and compared to the experimental results. The analytical response using AM-1 shows reasonable agreement with the experiment before cracking occurs; The initial stiffness is very consistent in both directions, however, it is in the longitudinal direction that the after cracking behavior differ visible, but while in the transverse direction the hysteretic response though is agreeable underestimates maximum force as well as displacement. Thus supporting that, the assumption of equivalence of responses between a rectangular cross sectional member and circular cross section member has large limitation in inelastic phase.

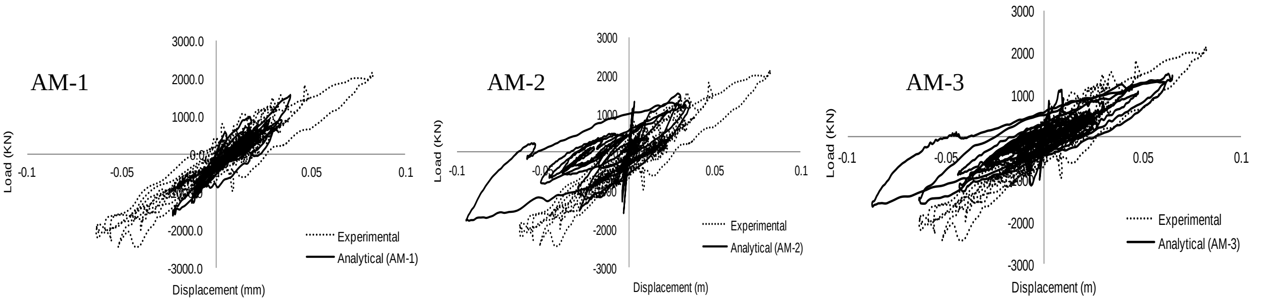
The analysis performed using the multi-directional polygonal lattice model presented in this paper show that largely better hysteretic agreement is obtained. However, before cracking of concrete response of AM-1 is softer than AM-2 and AM-3.

Looking at the response using AM-2 when consideration for reduction of shear area of concrete is used, it is visible that the prediction of maximum displacement and lateral load is reasonably performed, especially in the longitudinal direction. However the analysis overestimates the deformation in the hysteretic response in the transverse direction.

On the other hand, the response of AM-3 is softer than AM-2, and shows the most acceptable prediction of response. The response is highly dependent on coupling of loading directions, maximum acceleration and time history of the input ground motion, and also boundary conditions; in that way the softer response in AM-3 in longitudinal direction is coupled to the softer response in transverse direction. It is nonetheless important to mention that using the multi-directional polygonal lattice models the initial stiffness before cracking of concrete occurs is highly



(a) Longitudinal direction (x axis direction)



(b) Transverse direction (z axis direction)

Fig.9 Load-displacement relationship computed using analytical models

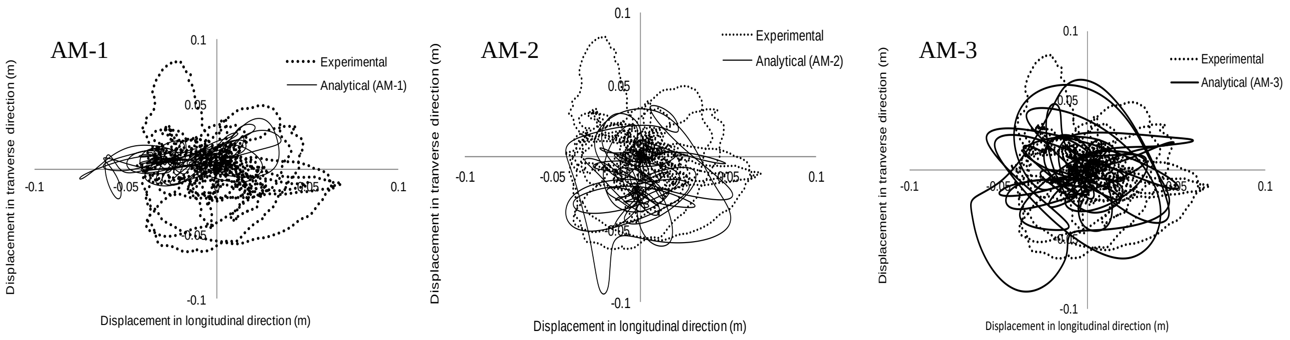


Fig.10 Displacement relationships computed using analytical models

estimated. Behind that is the elevated number of elements necessary to describe the element, when compared to the rectangular lattice model. With that in elastic range, regardless of the approach chosen, that is AM-2 and AM-3, the model consistently has large initial stiffness. This should be object of attention in further research on the topic. A detailed look at Fig.10 shows that AM-2 and AM-3 present more acceptable agreement with the experiment in comparison to AM-1; this comes from the more realistic discretization of target that AM-2 and AM-3 propose.

4. CONCLUSIONS

(1) The applicability of the 3D lattice model to perform dynamic analysis for reinforced concrete columns with circular cross section is proposed. The analytical results show that before cracking, the geometrical equivalence between circular and square cross sectional areas produces softer and more acceptable results, especially with relation to initial stiffness. On the other hand, in the inelastic range, the multi-directional polygonal 3D lattice model shows reasonable accuracy in hysteresis analysis, either considering the reduction of shear resisting area of concrete or reduction in the flexural stiffness that occurs in seismic response of RC members.

(2) In this research, the analysis is focused on maximum load and displacement and evaluation of initial stiffness. The validity of the model is verified under a limited scope, and further improvements should include other items such as analysis on residual displacement, more sophisticated hysteresis in the stress-strain relationship of concrete.

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REFERENCES

[1] Merta, I., “Shear strength model of reinforced concrete circular cross-section members”, Proc. of 5th International Conference on Fracture Mechanics of Concrete and Concrete Structures, 2004, pp. 1297-1302,

[2] Niwa, J., Choi, I. C. and Tanabe, T., “Analytical study for shear resisting mechanism using lattice model”, Concrete library of JSCE, 26, 1995, pp. 95-109.

[3] Miki, T. and Niwa, J., “Nonlinear analysis of RC structural members using 3D lattice model”, J. of advanced concrete technology, Vol.2, No.3, 2004, pp. 343-358.

[4] Ang, B. G., Priestley, M. J. N. and Paulay, T., “Seismic shear strength of circular RC columns”, ACI structural journal, Vol. 86, No.1, 1989, pp. 45-59.

[5] Merta, I. and Kolbitsch, A., “Shear area of RC circular cross-section members”, proc. of 31st Conference on our World in Concrete and Structures, 2006

[6] Pique, J. R., Burgos, M., “Effective rigidity of reinforced concrete elements in seismic analysis and design”, proc. of 14th World Conference on Earthquake Engineering, 2008

[7] Mander, J. B., Priestley, M. J. and Park, R., “Theoretical stress-strain model for confined concrete”, J. of Structural Engineering, ASCE, Vol. 114, No.8, 1988, pp. 1804-1826.

[8] Vecchio, F. J. and Collins, M.P., “The modified compression-field theory for reinforced concrete elements subjected to shear”, ACI Journal, vol. 83, No.2, 1986, pp. 219-231.

[9] Rokugo, K., Iwasa, M., Suzuki, T. and Koyanagi, W., “Testing method to determine tensile strain softening curve and fracture energy of concrete”, J. of Fracture toughness and fracture Energy, 1989, pp. 153-163.

[10] Fukuura, N. and Maekawa, K., “Computational model of reinforcing bar under reversed cyclic loading for RC nonlinear analysis”, J. of Materials, Concrete Structures and Pavements, JSCE, Vol. 564, No.35, pp. 1997, 291-295.

[11] Kawashima, K., Sasaki, T., Ukon, H., Kajiwara, K., Unjoh, S., Sakai, J., Kosa, k., Takahashi, Y., Yabe, M., and Matsuzaki, H., “Evaluation of seismic performance of a circular reinforced concrete bridge column designed in accordance with the current design code based on E-Defence excitation”, Journal of JSCE, Vol.66, No.2, 2010, pp. 324-343.