

# PROPOSAL OF LOCAL INDEX FOR DAMAGE EVALUATION OF RC BEAM FAILED IN SHEAR AND FLEXURE MODE

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## ABSTRACT

This paper examined a local index to use as failure criterion for RC beams in nonlinear finite element analysis. First, the requirement for local index such as mesh size independency was discussed. Then, local element deformation of a localized element in compression zone was proposed as the local index. Moreover, the value of the local index was proposed to evaluate maximum load point. The applicability of proposed index was verified by analyzing RC beam failed in shear and flexural behavior and the reasonableness was conformed.

**Keywords:** local index, local element deformation, mesh size dependency, maximum load

## 1. INTRODUCTION

Finite element method is powerful tool, and has significant importance for performance verification to understand load carrying capacity and failure mode of concrete structures. In the performance verification, it evaluates global behavior such as load-displacement relationship and local damage such as local strain. It is also possible to understand clearly the stress-strain behavior and the cracking behavior. However, in recent time, in order to evaluate structure member behavior, importance of local index is increasing. Moreover, the requirement moves toward evaluation of the structure behavior by using local information combining with the damage evaluation [1].

The necessity of local index in FE analysis is to evaluate the localized region damage of different members of several types of RC structures. In the global loading response of structures, in which we will have load-displacement relationship, it will be possible to evaluate the local member damage by local index that correspond to the global response like maximum load.

An experimental and analytical study conducted by Ueda et al. [2] has proofed that averaged local strain information taken in a specified damage region can be used as a local index for damage evaluation of RC structures. The concept was used by a local index proposed by Saito et al. [3] and their index was adopted in JSCE standard specification [1]. The concept takes different elements local information in the damaged area and the averaged value of each local information will be used as the local index for failure criterion of a structure. The reason why the averaged value used is that the local value varies in an element and the value depends on the mesh size. However, the method is complex because it requires the averaging technique of the local information

of different elements in a specific damaged area, and for analysis result of large mesh sizes, averaging length will less than the mesh size which cannot be solved. Therefore, local index is required, which is defined by only local information in an element in order to evaluate damage of RC structures. Moreover, the local index should be mesh size independent and consistent, that should be used for various types of RC structure members.

In this paper, a local index to use as failure criterion for RC beams in nonlinear finite element analysis is examined. First, the requirement for local index such as mesh size independency is discussed. Then, local element deformation of a localized element in compression zone is proposed as a local index to satisfy the requirement. Moreover, the value of the local index to evaluate maximum load point is discussed. Finally, the applicability of proposed index is verified by analyzing RC beam failed in shear and flexural behavior.

## 2. ANALYTICAL METHOD

3D non-linear finite element analysis was used in the analysis, which uses eight nodes in iso-parametric elements. In material model, the Lattice Equivalent Continuum Model (LECOM) developed in Nagoya University [4], which is one of the fixed smeared crack models and consists of combination of tension, compression and shear lattice, as shown in Fig.1, and uniaxial stress-strain relationships are adopted in each lattice. The stress-strain relationship in compression is modeled by compression softening model. In the model, Saenz equation [5] is used up to the compressive strength and a linear softening branch is assumed as shown in Fig.2. The slope of linear softening branch is defined by considering the compression fracture energy ( $G_{fc}$ ) and

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element size ( $L_{eq}$ ) in order to avoid element size dependency for load-displacement relationship as proposed by Nakamura and Higai [6]. The stress-strain relationship in tension is modeled by  $\frac{1}{4}$  tension softening model as shown in Fig.3. The tension fracture energy and element size ( $L_{eq}$ ) are used based on similar concept with compression model.

Reinforcement is modeled discretely by truss element. Bi-linear model is used as stress-strain relationship and stiffness after yielding is assumed by  $1/1000$  of initial stiffness. Bond behavior between concrete and reinforcement is model by setting link element, in which bond stress and slip relationship is introduced.

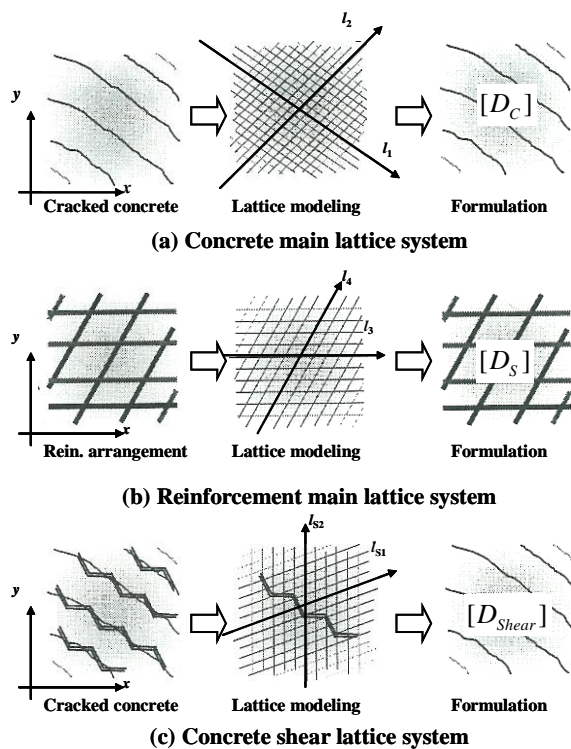


Fig.1 Diagram of lattice equivalent continuum model

### 3. LOCAL INDEX FOR COMPRESSION FAILURE

The evaluation of concrete structure damage based on local information as a local index is required in order to understand the structure behavior in loading history. Proposing a local index that can be used to

is necessary. The index can be calculated from the local information taken from a single localized part of a structure. In concrete structures, local strain value is one of the parameters that tells about the damage level at a specified region in a structure. However, the local strain is quite sensitive for mesh size variation and the result is inconsistent and mesh dependent in loading history of a structure. This tells that using local strain value as a local index is not adequate and will not evaluate different structures uniquely. As a result, the requirement for the local index is that the parameter does not show mesh dependency behavior and the index should have consistent behavior over the loading history of the structure. If the index fulfilled the requirement, the value

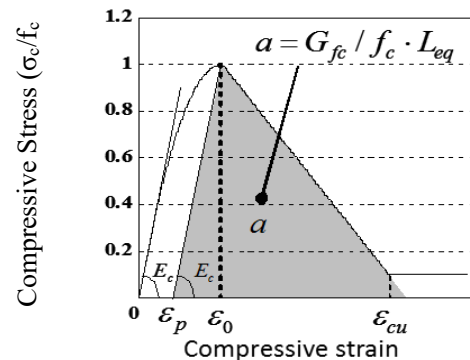


Fig.2 Uniaxial compression stress-strain

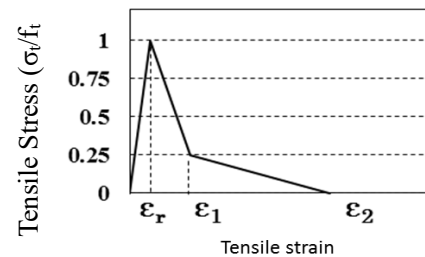


Fig.3 Tensile stress-strain

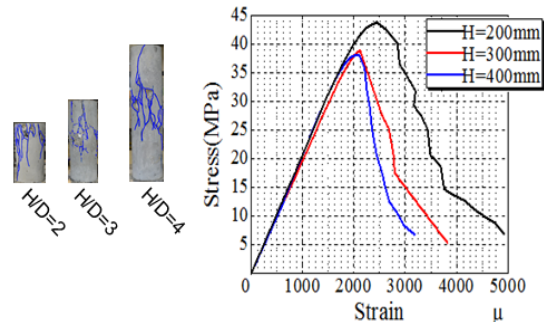


Fig. 4 Stress-strain relationship of concrete cylinders

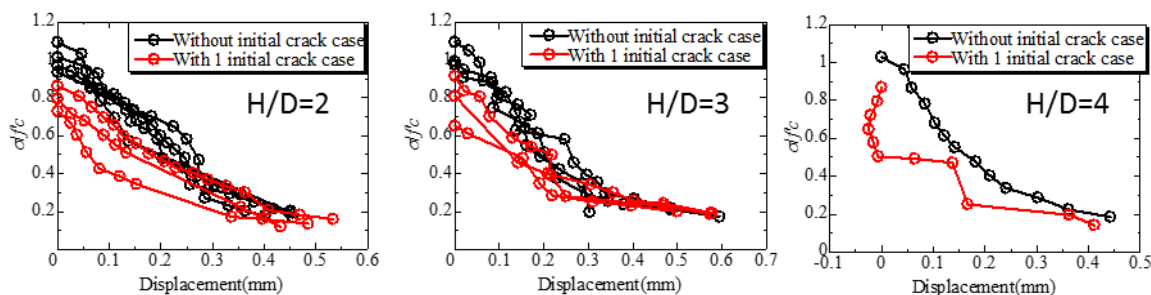


Fig. 5 Stress-deformation curve in post peak region of different size concrete cylinders

evaluate, for example, maximum load point of structures of the index can be related with damage progress such as

maximum load point.

The results of test series conducted in our laboratory for uniaxial compression test with different sizes are shown in Fig.4 by its stress-strain curve. Similar problem with mesh dependency in finite element analysis is observed for size dependency in test. To overcome this problem, normalizing the stress-deformation curve by each peak stress is effective after peak stress. Fig.5 shows the stress-deformation curve in post peak region, which include the results of uniaxial compression test of concrete cylinder with initial crack along loading direction [7]. The behavior of this stress-deformation curves of different specimen is similar. In the stress-deformation curve, for  $H/D=4$  with initial crack case, even though the behavior seems different from other cases up to 0.05mm deformation, the deformation behavior observed after that is still same and maximum damage is similar with other cases. This similarity indicates that the structure absorbs same energy and the local damage in post peak is same irrespective of structure size and initial crack. This local damage in experiment is analogy with localized element in finite element analysis. This experimental evidence would lead to propose a local index from Localized Element Deformation (LED) in an element for finite element analysis with different mesh size.

The reasonableness of the concept of LED and the problem of local strain are investigated by the simulation of shear failure beam tested by Iwamoto et al. [8]. The detail of the beam is shown in Fig.6. Modeling are done for the beam with element size of 20mm cube and 40mm x 37.5mm x 37.5mm.

The load-displacement relationships of the analytical results are compared with the experiment result as shown in Fig.7. The relationships for different element sizes are close to the experimental result and the analysis result before the marked point (○) do not show any mesh dependency behavior due to the fracture energy effect. Even though there is slight mesh dependency after the marked point, the difference is not significant which is the effect of the introduced fracture energy in the concrete modeling. Fig.8 shows the minimum principal strain distribution with different element size taken from the load value marked by ○ in Fig. 7. The strain localized in an element near to the loading point (the single top and nearest element to loading plate encircled by red line) because the beam failed under shear compression mode. In Fig.7, the averaged minimum principal local strain behavior in localized elements has drawn with load-displacement relationships in order to understand the strain behavior for damage evaluation, like maximum load. However, the local strain behavior is sensitive for mesh size variation and the value corresponding to the maximum load is quite different for different mesh sizes. This shows that the local strain can not be used as local index to evaluate damage or maximum load point.

LED is calculated by multiplying the element size ( $L_{eq}$ ) and the averaged minimum principal local strain of all gauss points in a localized element. The value can be calculated by using only the information in an element,

which is an easier technique than averaged technique [1][2][3], in a specified damage region independent on mesh discretization.

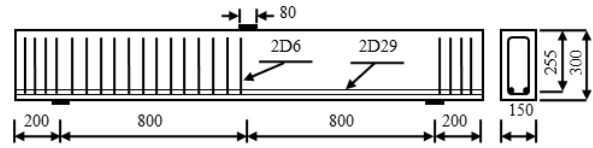


Fig. 6 Specimen outline and dimension (mm)

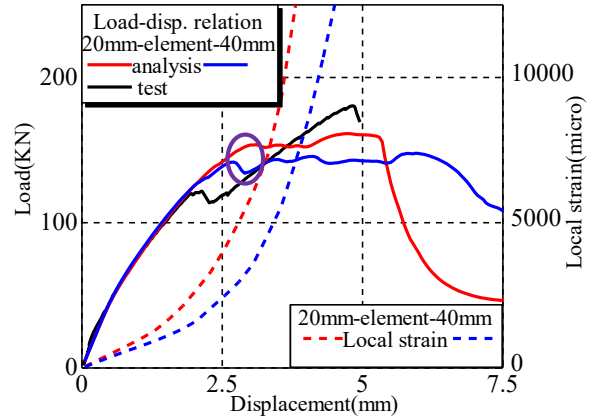


Fig. 7 Load-displacement relationship with local strain behavior of localized elements

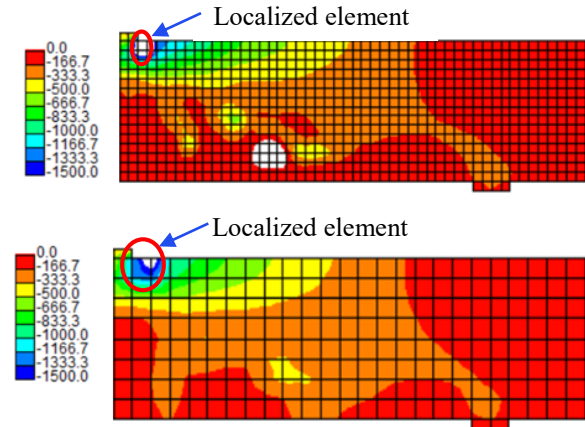


Fig. 8 Localization and strain distribution for mesh size of 20mm (top) and 40mm (bottom)

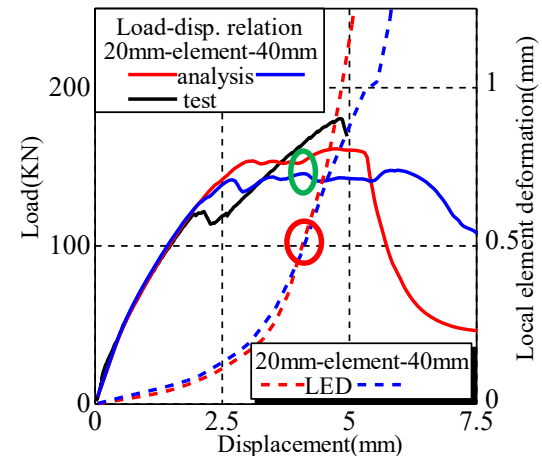


Fig. 9 Local element deformation with load-displacement relationship

Fig.9 shows the change of LED at localized element together with load-displacement relationship. The LED shows unique behavior independent on mesh size, which satisfy requirement of local index. LED is localized deformation of an element, which is identified with localized deformation of concrete cylinder as shown in Fig. 5. The ultimate state deformation in post peak region of concrete cylinder can be defined as for example 0.5mm from Fig.5. When the value applies to the LED in Fig.9 as shown by the red circle, it corresponds to nearly maximum load point which is shown by green circle. The value is 0.54 for 20mm element size and is 0.58 for 40mm element size, because the deformation

until peak stress (corresponding strain is 0.002 at peak stress) is added to deformation from peak stress. This result indicates that the value of LED can use to evaluate maximum load point with clear mechanical meanings.

#### 4. APPLICATION OF LOCAL INDEX

The applicability of proposed local index, LED, is verified by different shear failure beams and flexural failure beams.

##### 4.1 Shear Failure Beams

###### (1) Analytical model

Four beams tested by Yamaya et al. [9] as shown in Fig.10 are analyzed to investigate the applicability of

LED for beams failed in shear mode. Two beams are deep beam with shear span depth ratio of 1.92. Another two beams are slender beam with shear span depth ratio of 3.85. The stirrup is provided with spacing of 130mm in one of two beams for both cases. The material properties of the beams are listed in Table 1. 25mm and 50mm mesh size cube elements are used in the modeling of 3D non-linear finite element analysis.

###### (2) Analytical result

Fig.11 shows load-displacement relationship of test and analytical results with different element size. The change of LED are also shown in the figure. Load-displacement relationship in the pre-peak region shows mesh independent behavior in all analysis results. The validity of fracture energy in constitutive model is confirmed.

LED is gradually increase for all cases. And the increase rate increase when LED is more than 0.5. The behavior of LED in all graphs shown is unique and mesh independent. The applicability of LED as unique local index independent on mesh size for deep and slender beams with and without stirrups is confirmed.

In order to evaluate maximum load point based on the proposed value, LED values is 0.55 for 25mm element size and 0.6 for 50mm element size. For the deep beam without stirrup, LED value evaluate maximum load point reasonably. Moreover, LED value

Table 1 Material property of shear failed beams

Code	a/d	Rebar property (D10, D29)			Stirrup property (D6)			Concrete $f_c'$ (MPa)
		pt(%)	fsy(MPa)	Es(GPa)	pw(%)	fwy(MPa)	Ew(GPa)	
DB-N1	1.92	3.71	358	182	0.00	358	179	23.0
DB-N2					0.25			24.4
SB-N1	3.85	3.71	358	182	0.00	358	179	22.7
SB-N2					0.25			23.7

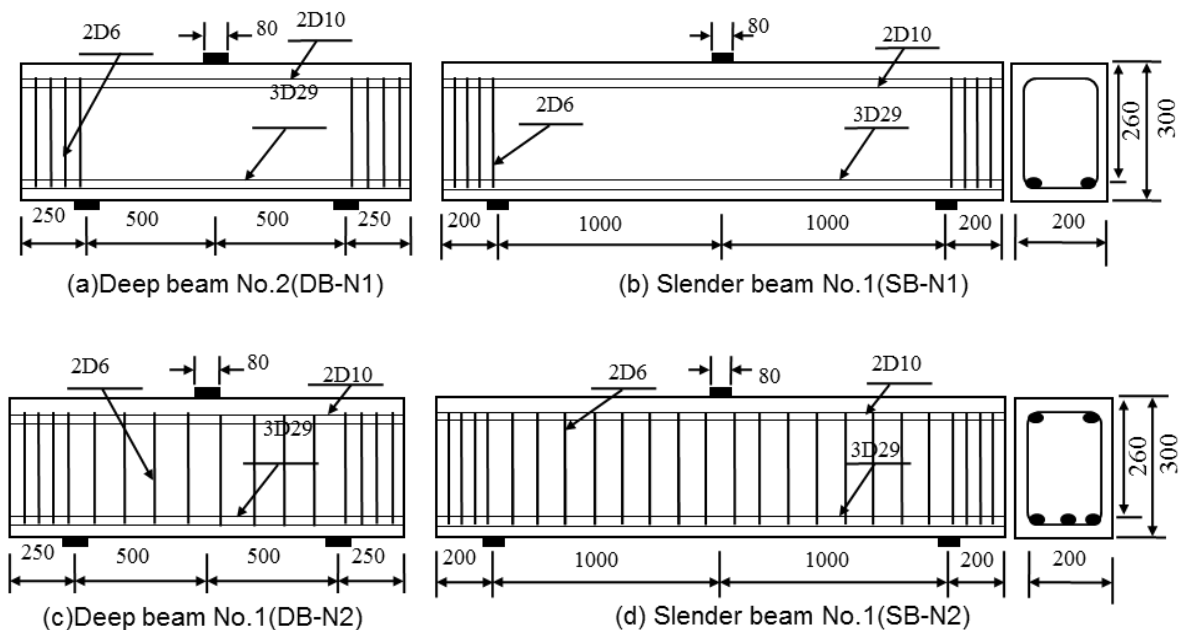


Fig.10 Shear compression failed specimen without and with stirrup (all dimension in mm)

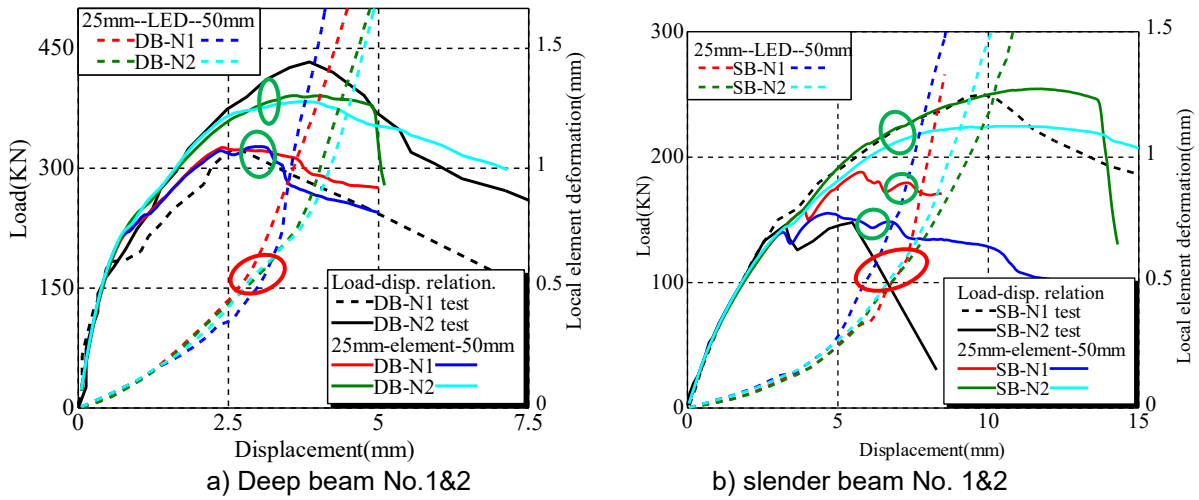


Fig. 11 Local element deformation with load-displacement relationship

Table 2 material property of flexural beam

Main rebar dia.	Main rebar	Main rebar ratio (%)	Concrete property		Main rebar property	
			$f'_c$ (Mpa)	$E_c$ (GPa)	$f_y$ (MPa)	$E_s$ (GPa)
2D-10		0.634	34.7	20	358	200
2D-13		1.126	31.8	20	361	200
2D-16		1.765	36.2	20	362	200

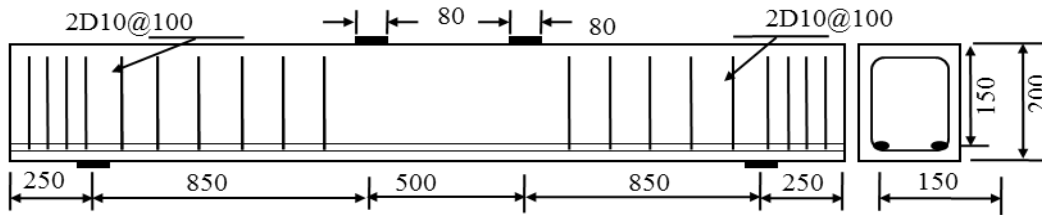


Fig.12 Flexural specimen dimension (mm)

also evaluate maximum load point of the beam with stirrup. For the slender beam without stirrup, maximum load point evaluated by LED value slightly beyond maximum load point in load-displacement relationship. This is the reason that the tension behavior of concrete is more important. For slender beam with stirrup, LED value underestimate the displacement corresponding to maximum load point. This may suggest the modification of LED considering with stirrup effect. But from all results, the concept of LED is applicable for shear failure beam and the proposed value of LED evaluate near the maximum load points.

## 4.2 Flexural Failure Beams

### (1) Analytical model

Three beams tested by Okada et al. [10] as shown by typical sample in Fig. 12 are analyzed to investigate the applicability of LED for beams failed in flexure mode. All beams are provided different rebar ratio as listed in Table 2 with the material property of the beams. 25mm mesh size cube and 40x40x37.5mm mesh size elements are used in the modeling of 3D-nonlinear finite element analysis. D10 stirrup provided with 100mm spacing which has young's modulus of 200GPa and yield strength of 362MPa.

### (2) Analytical result

Fig.13 shows load-displacement relationship of analytical results for different element size. The changes of LED also shown in the figure. Load-displacement relationship in the graph shows mesh independent behavior in all analysis results. The validity of fracture energy in constitutive model also confirmed in flexure failed beam.

LED is suddenly increase when it exceeds certain value. This may indicate that compression behavior is

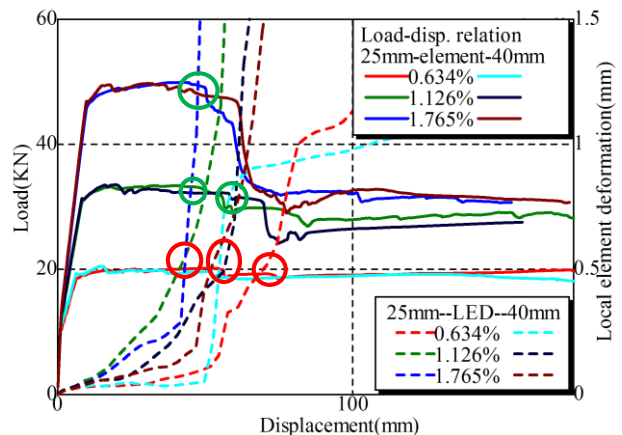


Fig.13 Local element deformation with load-displacement relationship

dominant for flexural failure more than shear failure. After the rate increase, the behavior of LED in all beams remains similar with insignificant mesh dependency behavior. The applicability of LED as a local index independent on mesh size for flexure beams with different rebar ratio is confirmed.

In order to evaluate maximum load point based on the proposed value, LED values are 0.55 for 25mm element size and 0.58 for 40mm element size. For all the three flexure failed beams, LED value evaluate maximum load point reasonably. From all results, the concept of LED is applicable for flexure failure beam and the proposed value of LED evaluate near the maximum load points.

## 5. CONCLUSIONS

- (1) The observed local strain behavior is inconsistent and quite mesh sensitive. Due to this reason, it is difficult to use it as a local index parameter in order to evaluate failure criterion of RC structures.
- (2) A local index defined by localized element deformation (LED) in a localized element in compression was proposed. The observed behavior is similar for different element sizes.
- (3) The value of LED to evaluate maximum load point was proposed. The value corresponds to the ultimate stage of concrete cylinder in post peak region and the mechanical meanings of the value is clear.
- (4) The proposed value of LED can evaluate maximum load point reasonably for RC beams failed in shear. Also, it is shown the possibility of using LED to evaluate beams failed in flexure with further comprehensive studies.

Further studies will be conducted to consider different parameters and structure types in order to propose reasonable and consistent LED value. Stirrup effect, span length and failure mode conditions for different types of structures will be also investigated.

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