- Technical Paper -

REINFORCING EFFECTS OF STIRRUPS ON CONCRETE CONTRIBUTION IN SHEAR

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ABSTRACT

This research investigates the reinforcing effects of stirrups on contribution of concrete in shear, focusing on arch action and beam action. Static bending tests were conducted on RC beams with various stirrup ratio and shear span. In this study, the contribution of arch action was estimated based on the strain of tensile steel bar at the support point. The results showed that while contributing directly, stirrups also improve the contribution of arch action, and maintain the contribution of concrete in beam action. In addition, the importance of regulation in stirrup spacing was also discussed.

Keywords: *RC beam, shear, arch action, beam action, stirrup ratio, shear span*

1. INTRODUCTION

In the calculation of shear capacity of RC beams with stirrups, such as in the current Japan Society of Civil Engineers Standard Specifications for Design and Construction of Concrete Structures (JSCE Standard Specification) [1], modified truss theory is often applied. In this theory, the shear capacity of RC beams is calculated as the sum of stirrups contribution and concrete contribution. As stirrups amount changes in the same shear span, it is assumed that only the stirrups contribution changes, while the concrete contribution remains fixed. However, stirrups affect the opening and propagation of diagonal cracks, which are highly related to concrete contribution because it includes mechanisms such as the dowel action and the aggregate interlock. Thus, stirrups might also affect concrete contribution.

Also, this theory can only estimate the occurrence of stirrups yielding, while the shear failure usually occurs unrelated to this. Shear resisting mechanism can be considered to improve this condition. One mechanism known is the beam action which includes stirrups and mechanisms of concrete mentioned above. Another mechanism is the arch action which is formed by concrete compressive strut and tensile steel bar as tie material. Nakamura, E. and Watanabe [2] and Iwamoto, et.al [3], using different method, showed that the arch action is dominant after the initiation of diagonal crack until failure. Explanations above indicate requirement to investigate the effects of stirrups on contribution of arch action and other mechanisms in concrete, in order to improve the calculation of shear capacity.

Here, the quantitative evaluation of each contribution from loading tests is essential. However, evaluation methods often include several assumptions and complicated calculations. Nakamura, M., et.al [4] proposed a method where the contribution of arch action is calculated based on the strain of tensile steel bar at a support. This method is simple, and have good agreement with other previous researches.

Table 1 Specimens' details

Name	d (mm)	b (mm)	a (mm)	a/d	p_w (%)	r_w (%)	s (mm)
2.5-0.00	300	150	750	2.5	1.69	0.00	-
2.5-0.17						0.17	250
2.5-0.28						0.28	150
2.5-0.38						0.38	110
3.5-0.00			1050	3.5		0.00	-
3.5-0.17						0.17	250
3.5-0.28						0.28	150
3.5-0.38						0.38	110
3.5-0.53						0.53	80
3.5-0.65						0.65	65
3.5-0.38-D10						0.38	250
3.5-0.53-D10						0.53	180

d: effective depth, b: web width, a: shear span length, p_w: tensile steel bar ratio, r_w: stirrup ratio, s: stirrup spacing

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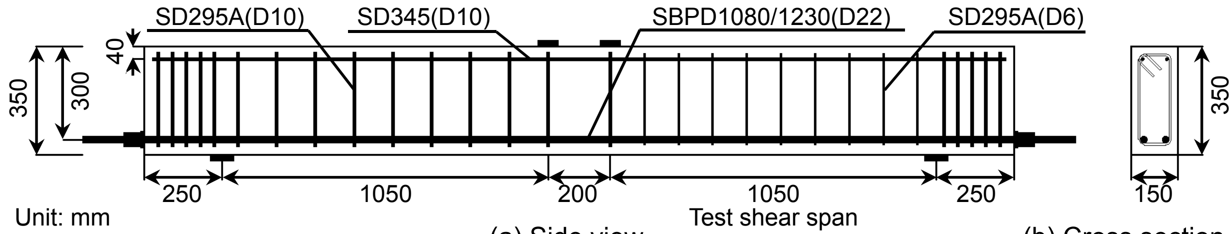


(a) Side view (b) Cross section

Fig. 1 Details of specimen 3.5-0.38

Table 2 Mechanical properties of steel bars

Name	Tensile steel bar			Compression steel bar			Stirrups		
	Size	Yield strength* (N/mm^2)	Elastic modulus (kN/mm^2)	Size	Yield strength* (N/mm^2)	Elastic modulus (kN/mm^2)	Size	Yield strength* (N/mm^2)	Elastic modulus (kN/mm^2)
2.5-0.00	D22	1197	201	D10	350	199	-	-	-
2.5-0.17							D6	339	193
2.5-0.28		1170			349	184		395	174
2.5-0.38		1197			350	199		339	193
3.5-0.00		1152			348	197	-	-	-
3.5-0.17							D6	339	193
3.5-0.28		1170			349	184		395	174
3.5-0.38		1152			348	197		339	193
3.5-0.53					343	196			
3.5-0.65									
3.5-0.38-D10							D10	348	197
3.5-0.53-D10									

*determined using the offset method, except for D10 stirrups

In this research, static bending tests on RC beams with various stirrup ratio and shear span were conducted, and the contribution of arch and beam actions was evaluated using the method proposed by Nakamura, M., et.al.. Based on these results, reinforcing effects of stirrups on concrete contribution in shear were discussed.

2. TEST PROGRAMS

2.1 Test Specimens and Materials

Details of 12 tested specimens, including stirrups arrangement are summarized in Table 1, and illustrated in Fig.1. All specimens were designed to fail in shear in the test shear span. The cross section was the width of 150 mm and the height of 350 mm, while the effective depth was 300 mm. The specimens were divided into two series with shear span ratios 2.5 and 3.5. Stirrups were arranged with stirrup ratio ranging 0.00% ~ 0.38% for shear span ratio 2.5 series, and 0.00% ~ 0.65% for shear span ratio 3.5 series in the test shear span. In the other span, stirrup ratio was fixed at 0.76%. Concrete with design cylinder compressive strength of 40 N/mm^2 was used. Water cement ratio was 47%, and unit water content was 175 kg/m^3. Maximum size of aggregate was 20 mm. Table 2 summarizes the mechanical properties of steel bars used. The yield strength of most steel bars was determined using the offset method.

2.2 Loading Method

A four-point bending test with simply-supported condition was provided to all specimens. Steel plates of 75 mm width were placed on the supports. Teflon sheets and grease were inserted between the specimen and the supports to prevent the horizontal friction. At the loading points, steel plates of 65 mm width were placed. During loading tests, applied load, mid span displacement, and strain of steel bars were measured. The strain of tensile steel bars was measured at the middle span and support points. The strain of each stirrup at the test shear span was measured at 1~3 points (75 mm, 175 mm, 275 mm from the top surface).

3. EXPERIMENTAL RESULTS

Figure 2 shows the load-displacement curves of all specimens. Table 3 shows the calculation and experimental results. The calculated shear capacity V_{y_cal} was the sum of concrete contribution V_{c_cal} and stirrups contribution V_{s_cal}, following the JSCE Standard Specifications [1]. V_{c_cal} was calculated with the equation proposed by Niwa, et.al [5], and V_{s_cal} was calculated based on the truss theory with the angle of diagonal compression of 45°. These are shown in Eqs. 1, 2 and 3.

$$V_{y\_cal} = V_{c\_cal} + V_{s\_cal} \tag{1}$$

$$V_{c\_cal} = 0.20\left(f_c' p_w\right)^{1/3}\left(d\right)^{-1/4}\left(0.75 + \frac{1.4}{a/d}\right)bd \tag{2}$$

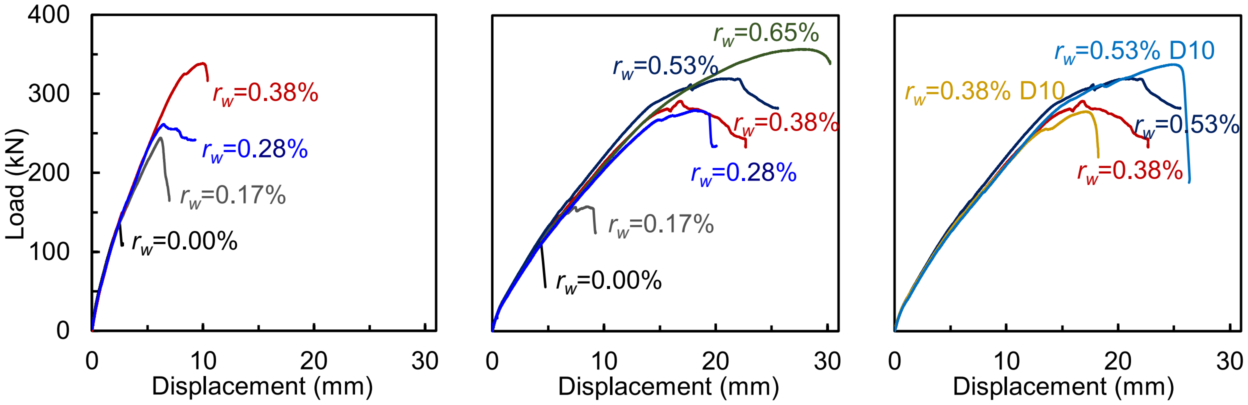
$$V_{s\_cal} = A_w f_{wy} \frac{z}{s} \tag{3}$$

where, f_c': compressive strength of concrete (N/mm^2), p_w: tensile steel bar ratio (%), d: effective depth (m), A_w: cross sectional area of 1 set of stirrups (mm^2), f_{wy}: stirrup yield strength (N/mm²), z: lever arm length (=d/1.15) (mm), s: stirrup spacing (mm), b: web width (mm), a: shear span length (mm)

In all specimens, at the peak load, almost all stirrups at test shear span were yielded. After the peak load, diagonal cracks at test shear span became more prominent, indicating the failure in shear. Fig. 3 Shows crack patterns at failure. A crack with the biggest opening was considered as the critical diagonal crack.

In specimens without stirrups, the load dropped as a diagonal crack initiated. In specimens with $r_w \leq 0.38\%$, the failure was caused by diagonal crack penetrating the compression fiber, as shown in Fig. 3 (a)~(f). In specimens with $r_w \geq 0.38$, it was caused by concrete crush near the loading point, as shown in Fig. 3 (g)~(i).

For specimens 2.5-0.00, 3.5-0.00, and 3.5-0.17, V_{u_exp}/V_{y_cal} was approximately 1.0. For other specimens, it was ranging between 1.2~1.4. When a/d = 2.5, V_{u_exp}/V_{y_cal} became larger as r_w increased. When a/d = 3.5, the largest V_{u_exp}/V_{y_cal} was when r_w = 0.28%.



(a) a/d=2.5 (D6) (b) a/d=3.5 (D6) (c) a/d=3.5 (D10)

Fig. 2 Load-displacement curves

Table 3 Mechanical properties of concrete, calculation and experimental results of shear capacity

No	f_c' (N/mm²)	E_c (kN/mm²)	V_{c_cal} (kN)	V_{s_cal} (kN)	V_{y_cal} (kN)	V_{u_exp} (kN)	V_{u_exp} / V_{y_cal}
2.5-0.00	46.0	33.0	68.4	0.0	68.4	68.1	1.00
2.5-0.17	42.7	32.2	66.7	22.4	89.1	122.2	1.37
2.5-0.28	40.8	32.2	65.3	43.5	108.8	130.7	1.20
2.5-0.38	45.0	31.2	67.9	51.0	118.9	169.1	1.42
3.5-0.00	41.7	31.7	58.1	0.0	58.1	56.1	0.96
3.5-0.17	42.3	30.1	58.4	22.4	80.8	78.9	0.98
3.5-0.28	42.4	31.3	58.1	43.5	101.6	139.9	1.38
3.5-0.38	41.7	32.0	58.1	51.0	109.1	145.8	1.34
3.5-0.53	41.4	30.6	58.0	70.1	128.1	160.0	1.27
3.5-0.65	42.4	31.2	58.4	86.3	144.7	178.6	1.25
3.5-0.38-D10	41.3	31.7	57.9	51.8	109.7	139.1	1.27
3.5-0.53-D10	49.1	34.6	61.4	71.9	133.3	169.0	1.23

f_c': compressive strength of concrete, E_c: elastic modulus of concrete, V_{y_cal}: calculated shear capacity, V_{c_cal}: concrete contribution in V_{y_cal} (N), V_{s_cal}: stirrup contribution in V_{y_cal}, V_{u_exp}: experimental shear capacity

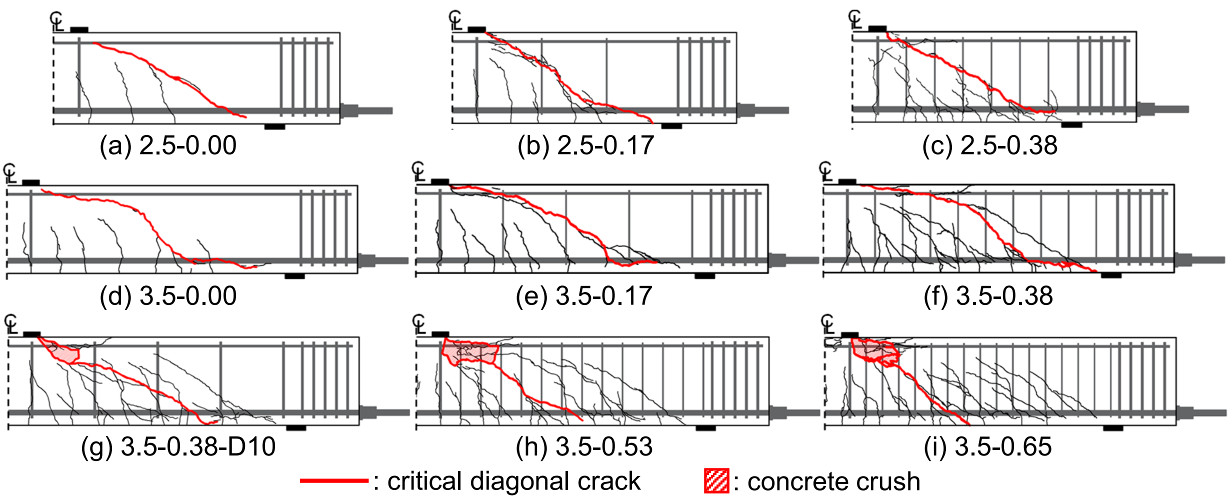


Fig. 3 Crack patterns

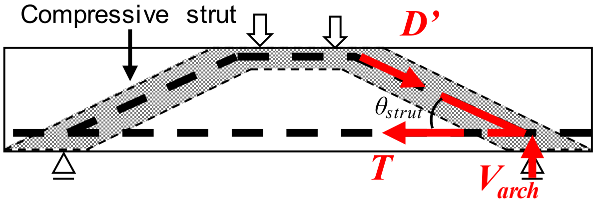


Fig. 4 Illustration of arch action

4. SHEAR RESISTING MECHANISM

4.1 Evaluation Method

Estimation method proposed by Nakamura, M., et.al [4] was applied to evaluate the shear resisting mechanisms. The applied shear force V is thought to be resisted by the contribution of arch action V_{arch}, and the contribution of beam action V_{beam}. V_{beam} can be separated into the contribution of stirrup V_{sbeam}, and the contribution of concrete V_{cbeam} which is considered as the dowel action and the aggregate interlock after initiation of diagonal crack.

V_{arch} can be obtained based on the equilibrium at the support, where as shown in Fig. 4, consists of the tensile force of tie member T, the compressive force from the compressive strut D', and the contribution of arch action in shear capacity V_{arch}. T can be obtained from the strain of tensile steel bars at a support point ε_s. V_{sbeam} is considered as the total tensile force of stirrups that intersect with the critical diagonal crack. Here, the region where the critical diagonal crack intersects with the tensile and compressive steel bars is considered. Finally, V_{cbeam} is obtained by subtracting V_{arch} and V_{sbeam} from V. For comparison, V_{cbeam} added by V_{arch} is defined as the concrete overall contribution $V_{concrete}$. Explanations above are shown in Eqs. 4 ~ 11.

$$V = V_{arch} + V_{beam} \tag{4}$$

$$V_{beam} = V_{sbeam} + V_{cbeam} \tag{5}$$

$$V_{arch} = T \tan\theta_{strut} \tag{6}$$

$$T = A_s E_s \varepsilon_s \tag{7}$$

$$\tan\theta_{strut} = z/a \tag{8}$$

$$V_{sbeam} = \sum (A_w \sigma_w) \tag{9}$$

$$\sigma_w = \begin{cases} E_w \varepsilon_w (E_w \varepsilon_w < f_{wy}) \\ f_{wy} (E_w \varepsilon_w > f_{wy}) \end{cases} \tag{10}$$

$$V_{arch} + V_{cbeam} = V_{concrete} \tag{11}$$

where, θ_{strut}: angle of compressive strut (°), A_s: total area of tensile steel bars (mm^2), E_s: elastic modulus of tensile steel bars (N/mm^2), ε_s: strain of tensile steel bars at support, z: lever arm length (=d/1.15) (mm), a: shear span length (mm), A_w: cross sectional area of 1 set of stirrups (mm^2), σ_w: tensile stress of stirrup (N/mm^2), E_w: elastic modulus of stirrup (N/mm^2), ε_w: strain of stirrup, f_{wy}: yield strength of stirrup (N/mm^2).

4.2 Transition of Shear Resisting Mechanisms

Based on researches conducted by Nakamura, E. and Watanabe [2], Iwamoto, et.al [3], and Niwa, et.al [6], the transition of shear resisting mechanism can be predicted as following. Before the initiation of diagonal crack, shear force is resisted mostly by the contribution of concrete in the beam action. After the initiation of diagonal crack, stirrups will start contributing, followed by the contribution of arch action, replacing concrete in the beam action. Stirrups can only resist shear force up to the yielding point, and after this point the increase of shear force will be resisted by arch action. Also, due to the contribution of arch action, overall contribution of concrete will be maintained approximately equivalent to the calculated value V_{c_cal}.

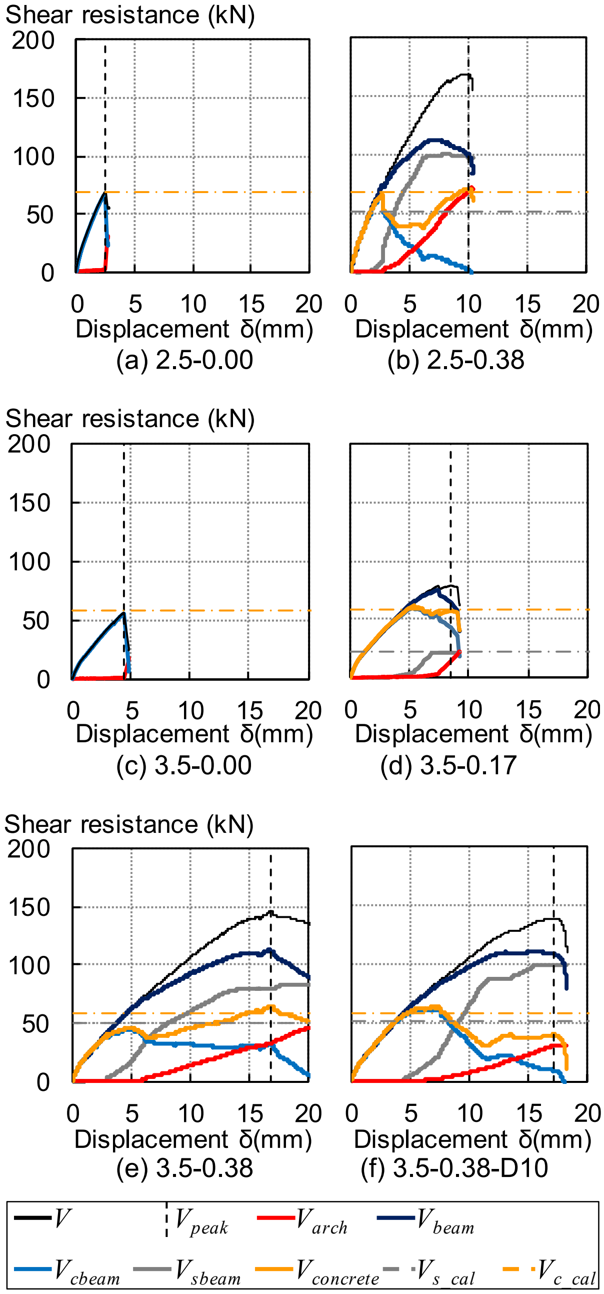


Fig. 5 Transition of shear resisting mechanisms

Figure 5 shows the transition of every shear resisting mechanisms during a loading test, obtained by applying explained evaluation method to each specimen. At the beginning, V_{cbeam} was the only one increasing along with V. Then, after V reached approximately V_{c_cal}, V_{cbeam} started decreasing, while V_{sbeam} started increasing significantly. This timing generally matched with the observed timing of the initiation of diagonal cracks. Shortly after, V_{arch} started increasing considerably. Similar tendency was kept until the increase of V_{sbeam} became slight. Here, yielding in most stirrups in a test shear span was observed. Finally, V_{arch} was the only one increasing together with V until V reached V_{peak}. Excluding 3.5-0.38-D10, $V_{concrete}$ was maintained approximately equivalent to V_{c_cal} at the peak shear force. From the explanations above, results obtained by applying evaluation method proposed by Nakamura, M., et.al showed a good agreement with the previous researches up to the peak shear force.

Regarding the behavior after the peak shear force, Iwamoto, et.al [3] reported that the contribution of arch action will reach its peak in the same time with the applied shear force. In the results of this research, V_{arch} was still increasing after the peak shear force, and the peak of V_{arch} could not be recognized in most specimens. This is most likely because V_{arch} was obtained from the strain of tensile steel bar at the support. After the peak shear force, even though the strain of tensile steel bar at the mid span decreased, the strain at the support did not decrease. This is thought to be due to the concentrated deformation along the critical diagonal crack, which includes the area near support point.

Hence, it can be said that this evaluation method is applicable to evaluate shear resisting mechanisms only until the peak shear force.

4.3 Contribution at Peak Shear Force

Contribution of mechanisms at peak shear force V_{arch_peak}, V_{beam_peak}, V_{sbeam_peak}, V_{cbeam_peak}, $V_{concrete\_peak}$ in all specimens are summarized in Fig. 6. The graphs were arranged in increasing r_w order.

In specimens with a/d = 2.5, as r_w increases, increasing tendency in both V_{arch_peak} and V_{beam_peak} was observed. However, when V_{beam_peak} was divided into V_{sbeam_peak} and V_{cbeam_peak}, the increase in V_{sbeam_peak} and the decrease in V_{cbeam_peak} was observed. Since almost all stirrups were yielded, the increase in V_{sbeam_peak} indicates the increase in the number of stirrups intersecting the critical diagonal crack as shown in Fig. 3 (b)(c). Meanwhile, the decrease in V_{cbeam_peak} was significant, and in specimen 2.5-0.38, V_{cbeam_peak} was close to zero. V_{arch_peak} in specimen 2.5-0.28 was lower than 2.5-0.17, which has lower stirrup ratio, and this indicates that the contribution of arch action might be unstable with these amount of stirrups.

In specimens with a/d = 3.5, the increase in V_{arch_peak} was observed when r_w increased from 0.00% to 0.28% (specimen 3.5-0.00, 3.5-0.17, and 3.5-0.28). When r_w increased higher than 0.28%, V_{arch_peak} stabilized at a certain value, indicating the existence of maximum possible contribution of arch action. As shown in Fig. 3 (g)(h)(i), the specimens with high r_w failed due to the concrete crush near the loading point. Thus, there might be a relationship between this failure mode and the maximum contribution of arch action. Nakamura, E. and Watanabe [2] reported that the compressive strength of concrete affects the contribution of arch action, and similar tendency was observed, as 3.5-0.53-D10 which had high compressive strength showed higher contribution of arch action than other specimens.

Meanwhile, tendencies regarding V_{beam_peak}, V_{sbeam_peak}, and V_{cbeam_peak} also showed difference between when r_w increased from 0.00% to 0.28% and when r_w increased higher than 0.28%. When r_w increased from 0.00% to 0.28%, similar to a/d = 2.5, V_{sbeam_peak} increased while V_{cbeam_peak} decreased, thus resulting in increasing V_{beam_peak}. When r_w increased higher than 0.28%, the increase in V_{sbeam_peak} became less significant, and V_{cbeam_peak} stabilized at a certain value, resulting in steadily increasing V_{beam_peak}.

The difference in increasing tendency of V_{sbeam_peak} is related to the angle of the critical diagonal crack. As shown in Fig. 3 (e)(f), when r_w was small, the angle of the critical diagonal crack was gently sloped, and as shown in Fig. 3 (h)(i), when r_w was higher, the angle became closer to 45°. This clearly affected the number of stirrups intersecting. Stabilized V_{cbeam_peak} indicates the ability of the beam to maintain the minimum contribution of concrete in beam action. As V_{cbeam_peak} represents mechanisms such as the dowel action and the aggregate interlocking, it is most likely to be related to

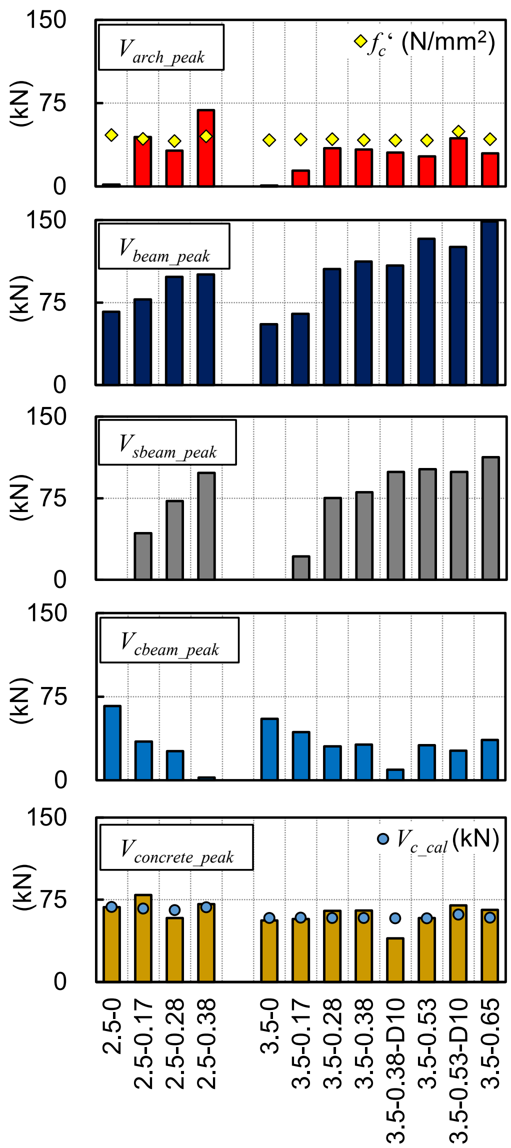
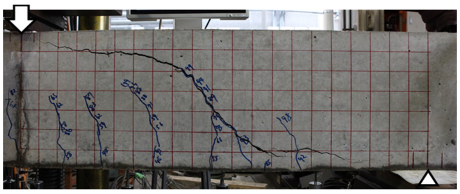
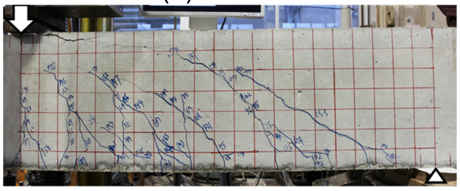


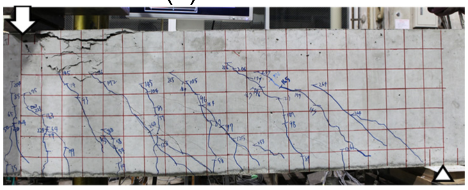
Fig. 6 Contribution at peak shear force



(a) 3.5-0.00



(b) 3.5-0.38



(c) 3.5-0.65

Fig. 7 Diagonal cracks at peak shear force

the diagonal cracks. As shown in Fig. 7(b)(c), specimens in this range had similar crack pattern at the peak shear force, and crack opening was slight.

Based on the observed results of specimens with a/d = 3.5, it can be predicted that in specimens with a/d = 2.5, if r_w increased even more than 0.38%, similar tendencies where V_{arch_peak} and V_{cbeam_peak} stabilize at some value may also happen. Even though, in specimen 2.5-038 the value of V_{arch_peak} was much larger than in specimen 3.5-0.38, while V_{cbeam_peak} was close to 0. Thus, the stabilized values are most likely to be different to those of specimens with a/d = 3.5.

From explanations above, it is clear that with the change in r_w, not only in V_{sbeam_peak}, changes in V_{cbeam_peak} and V_{arch_peak} were also observed. Especially when r_w is relatively small, the increase in r_w may significantly increase V_{arch_peak}, while the decrease of V_{cbeam_peak} may also occur. When r_w is relatively high, stirrups can guarantee the maximum V_{arch_peak} and maintain the minimum V_{cbeam_peak}.

4.4 Importance of Stirrup Spacing

The importance of stirrup spacing was observed from specimens 3.5-0.17 and 3.5-0.38-D10. In these specimens, stirrup spacing was bigger than half of the effective depth, thus, regulation of JSCE Standard Specifications was not satisfied [1].

From Table 3, 3.5-0.17 was the only specimen with stirrups of which experimental shear capacity was equivalent to the calculated result based on Modified Truss Theory. From Fig. 3 (e), it can be observed that there was only 1 stirrup contributing, which is similar as assumed in calculation. Also, V_{arch_peak} was relatively small compared to other specimens, showing the inability of the specimen to manifest enough contribution of arch action.

From Fig. 6, 3.5-0.38-D10 had relatively small V_{cbeam_peak} and $V_{concrete\_peak}$ lower than V_{c_cal}. From Fig. 5 (f), V_{cbeam} in 3.5-0.38-D10 decreased significantly after the initiation of diagonal crack. This is most likely related to the gently sloped critical diagonal crack as shown in Fig. 3 (g), which might result in the inability to maintain the minimum contribution of concrete.

Hence, the regulation in JSCE Standard Specifications about the maximum stirrup spacing is considered to be rational, to guarantee the maximum V_{arch_peak}, to maintain the minimum V_{cbeam_peak}, and to maintain $V_{concrete\_peak}$ equivalent to V_{c_cal}.

5. CONCLUSIONS

In this research, loading tests on 12 RC beams with different stirrup ratio, and shear span were conducted to investigate the effect of stirrups on concrete contribution in shear, focusing on arch action and beam action. From the results, following conclusions may be deduced.

(1) Evaluation method using the strain of tensile steel bar at the support is applicable to evaluate arch and beam action. However, it is applicable only until the peak shear force.
(2) With the increase in stirrup ratio, along with the increase in stirrups' direct contribution in shear, the increase in contribution of arch action and the decrease in contribution of concrete in beam action were also observed.
(3) When stirrup ratio was increased more than a certain amount, the maximum value of contribution of arch action can be guaranteed, and the minimum value of contribution of concrete in beam action can be maintained. The failure was due to concrete crush near the loading point.
(4) The regulation in JSCE Standard Specifications regarding the maximum stirrup spacing is rational.

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